

## DISCRETE DYNAMIC PROGRAMMING FOR OPTIMAL WATER ALLOCATION

Dr.Abida Taheran:

### INTRODUCTION

Single and multiple objective simulation models are in use for quite some time for studying different water allocation policies. Expert judgement is used to depict a few "most likely" scenarios consequences of which are then simulated and evaluated. But this approach may very often fail to identify the global optimal solution. Researchers have also used linear programming to optimize the system and its components. But the requirement of having linear and concave objective function and so on make it extremely difficult to model a realistic scenario. In this context, dynamic programming technic offers a truly helping hand. And this paper attempts to show how this powerful tool can be used to answer a number of frequently asked questions in relation to water allocation within a command area of an irrigation project (it will also hint as to how the model can be extended for the entire project).

### THE CONCEPTUAL MODEL FOR THE OPTIMAL WATER ALLOCATION POLICY

A discrete dynamic optimization model has been developed below to determine the optimal water allocation policy and corresponding net return. The model is to be applied to a command area (CA) served by a single outlet of the main canal for a particular cropping season. Since the distribution system within the CA is typically a complex network of primary and secondary canals and field ditches, a number of simplifying assumptions are necessary to keep both the theoretical and practical (such as the extensive and costly data base) aspects of the model manageable.

### ASSUMPTIONS

i) The existing water distribution network is taken as given. So the model will not try to redesign the distribution

system. However, the model will allow incorporation of benefits from better maintenance activities or from canal lining (by reducing the system loss).

ii) It has been assumed that the potential benefit from irrigation could be measured and related to the amount of water applied. The model will use benefits and costs of irrigation on a seasonal basis to determine the optimal water allocation. So the implicit assumption is that appropriate operational schedule can be figured out to accommodate the optimal policy. This is not an unrealistic assumption given the fact that most of the canals and ditches currently carry less than full capacity water and flow can be significantly improved through canal improvements and lining.

iii) The cropping pattern within the command area has been assumed to remain more or less the same. It is a necessary assumption only if the same benefit and cost functions are to be used for a number of seasons. If significant changes are anticipated, the functions have to be adjusted but the basic model still remains the same.

iv) Only the primary canal has been explicitly considered in the model although the same procedure can be readily extended to secondary canals and field ditches. The model will accept the flow available at the primary canal inlet as given and use a forward recursion scheme to trace out the optimal (economically efficient) allocation. The output from this idealistic simulation will serve as the bottom line for evaluation of subsequent outputs from more realistic scenarios.

v) The primary canal has been divide into a series of discrete intervals (Fig.1). It has been assumed that a functional relationship (or a look up table) is available which gives the conveyance loss for a given inflow into the canal segment. Note that a single measure of efficiency loss is not adequate since the amount of water lost will vary widely for different inflows. So water loss should be specified for at least three to four flow intervals and linear interpolation may be used for flows within a particular interval.

vi) The model optimizes from the local water manager's point of view in the sense that only the direct benefits and costs within the CA are considered. However, this is not

FIGURE 1

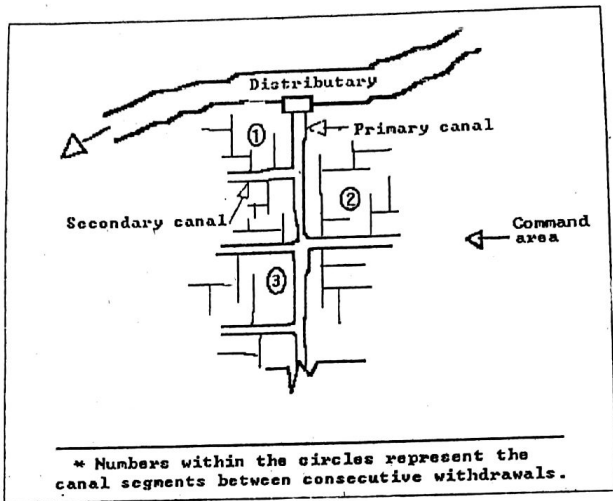
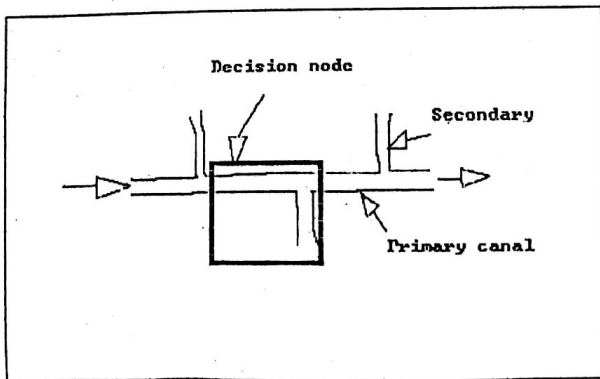


FIGURE 2



a necessary condition. Broader definition of benefit and cost could be easily introduced although that is more commonplace in a project wise or regional CBA analysis. In fact the model could be easily extended to optimize water allocation for the entire project where the outputs from the primary canal based simulations will constitute the net benefit functions which in turn will serve as inputs for a higher level optimization.

