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# ISLAMIC ART, MATHEMATICS AND HERITAGE OF SINDH

## Abstract

*This study first reviews the links between Mathematics and Islamic Art. A prominent feature of Islamic Art is the use of some very basic geometrical ideas like squares, circles and triangles. Then a mathematical study of the geometrical patterns of the Shah Jahani Mosque at Thatta, Pakistan is presented. The mathematics of some patterns present on the dome, walls and arches (interior) of the aisles of the mosque is described. No such mathematical study is available yet. This study will also contribute in the literature related to Teaching of Mathematics, Architecture, Arts and Tourism.*

## Keywords

*Islamic Art, Teaching Mathematics, Conic Section and CAS*

## Introduction

Patterns in Islamic Art (demonstrations of visual arts in view of basic Islamic principles) have their importance in teaching of mathematics as these patterns demonstrate a lot of symmetry providing a good range of examples (Abas, 2001). At very early age children are introduced to identify colors and shapes (similarity, symmetry and differences). At middle levels students are introduced to geometry and measures and properties of geometrical shapes and objects (squares, circles, triangles, conic sections). At further advanced college and university levels, transformation (symmetry, rotations, reflections and translation) geometry is introduced. These concepts involve a lot of algebra and other areas of mathematics. Mathematics of nature also involves a lot of geometrical patterns.

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Architecture through ages possesses a large stock of geometrical patterns and shapes. It is particularly true for Islamic Art. Geometrical shapes and objects (squares, circles, triangles, and conic sections) are abundant in the architectural and artistic heritage of Islamic era because the teachings of Quran always invite people to think over the wisdom lurking in the structure of cosmos and the celestial objects like skies, constellations, stars, sun, moon and the earth itself (Mashayekh, 2004) and (Elizabeth, 2008). Resultantly, knowledge of geometrical shapes and the underlying mathematics excelled in an era that ranges from Umayyads (in Arabia) to Mughals (in India) through Abbasids, Seljuks, Persians and Uthmanis(The Metropolitan Museum of Art, 2004) and (Andrew, 2002). Scholars in these eras enhanced the Euclidian geometry in its foundations and applications.

One problem of architectural and artistic importance was the problem of transition of a square into a circle. Squares, circles, triangles and polygons were used to generate various shapes and designs found in architectural and artistic works in medieval (and later) Islamic eras (Michael Field, 2009), (jairazboy, 2000), (Abdi Omar Shuriye et. al 2011), and (Salman , 2012). The patterns are made up of a small number of repeated geometric elements. The simple forms of the circle, square, and straight line are the basis of the patterns.

These elements are combined, duplicated, interlaced, and arranged in intricate combinations. In the study of geometrical patterns of the Shah Jahani Mosque at Thatta, Pakistan (Hassan Dani, 1982) it is found that patterns present on the dome, walls and arches (interior) of the aisles of the mosque are a very good example of the use of geometry and provide wonderful examples at all levels for the teachers of mathematics. Particularly when using CAS (Computer Algebra System) tools (Ansari et. al 2013). For a detailed study of Islamic art in the Indo-Pak subcontinent one can refer to (Barbara, 2002), (GhazalaMisbah, 2003) and (MariumGul, 2011).

Section: 1 provides a general description of patterns in Islamic Art and their role in teaching of mathematics. Section: 2 gives a brief description of Islamic Mathematics. Section: 3 is the detailed study of the basic characteristics of patterns in Islamic Art exemplifying with the help of the geometrical patterns found in mosques.

Mosques, being most prominent and essential manifestation of Islamic Culture are a prominent example of the use of geometry in Islamic Art. We have studied almost all the elements of a mosque and the geometrical patterns inscribed therein considering the Shah Jahanani Mosque, Thatta, Pakistan. The mathematics behind these patterns is also discussed in detail. This is the main task of Section: 4 of this study. Section: 5 conclude the study.

## *Features of Patterns of Islamic Art*

The term Patterns of Islamic Art usually means the Islamic Visual Art produced from the 7th century onwards by people who lived in the populations ruled by Muslim rulers or within the territories culturally occupied by Islamic populations. But it has no solid limits and examples can be found in earlier eras as well. It is interesting to note that patterns of Islamic Art demonstrate a lot of symmetry providing a good range of examples helpful in teaching at all levels. As it is said earlier, at very early age children learn to identify colors and shapes in terms of similarity, symmetry and differences.

