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Numerical Second Order Method of Numerical Techniques for Solving Nonlinear Equations

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Abstract: Various iterated methods have been recommended to solve nonlinear equations. This study is suggesting a Numerical Method for solving nonlinear problems. This Numerical method has order of convergence is two, and it is derived from Taylor series expansions and Adomian's decomposition technique. Numerous numerical illustrations to demonstrate the competence of the proposed method by the Assessment of Steffensen method and Newton Raphson Method.

Keywords: Nonlinear Equations, Numerical Techniques, Open Methods, Convergence Analysis, Absolute Error

1. INTRODUCTION

One of the oldest and most basic problems in mathematics is that of solving nonlinear equations $f(x) = 0$, which arises in a wide variety of practical applications in Physics, Chemistry, Biosciences, Engineering (Biswa, 2012) and (Yasmin and Janjua, 2012). for example: Distance, rate, time problems, population change, Trajectory of a ball etc.In fact, the world is occupied of such physical phenomena that can be modeled mathematically, finding solutions to those physical problems is worth knowing. It is often the case while finding out solutions to the physical world problems by applying mathematics in the form of an equations, that the exact formulas or analytical schemes are not capable of handling complexity of such problems, there numerical techniques are employed to get the solutions (Iwetan *et al*, 2012). For solving these kinds of application nonlinear problems, we can use one of the most powerful and well-known iterative methods known to converge quadratically that is Newton Raphson method $x_{n+1} = x_n - \frac{f(x_n)}{f`(x_n)}$. Newton Raphson method is often used to obtain the approximate solution of such problems because it is not always possible to obtain its exact solution by usual algebraic process (Akram and Ann, 2015). This method is fast converging numerical techniques but are not reliable because keeping pitfall (Soram, 2013). However, it is most useful and vigorous numerical techniques. Recently in literatures few modifications in Newton Raphson method had been developed by using many different techniques including Taylor series, decomposition method, homotopy techniques, quadrature formulas (Golbabai and Javidi, 2007), (Darvishi, and Barati, 2007), (Cordero and Torregrosa, 2007), (Somroo *et al*, 2016) and (Qureshi *et al*, 2017). Likewise, the design of proposed iterative formula for solving nonlinear equations $f(x) = 0$ is a very interesting and important task in numerical analysis. The mainpurpose of this research is the development of anumerical iterated algorithm for solving nonlinear equations by considering on Taylor series expansions and Adomian's decomposition technique (Basto *et al*, 2006) (Jishe Feng, 2009), and it is convergedquadratically. Theproposed iterative formulas which converge more quickly than Steffensen Method and Newton Raphson method. Mathematical package C++/MATLAB is used to clarify the results of proposed second order iterated method.

2. NEW ITERATIVE METHOD

Considering the nonlinear equation, such as

$$f(x) = 0 \tag{1}$$

Now writing $f(x + h)$ in Taylor's series expansion about x, one obtains

$$f(x+h) = f(x) + hf`(x) + g(h) \tag{2}$$

$$g(h) = f(x+h) - f(x) - hf`(x) \tag{3}$$

Supposing $f`(x) \neq 0$, one searches for a value of h and $f(x+h) = 0$, such that

$$f(x) + hf`(x) + g(h) = 0 \tag{4}$$

This is equivalent to finding the following `h`

$$h = -\frac{f(x)}{f`(x)} - \frac{g(h)}{f`(x)} \tag{5}$$

(5) can be rewritten in the following form

$$h = c + N(h) \tag{6}$$

where

$$c = -\frac{f(x)}{f`(x)} \tag{7}$$

and

$$N(h) = -\frac{g(h)}{f`(x)} = \frac{-f(x+h) - f(x) - hf`(x)}{f`(x)} \tag{8}$$

Here c is a constant and N(h) is a nonlinear function. When applying Adomian's method to (6), we use the technique of Basto, such as

$$S = \frac{-N(c+S*)S + N(c+S)}{1 - N'(c+S*)} \tag{9}$$

when x is sufficiently close to the real solution of $f(x) = 0, S* \approx 0$. Thus (9) converts to

$$S = \frac{-N(c)S + N(c+S)}{1 - N'(c)} \tag{10}$$

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Applying the Adomian's method to (6), we obtain

$$A_0 = N(h_0) = N(c) = \frac{N(c)}{1 - N'(c)}$$
$$= \frac{f(x+c)}{2f'(x) - 2f'(x+c)} \quad (11)$$

Now we construct the iterative method. For $h \approx h_0 = -\frac{f(x_n)}{f'(x_n)}$, obtains $h + x \approx x - \frac{f(x)}{f'(x)}$, which yields Newton method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

For k = 1, one obtains $h \approx h_0 + N(h_0), h + x \approx x + h_0 + N(h_0)$, which suggests the following iterative method [12], we have

$$x = x_{n+1} - \frac{f(x_{n+1})}{2f'(x_n) - f'(x_{n+1})} \quad (12)$$

(12) can also be written as

$$x = x_n - \frac{f(x_n)}{2f'(x_{n+1}) - f'(x_n)} \quad 13)$$

Finally, using Newton Raphson Methodin (13), we get

$$x_{n+1} = x_n - \frac{f(x_n)}{2f'\left(x_n - \frac{f(x_n)}{f'(x_n)}\right) - f'(x_n)} \quad (14)$$

Hence (14) is a proposed numerical method.