### THE MODEL AND ITS COMPONENTS

The model along with its components can be best described with the help of Figure 2 and Figure 3. Figure 2 shows the physical configuration of a decision node of the dynamic model to be formed. Figure 3 shows the schematic representation of the same actually used by the model. The symbols used have interpretations as follows:

- $X_{i-1}$  = inflow to the current decision node - a state variable;
- $X_i$  = outflow from the current decision node, also a state variable;
- $L_i$  = canal loss inside a decision node (between two consecutive stages or points of withdrawal), a function of  $X_{i-1}$ , includes both spill and seepage loss;
- $ET_i$  = evapotranspiration loss between two consecutive stages, can be significant in arid and semi arid areas, otherwise can be omitted or lumped with the canal loss;
- $U_i$  = delivery to the secondary canal - a decision variable;
- $R_i(U_i)$  = the return function, seasonal net benefit reaped from the delivery  $U_i$  at state "i" of a particular stage.

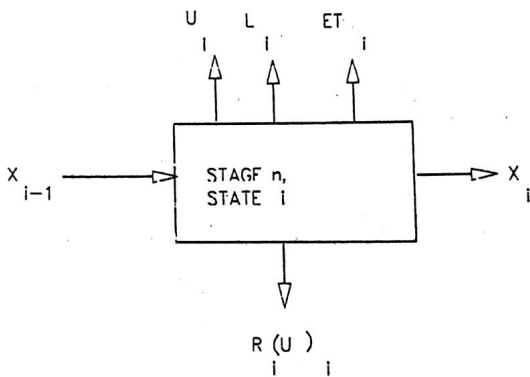
Now based on the schematization mentioned above, the overall objective would be to maximize the cumulative net seasonal benefit from the optimal water allocational scheme for the entire system. The equivalent recursion equation can be given as under:

$$F_i(X_i) = \text{MAX}_{U_i} \{R_i(U_i) + F_{i-1}(X_{i-1})\}$$

where:

$F_i(X_i)$  = maximized sum of net benefits up to

FIGURE 3 : SCHEMATIC REPRESENTATION  
OF A DECISION NODE



current state and stage;

$F_{i-1}^*(X_{i-1})$  = maximized sum of net benefits that could be achieved upto the previous stage if  $U_i$

decision is made at the current stage.

Note that a forward recursion scheme is used instead of the usual backward scheme because the flow at the main canal inlet has been assumed as given (initial condition known). Also a regular non inverted form has been used. The recursion has to be carried out subject to the following conditions:

**a) State Transformation**

$$X_i = X_{i-1} - U_i - L_i(X_{i-1}) - ET_i$$

**b) Initial Conditions**

$X_0$  = known, could be historical or forecasted;

$F_0(X_0)$  = zero when no delivery cost is attached and gravity flow is ensured at the primary canal inlet; a negative number when such cost is present and/or water has to be pumped or lifted from the distributary canal into the primary canal.

**c) Terminal Conditions**

$X_{N_i-1} > 0$ ,  $N$  is the total number of canal segments;

$R_i(U_i)$  for all stages,  $n > N+1 = 0$ . This means that water released from the tail end of the primary canal does not generate any benefit. If any tail end benefit or cost is identified, this condition has to be changed accordingly and can assume a non zero value if the model is extended to include the secondary canals as well. Then one secondary canal may discharge into another. Interconnection like this, where actual flow is distributed both in terms of time and space, makes the network extremely complicated and cause separability problem. Simplification based on experience and professional judgement has to be made to keep the problem solvable under such circumstances.

**d) Physical Constraint**

$X_i > 0$ . Essential to prevent  $X$  all from being negative when  $X_{i-1}$  small and/or  $L_i$  and  $ET_i$  are high. Also means that the flow direction can not be reversed, water can only be delivered to the secondary canal but can not be taken away.

**e) Demand Constraint**

$0 < U_i < U_{n-max}$  where  $U_{n-max}$  is the maximum demand for water at stage "n" (at the nth withdrawal point) that comes from the associated cropping pattern. The upper limit can be further extended if impacts of over irrigation by the farmers are to be simulated.

The model, when run with the conditions described above, will give the economically efficient water allocation scheme. This will serve as the base line scenario against allocation policies resulting from more realistic simulations.

**HOW TO USE THE MODEL? WHAT IF... ANALYSIS?**

The model can be used to answer a whole range of questions all of which can not be discussed in this brief paper. Only a few important possibilities are described below.