At middle levels students get intimated with geometry and measures and properties of geometrical shapes of squares, circles, triangles, conic sections etc. At college and university levels geometry of transformation that is symmetry, rotations, reflections and translation is introduced (Abas, 2001). These concepts involve a lot of algebra and other branches of mathematics. Mathematics of nature also involves a lot of geometrical patterns. A review of architecture through ages revels that a large stock of geometrical patterns and shapes is available for use. In particular, that the skills termed as Islamic Art are frequently available. Some prominent representatives of Islamic Art refer to Spanish, Turkish, Persian and Mughal destinies.

In fact, there is a basic difference between Islamic Art and the others. The use of living objects is prohibited in Islamic Art. One may consider it distracting but Muslim artists found their way in the realm of art and culture using geometrical patterns and explored new dimensions of artistry and perfection. Their needs consequently evolved many areas of mathematics like geometry and algebra. Al-Khwarizmi and Omar Khayyam are some well-known examples in this regard.

There are other dimensions of Islamic Art also for example calligraphy, the artistic inscription of religious scripts on structures, buildings, walls, roofs, paper and other media. Later it included the inscription religious and non-religious poetry as well. It is important to note that geometry that was mainly involved in resolving structural problems and designing and planning of the structures such as cities, mosques, palaces, ordinary homes, gardens, domes, fountains and resorts. Even it engulfed pottery, ceramics, carpets, play boards and maizes etc. This close relationship of scientific areas like mathematics, geometry, cosmology, and astrology and everyday life lead to awesome aesthetic, literary and poetic beauties maintaining high levels of craftsmanship.

It is mentioned earlier that geometrical shapes and objects like squares, circles, triangles, and conic sections are abundant in the architectural and artistic heritage of Islamic era that resumed just after the formation of the first Islamic government in Madinah. The inclusion of these geometrical object have their roots in the teachings of Quran that invite people to think over the wisdom lurking in the structure of cosmos and the celestial objects like skies, constellations, stars, sun, moon and the earth itself.

Obviously it calls for knowledge of geometrical shapes and the underlying mathematics which excelled in an era that ranges from Umayyads (in Arabia) to Mughals (in India) through Abbasids, Seljuks, Persians and Uthmanis. Scholars in this era added much in the Euclidian geometry including its foundation as well as applications. The list of problems having architectural and artistic importance includes the problem of transition of a square into a circle via triangles (Mashayekh, 2004). In fact squares, circles, triangles and polygons were used to generate various shapes and designs found in architectural and artistic works in medieval (and later) Islamic eras.

From the point of view of teaching of mathematics, works of art can serve as an affordable stimulation and visualization of various concepts in mathematics and geometry. This can be easily made possible to utilize in many ways (Abas, 2001).

A variety of mathematico-artistic works such as geometric patterns in textiles, ceramics, metalwork, architectural elements, and manuscripts is available and now it is becoming common to use these patterns as a teaching aid.

## *An overview of Islamic Mathematics*

Islamic Mathematics is the term used to refer to the mathematics done in the Islamic world between the 8th and 13th centuries AD. Mathematics from the medieval Middle East is very important in laying down the foundations of the mathematics we use today. While Europe endured its "Dark Ages," the Middle East preserved and expanded the arithmetic, geometry, trigonometry, and astronomy from the ancient Greek philosophers, such as Euclid. The most important contribution may be the invention of algebra, which originated in Baghdad in the House of Wisdom (Bayt Al-Hikma) (established 9th century in the era of Caliph Harun Al Rashid). Basically it was a library and a bureau of translation which in turn got converted into a citadel of knowledge, research and scholastics. Many classics available at that time were translated and republished along with precious original works (Elizabeth, 2008).