3. RATE OF CONVERGENCE

The following section will be shows that the New Developed Method is Quadratic Convergence.

proof

Let `a` be a simple zero of f. Then, by expanding $f(x_n)$ and $f'(x_n)$ in Taylor's Series about `a`, we have

$$f(x_n) = f'(a)(e_n + c_2 e^2{}_n + c_3 e^3{}_n + \cdots) \quad --(i)$$
$$f'(x_n) = f'(a)(1 + 2c_2 e_n + 3c_3 e^2{}_n + \cdots)(ii)$$

By using $c_k = \frac{f^k(a)}{k! f^{k-1}(a)}$,k=2,3, 4… and $e_n = x_n - a$

From(i)and(ii), we have

$$\frac{f(x_n)}{f'(x_n)} = e_n - c_2 e^2{}_n + 2(c^2{}_2 - c_3)e^3{}_n + \cdots \quad \text{(iii)}$$

From (iii), we get

$$y_n = c_2 e^2{}_n - 2(c^2{}_2 - c_3)e^3{}_n + \cdots \quad --- \text{(iv)}$$

Expandingf(y_n) and $f'(y_n)$ in Taylor's Series about `a` and using(iv), we have

$$f(y_n) = f'(a)[c_2 e^2{}_n + 2(c_3 - c^2{}_2)e^3{}_n + \cdots]$$

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$$f'(y_n) = f'(a)[1 + 2c^2{}_2 e^2{}_n + \cdots] \quad --- \text{(v)}$$

By using (i) and (v) in (14),

$$e_{n+1} = e_n - \frac{e_n f'(a)(1 + c_2 e_n + \cdots)}{f'(a)[2 + 4c^2{}_2 e^2{}_n - 1 - 2c_2 e_n - 3c_3 e^2{}_n \ldots]}$$

$$e_{n+1} = e_n - \frac{e_n(1 + c_2 e_n + \cdots)}{[1 - 2c_2 e_n - 3c_3 e^2{}_n + 4c^2{}_2 e^2{}_n \ldots]} \quad ---(vi)$$

For easier to solve we are ignoring higher order of term and c = c_2, thus (vi) become

$$e_{n+1} = e_n - \frac{e_n(1 + c_2 e_n)}{[1 - 2c_2 e_n - (3c_3 - 4c^2{}_2)e^2{}_n]}$$

$$e_{n+1} = e_n - e_n(1 + ce_n)[1 - 2c_2 e_n - (3c_3 - 4c^2{}_2)e^2{}_n]^{-1}$$

$$e_{n+1} = e_n - e_n(1 + ce_n)\,[1 + 2c_2 e_n + (3c_3 - 4c^2{}_2)e^2{}_n]$$

$$e_{n+1} = e_n - e_n[1 + 2c_2 e_n + (3c_3 - 4c^2{}_2)e^2{}_n + ce_n]$$

$$e_{n+1} = e_n - e_n[1 + 3ce_n + (3c_3 - 4c^2{}_2)e^2{}_n]$$

$$e_{n+1} = e_n - e_n - 3e^2{}_n - (3c_3 - 4c^2{}_2)e^3{}_n$$

$$e_{n+1} = -3e^2{}_n - (3c_3 - 4c^2{}_2)e^3{}_n$$

Hence this proves that the proposed numerical method has second order of convergence.

4. NUMERICAL EXAMPLES

In this section, we present some examples to illustrate the efficiency of the new developediterative methods in this paper. We compare the Newton Raphson method and Secant method. The following stopping criteria is used for computer programs:

$$|x_{n+1} - x_n| < \epsilon, \text{ for} \epsilon < 10^7$$

Mathematical package C++/MATLAB has used to examine the fallouts of proposed method. From the numerical results in Table-1, it has been observed that the numerical second order method isfalling iterations and accuracy perception, such as in **(Table-1).**

Table-1

Functions	Methods	Iterations	X	A E %
x^3 -9x+1	Steffensen method	3		0.0003980850
(0,1)	Newton Raphson Method	3	.111264	0.0003980850
	New method	3		0.0000000000
Sinx-x+1	Steffensen method	5		0.039410600
(0.5,2.5)	Newton Raphson Method	5	1.93456	0.000011921
	New method	5		0.000000000
e^x–5x	Steffensen method	4		0.00000298
(0,0.5)	Newton Raphson Method	4	.259171	0.00000298
	New method	4		0.00000000
2x-lnx-7	Steffensen method	3		0.001535
(3,5)	Newton Raphson Method	3	4.21991	0.000095
	New method	3		0.000000
x^3 -9x+1	Steffensen method	3		0.0003980850
(0,1)	Newton Raphson Method	3	.111264	0.0003980850
	New method	3		0.0000000000

5. CONCLUSION

In this study, we have recommended a numerical second order method for solving nonlinear equations. From numerical examples, we show that the efficiency of proposed method is about the superior thanexisting second order methods such as Steffensen method and Newton Raphson Method.The developed numerical method is fast converging to approaching the root. Hence the proposed method is execution well, more effectual and easier to employ with reliable results for solving non-linear equations.

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