**a) Simulation with distributional considerations**

The basic model as has been developed so far is too single minded in the sense that it only tries to maximize the single objective of economic efficiency. However, in any real world irrigation project, distribution of water among various user groups (such as small farmer vs. large farmer or head end farmer vs. tail end farmer) is an extremely important issue. So some minimum amount may have to be guaranteed for different user groups. This is particularly essential where the project is already in existence and the farmers are entitled to have water according to a fixed or variable rotation irrigation schedule. In such a case, any suggested policy should imply an improvement of supply for most of the farmers and should not result in drastically different water distribution.

So additional constraints are necessary to incorporate equity issues. Let:

$T_n$  = target delivery for stage n;

$D_i = (T_n - V_i) / T_n$  - relative deviation of the possible supply  $V_i$  from the target;

$W_n$  = weight associated to stage in where higher value implies higher priority and;

$R_i(U_i) = W_n \cdot D_i$  - modified return function.

Then the modified recursion equation which will minimize the maximum deviation is:

$$F_i(X_i) = \text{MIN} \{ \text{MAX} [ R_i(U_i); (U_i); F_{i-1} X_{i-1} ] \}$$

$U_i$

This formulation is slightly different than the original because it does not provide the maximized cumulative net benefit. But a separate subroutine can be easily added which will generate the desired cumulative net benefit by using the optimal values of the decision variable. The difference between the cumulative net benefits of the original and the current simulation is the "opportunity cost" of favouring a particular distributional pattern over the economically efficient one.

#### b) Long run net benefit

So far the inquiry has been centered on the water allocation policy for a given inflow at the primary canal inlet. But this inflow or amount of water available will vary from season to season. The project manager who manages all the command areas under an irrigation project may be interested in the average or long run net benefit from a particular CA. His decision about resource allocation will depend on the expected return from each of these CAs. The model presented above can be used to estimate such expected values. The following simple steps will suffice:

- i) Collect a series of about 15 to 20 (the more the better) data points (actual or simulated inflows to the primary canal).
- ii) Perform frequency analysis. Then determine the expected value of the net benefit (EVNB) as:

$EVNB = E_j \{ P_j \cdot NB_j (V_j) \}$ ,  $j = 1, 2, \dots, k$   
 where  $P_j$  is the probability of having an inflow of magnitude  $V_j$  (note that all the inflows and outflows are in volumetric terms) and  $NB_j$  is the corresponding net benefit and  $k$  is the number of inflows.

The procedure can be used for policies with or without distributional considerations.

### c) Channel improvement and optimal level of investment

In many parts of the world, only 40 to 60% of the water supplied for the command area actually reach the crop fields. The loss occurs due to spillover, seepage, ET (through weeds and trees growing along the canal) and insects and rodents (which dig holes on banks and bed of the canal). A significant part of the system loss could be reclaimed if cleaning and lining activities were undertaken. Then the inevitable question that arises is how much money (if any) should be spent on such channel improvement activities. The model presented here can be directly used to answer this question. The only changes necessary are the modifications of (or additions to) the look up table for loss estimation (this unfortunately may not be a trivial job) and calculation of a few new cost functions which will reflect the cost of improvements as well. Then the expected value of net benefit with regular maintenance and/or lining can be estimated as before and can be compared with the same when no such options were considered. Curves of total expected benefits and costs against different levels of channel improvement activities can be drawn for the CA or for the entire project. The optimum level of improvement activities (to be read off the x-axis) is the one corresponding to which the difference between the total benefit and total cost is the maximum.

### CONCLUSION

The paper has briefly shown the potential of dynamic programming in water allocation related decision process. It discussed about how to obtain economically efficient water allocation, how to include equity considerations, how to estimate long run or expected value of the net benefit and how

to determine optimal investment level for channel improvement. Many other questions can be answered using the same basic conceptual frame work. The model can get input from a similar submodel for secondary canals and serve a higher level model by its output. Thus the concept can really be incorporated into a multilayered integrated model which will perform complete horizontal and vertical optimization.

### REFERENCES

No book or published material has been directly quoted in this paper. However, classnotes from CE546 (offered at Colorado State University, a course on systems approach of water resources) and the handout prepared by Dr.Darrell Fontane et al. have greatly helped. Other texts consulted are listed below.

1. Major, David C.; Lenton, Robert L., *Water Resource Systems Planning*, New Jersey: Prentice-Hall, 1979.

2. Haimes, Y.Y.; Hall; W.A.; Freedoman, H.T., *Multiobjective Optimization in Water Resources Systems*, Amsterdam: Elsevier Publishing Company, 1975.

3. Sampath, R.K., Young, Robert A.; (ed.), *Social, Economic and Institutional Issues in the Third World Irrigation Management*, Boulder: Westview Press, 1990.

4. Larson, Robert E.; Casti, John L., *Principles of Dynamic Programming (Part-I)*, New York: Marcel Dekker, Inc., 1978.

5. Hillier, Frederick S., Lieberman, Gerald J., *Introduction to Operations Research*, (4th Ed.), Oakland: Holden-Day, 1986.