A large number of Muslim and non-Muslim Scholars were engaged there to work in free and peaceful environment. Resultantly, such high caliber creations emerged that influenced many coming centuries. One prominent name is that of Al-Khwarizmi (800-47 AD). He is considered to be the founder of Algebra (Abdi Omar Shuriye et. al 2011). His most famous book is Aljabr-wal-Muqabla). Harmonizing the links between Hindu and Arabic numerals he led towards the idea of considering numbers as number system by introducing zero and natural numbers. It was a fantastic work converting philosophy into a clever scientific tool. The other prominent name is that of Al-Kashi (1380-1429AD). He contributed to both algebra and geometry. He used his formulae to solve daily life problems. One of his contributions is the determination of the value of $\pi$ with sufficient accuracy (Al-Risala Al-Muhitiyya).

Iterative methods developed by him helped him much to attain better accuracy while calculating areas of plane figures, surface area and volume of pyramids, volumes of cones of the sphere with applications to domes, vaults and arches. His works in geometry were vastly utilized by architects in calculating the wages and the costs of materials used before or after constructions. Latter these formulae were used in medieval Italy and in Iran in the era of Safvids in seventeenth century (Abdi Omar Shuriye et. al 2011). Another famous name is that of Ibrahim ibnSinan (908-46 AD (Elizabeth, 2008). He is famous for his work on conic sections. He also elaborated the methods of constructing parabola, hyperbola and ellipse.

These constructions have great applications in Muslim architecture. In contrast to Muslim architecture (vastly utilizing conic sections) the Greek architecture was limited to flat surfaces only (Elizabeth, 2008). Another famous reference is that of Abu Nasr-Al-Farabi (870-950 AD(Elizabeth, 2008). He wrote his book "Spiritual Crafts and Natural Secrets in the Details of Geometrical Figures" through which artists solved their artistic problems for examples, to construct at the endpoint A of a segment AB, a perpendicular to that segment, without prolonging the segment beyond A, to construct a square in a given circle and to divide a line segment into any number of equal parts. The Islamic art of geometrical design lead to an expansion of geometrical proofs and constructions improving geometry to be able to help the creation of beautiful geometric designs of Islamic art.

## *A review of the role of Geometric Design in Islamic Art*

Geometric patterns, calligraphy, human and animal shapes and vegetal designs are the main features of Islamic art. They may be found to exist singly or in unison with the others depicting unity, logic and order. Compass and ruler were the basic instruments needed in such constructions (The Metropolitan Museum of Art, 2004). Usually, squares, triangles and circles lay the foundation of these designs. The circle had its importance in creating vegetal designs, calligraphic and many pointed stars designs.

*Basic Characteristics of Islamic Patterns*

Some of the basic characteristics of Islamic patterns and designs can be summarized as follows:

1.  They are made up of a small number of repeated geometric elements. The simple formsof the circle, square, and straight line are thebasis of the patterns. These elements arecombined, duplicated, interlaced, andarranged in intricate combinations.

2.  Usually the patterns are framed using grids of equilateral triangles, squares and hexagons. In fact in mathematical terminology these are regular tessellations. These grids may continue for any desired length.

3.  Sometimes patterns with a background pattern overlapping a foreground pattern are also used. This creates a flattening impact without depth.

4.  The patterns are not limited to any particular frame and are infinitely expandable. The patterns give rise to arbitrary frames showing self-similarity, one basic pattern or arrangement of patterns predicting and projecting similar patterns or arrangements at different scales.

5.  Sometimes a combination of a pattern and calligraphy is used involving principles of symmetry.

6.  Symmetric tiling is frequently used for decorating buildings.

7.  Tessellations are also one of the prominent features of the Islamic patterns. These are puzzle like designs having different polygonal shapes as components fitted together leavingno gaps without requiring any foreground-background combination.

8.  Combinations of interlacing or weaving plantlike forms and vegetal patterns are also used. The former set as foreground and the latter as contrasting background. The contrast can be created by varying the shades of the same color or by varying the colors.

Figures 1-8 clearly demonstrate these characteristics



Fig 1    Fig 2    Fig 3    Fig 4    Fig 5    Fig 6    Fig 7    Fig 8

Figures 1, 6 and 7 are from the graveyard of Maklinear Shah Jahani Mosque Thatta, era 1352 AD. Fig 1depicts a pattern at a wall of the tomb of Amir Sultan Muhammad (died in 1556 AD) (Hassan Dani, 1982 and MariumGul, 2011).Figure 6 is a pattern on the walls of one of the tombs (MariumGul, 2011). Figure 7 is a pattern on the wall of the tomb of Jam Nizamuddin (died in1508 AD) (Hassan Dani, 1982, MariumGul, 2011).

Figures 2-5 and 8 are from the Shah Jahani Mosque Thatta Era: Mughal Emperor Shah Jahan (1627-1658 AD). These depict patternsondifferent walls of the mosque (Brabara, 2002).

## *Mosque as a Prominent Example of the Use of Geometry in Islamic Art*

The word Mosque comes from the Arabic word Masjid through the Spanish word mesquite. The word Masjid itself drives from the Nabatean word Mesgedha (place of prostration). The word Jami appeared in the 10th century, and from 1092 AD every large mosque is called Jami in the inscriptions, while Masjid is known as secondary mosque (Jairazboy, 2000).

Lahore is famous for its Mughal architecture. Among them the most important is the Badshahi Mosque which was built in 1674 by Emperor Aurangzeb (1658-1707 AD) (Brabara, 2002). The mosque is generally same as the Jami Masjid in Delhi but the Badshahi Mosque is much larger in area. This mosque contains all features of the Islamic art such as calligraphy, geometric designs and plant like designs (Andrew, 2002).

Another fine architectural example from Lahore is Wazir Khan mosque, one of the famous mosques of the Mughal architectures built in (1634-1635 AD) in the era of Mughal Emperor Shah Jahan(1627-1658 AD) (Brabara, 2002). This mosque contains all the tastes of the Islamic architecture for example calligraphy, flower patterns and geometric patterns including mosaic tiles with contrast of the colors in background which make it beautiful and elegant (Andrew, 2002). A comparison of the prominent parts and inscribed patterns depict the following.

### Minarets

The Badshahi mosque has eight small and large minarets in total, four large tapering minarets with several pavilions, hujras and passages, one at each corner of the sehn (courtyard). The otherminarets stand at each corner of the prayer hall (GhazlaMisbah, 2003). The Wazir Khan mosque has four thick octagonal minarets and all the surfaces are covered with floral and geometric designs in bright colored background tiles (Andrew, 2002). Geometric patterns are made with six pointed stars and hexagons on the surface of the square based minaret (GhazlaMisbah, 2003).

### Domes

The Badshahi mosque contains three externally white colored bulbous domes among them two cover the prayer chamber. It is decorated with floral designs. The ceiling of the domes in the prayer chamber of the Wazir Khan Mosque contains calligraphic patterns (GhazlaMisbah, 2003).

### Mihrab

The mihrab of Wazir Khan Mosque is framed in calligraphic patterns and plant like patterns. (GhazlaMisbah, 2003).The mihrab of Badshahi mosque is made up of two superimposed arches which contain the vegetal designs.

## Walls

The walls of Wazir Khan Mosque are rich in calligraphic patterns and plant like designs, some of these patterns are in red and yellow colors and the calligraphic patterns are framed with plant like patterns or geometric patterns and at some surfaces plant like patterns are framed with geometric patterns such as squares, hexagons and many pointed stars patterns. Geometric patterns are made with squares, rectangles, hexagons, hexagonal and octagonal stars Patterns. The walls of Badshahi mosque are adorned with vegetal designs and many pointed stars patterns. These patterns are in fact, carvings. On the walls marbles is also used (GhazlaMisbah, 2003)**.**

## Floor

The floor of Wazir Khan Mosque is made by backed bricks and embellished with simple and complex geometric designs for example the floor of upper level courtyard has complex design (Salman, 2012). The floor of Badshahi mosque at some places is inlaided with white marble (GhazlaMisbah, 2003).

## Shah Jahani Mosque Thatta, Sindh

The Shah Jahani mosque Thatta was built in 1644 AD by the Mughal Emperor Shah Jahan(1627-1658 AD.(Brabara, 2002). This mosque is Jami mosque. It was first repaired in 1692 AD during the reign of the Mughal Emperor Aurangzeb (1658-1707 AD (Brabara, 2002)(Hassan Dani, 1982). Discussing the prominent parts of this mosque it was found that a unique feature of this mosque is that it has no minaret at all (Hassan Dani, 1982). The main Dome surmount the mihrab chamber, the octagonal cylinder provide circular base to main Dome. Interlace arches are made at the walls of this cylinder which contained tiles of geometric patterns and vegetal patterns separately that beatify the mihrab chamber.

Geometric patterns and vegetal designs (their arrangement forms hexagon and circular patterns) are made to embellish the Domes' ceilings (Hassan Dani, 1982). The mihrab of the Shah Jahani mosque contained in double rectangular frame, the upper frame is contained in arch and the lower rectangular frame contained the mihrab.

Calligraphic patterns are written within the space between upper and lower frame of the mihrab. The spandrels of mihrab and upper arch that contain the frames of mihrab have vegetal patterns (Hassan Dani, 1982). In the mihrab a lattice of hexagons (honey-comb tiling [6]) is carved at the dado level and the same patters are made at the upper part of mihrab with white plaster (Hassan Dani, 1982).The mihrab of the Shah Jahani mosque is contained in double rectangular frame, the upper frame is contained in arch and the lower rectangular frame contained the mihrab.

Shah Jahni mosque is built with red bricks, some places including upper part of the mihrab and Domes from the outside are white plastered and Like the Wazir khan mosque all the surfaces of Shah Jahani mosque embellished with the different geometric patterns, plant like designs and calligraphic patters with the variations of red and blue background. Arches are framed with rectangular patterns (Hassan Dani, 1982). At the upper part of the corner squinches in the main Dome chamber cobweb (spider's web) geometric pattern are carved with different coloures bands (jairazboy, 2000). The floor of Shah Jahani mosque is very simple. It was made with stone in 1657AD about tenth year from its building time (Hassan Dani, 1982).

## Creating Geometric Pattern of Art

In the perspective of our above discussion we present our findings about our main subject that is the Shah Jahani mosque, Thatta, Sind, Pakistan.

## Pattern with Circles

Circle is considered as the fundamental geometrical shape in Islamic art therefore it is found in most of the Islamic patterns. Different arrangements of several circles make different patterns, which also contained triangles, quadrilaterals and polygon in them, for example four pointed stars, six pointed stars and twelve pointed stars.
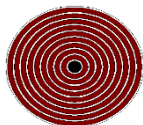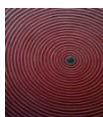


Fig: 9                    Fig: 10

Figures 9 and 10 are the concentric circles carved on inside of one of the domes of Shah Jahani mosque in the northern aisle's chamber. White coloured lines show circumferences carved in domes and the centre is covered with a piece of iron metal, these represent the whirls formed as a water drop falls into a water reservoir. Figure 9 is computer generated. The equation of concentric circles with changing values of c, keeping the centre to be fixed is $x^2 + y^2 + 2gx + 2fy + c = 0$

If we want to find out the radius of any circle in the above pattern we should know the difference in radii of any two consecutive circles and the radius of the largest circle in this pattern, with the help of the following formulae we will be able to calculate the related mathematics to this pattern,

Radius of any circle in this pattern $= r_n = R - (n-1)r'$

Where $R$ a radius of the largest circle, $n$ is the required radius of the circle and $r'$ is a difference of any two consecutive circle's radii.

Number of circles contained in this pattern $= n_c = \dfrac{R}{r'}$

Perimeter of concentric circles $=$

$P_t = 2\pi[R + (R-r') + (R-2r') + (R-3r') + ..... + (R-nr')]$ Units.

Perimeter of any circle in the pattern $= P_n = 2\pi[R - (n-1)r']$ units.

Area of any circle in in the pattern $= A_n = \pi[R - (n-1)r']^2$ sq. units.

## Pattern with Eight Pointed Stars

Eight pointed star is one of the most important pattern in Islamic art as it is found in most of the Islamic architectures. In Shah Jahani mosque sequels of eight pointed star patterns made inside of few domes of the aisle's chamber and in northern and southern bays to the mihrab chamber. The lattice made by eight pointed stars is carved at the walls of the aisles.
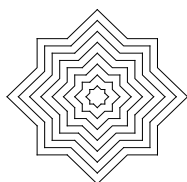

Fig: 11


Fig: 12


Fig: 13

Figure: 13 is one of the Domes of Shah Jahani mosque taken from inside. This shows a sequel of growing eight pointed star patterns that covers a Dome completely, Fig: 11 is computer generated diagram, this shows resemblance with photo on the right and same diagram with coloured is placed in the middle. Eight pointed star can also be obtained by two same squares if one rotates by $45°$ on the other square. Mathematics related to this pattern is as follows.

Perimeter of eight pointed star $= P = 4s$ units, Where $s$ is a length of a side.

Total Perimeter of $n$ numbers of eight pointed stars

$P_t = P+(P-8s')+\{P-8(2s')\}+\{P-8(3s')\}+...+(P-n8s')$ Units

Where $P$ the perimeter of the largest star, $n$ is numbers of stars contained in a pattern and $s'$ is the difference between any two consecutive stars.

Perimeter of any eight pointed star in the patterns $=P_n = P-8(n-1)s'$ units

Where $n$ number of required star's perimeter in the patterns, $n$ is natural number and $s'$ is the difference between any two consecutive stars contained in a patterns.

Numbers of eight pointed stars contained in a pattern $=n=\dfrac{s}{s'}$

 Where $s$ is the length of a side of the largest eight pointed star in a pattern.

Area of eight pointed star $= A=\dfrac{5s^2}{4}$ sq. units

Area of any star in the "eight pointed star patterns $=$ $A_n =\dfrac{5}{4}\{s-2(n-1)s'\}^2$ sq. units

Where $A_n$ is the required star's area, $s$ is length of each side and $s'$ is the difference of any two consecutive stars in the pattern

## Lattice of Eight pointed stars

Lattice of eight pointed star is carved at the walls of aisles as shown in the following figures, These designs cause to enter the sun light into the mosque that is why no artificial light sources are needed during the day for inside the mosque but for the night artificial sources of light are connected in various places inside and outside of mosque.
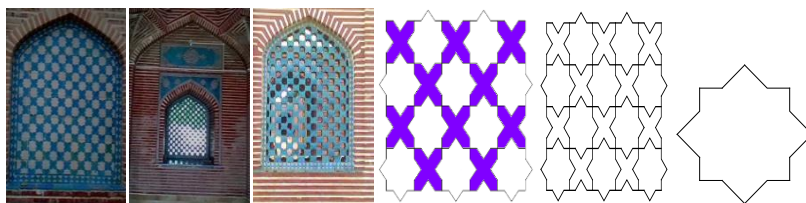


Fig 14       Fig 15       Fig16Fig17       Fig18       Fig19

Figures 14-16are taken from the walls of Shah Jahani mosque in the aisles chamber. These show lattice of eight pointed stars.

The lattice patterns of eight pointed stars can also be obtained by joining stars side by side together. Figures 17-19 depict computer generated patterns. We can obtain lattice of eighth pointed stars by the help of coordinate sequences.

## Conclusion, Discussion and Proposals

This study explored the prominent features of Islamic Art in the perspective of geometrical structures. Considering mosque as one of the prime demonstrations of Islamic Art, Shahjehani mosque Thatta, Sind, Pakistan is studied and the mathematics of the geometrical patterns found therein is explored.

Investigating the prominent elements of Shahjehani mosque that is the domes, mihrab, walls and floors (note that one more element the minaret are not present in this mosque) it is found that full use of geometry is made in the designs carved on domes, mihrab, walls and floors. This involves the use of symmetry, repetition of basic geometrical designs, grids of equilateral triangles, overlapping one pattern over the other, regular tessellations and interlacing of geometrical patterns.

Some patterns are generated by using self-similarity that is expanding, repeating and projecting the same design or one basic pattern or arrangement of patterns at different scales (it is the principal which gave rise to most modern branch of geometry called fractal geometry). Also new patterns are generated by combining geometrical patterns with calligraphy (with or without involving principles of symmetry). It is interesting to note that symmetric tiling is generally used for decorating the building itself. Note that tessellations are puzzle like designs having different polygonal shapes as components.

Now we summaries our findings regarding some of the geometrical patterns of Shah Jahani mosque, Thatta, Sind, Pakistan as follows.

Circles are considered as most fundamental geometrical shapes. Concentric circles are found on the domes of the mosque. Sequels of eight pointed star are carved inside the domes. The lattice made by eight pointed stars is carved on the walls of the aisles.

It is proposed that other historical heritage of Sindh should be studied on the same lines. Sindh is rich in such heritage but most of it lack scientific study.

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