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Teaching of Abstract Algebra at Undergraduate Level

Abstract

This study suggests that how the techniques of teaching abstract algebra at undergraduate level can be improved. With the help of examples it proposes that a better understanding of abstract algebra can be developed by adopting computer algebra system (CAS). It also, proposes a framework which describes some objectives of teaching algebra and suggests some of the hardware and software to be utilized.

Keywords:

CAS (Computer Algebra System), MATLAB, Teaching, MATHEMATICA and HEC (Higher Education Commission)

Introduction

The traditional tools for teaching and learning algebra include text books, lecture notes, lectures, assignments and examinations. The existing tools with their entire shortcoming are commonly used and recognized by the teaching community for teaching Mathematics. In the following we will discuss some aids to enhance the impact of these existing tools so that the same tools can be utilized to develop a better understanding of the subject in the students.

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We will proceed further by stressing on an analysis and study of the lectures delivered in the class rooms. In our view a good lecture must be clear, systematic, complete and precise. However, despite all its qualities a lecture is a bad teaching instrument if it is dull. A good lecture contributes to assist the students at all the stages of acquiring knowledge. Thus just lecturing in any manner and demonstrating any material may hardly be of any use to the students. So, for any prospective teacher it becomes essential to understand and know that how students learn Mathematics. Students learn Mathematics through intuition, induction, contrast, analogy, simulation and analysis. All these stages are related to the mental level of the audience. As a first step a good teacher must keep care of the mental level of individual students because every student has a different ability to adopt and understand and it is the duty of the teacher to help the students to overcome their difficulties. So a lecture should be planned to cover the needs of all the students in the class. Some good references in this regard are Benijamin (Benijamin, 2001) and Zhang (Zhang, 2004).

Teaching of abstract algebra can be well handled in light of the above discussion but the problem is that the subject requires more use of intuition. As we have mentioned above, the mental caliber of all the students is not the same so on occasions many students may loose interest and they start considering the lecture as dull. The teacher should remain alert to counter the situation. A temporary solution may be to lower the tempo of the lecture and to use some repetition. But a permanent solution may be to use simulation to enhance the intuition. Visualization via simulation can be a good support to enhance the intuition (Dikovic, 2007), (Kaltofen, 2008). For this purpose an algebra

lecture may be equipped with techniques of Computer Algebra System (CAS) which we will briefly describe in the next section.

The first section of the paper gives an introduction of tool assisted teaching of algebra and CAS. The second section exemplifies that how CAS can be applied to some specific algebraic computations. The final and the third section proposes that how to improve techniques of communicating algebra in the class room. It also includes a framework describing some objectives of teaching algebra and some suggestions regarding hardware and software to be utilized.

1. Tool Assisted Teaching and Computer Algebra System

Our existing scenario of an algebra class consists of a callous chalk-blackboard teacher, hushed paper-pen students hammered with long proofs of harsh theorems, nailed with cumbersome calculations, all fixed, solving fatiguing examples and annoying exercises.

In contrast consider a class room with handy programmable /graphical calculators or a set of computers, performing related graphics, demonstrating functions and their characteristics, visualizing behavior of the solutions of equations and signifying the involved computations which can briefly be termed as tool assisted teaching.

A computer algebra system (CAS) is a tool assisted system which is capable of performing symbolic computation in addition to numeric computation. For an introduction to CAS one can refer to Ansari et al (Anasri, & Ahmed, 2005) and Krishnamani et al (Krishnamani, &

Kimmus, 1995) and references therein. Softwares such as *MATHEMATICA* or *MATLAB* can be easily employed to assist. As regards hardware Texas Instruments graphing and CAS calculators TI-92, TI-Nspire or computers can be used. In a country like Pakistan availability of programmable calculators for each individual in the class is difficult. So instead of using CAS calculators like TI-92 or TI-Nspire computers will be a better option. Low priced refurbished computers are easily available in the market.

2. *A Review of BS Courses in Algebra in the Perspective of Adopting CAS*

For the current BS curriculum suggested by Higher Education Commission (HEC) of Pakistan (Curriculum of Mathematics Revised, 2005), algebra introduced in the Higher Secondary Certificate (HSC) courses forms the base. At the BS level a single course is provided in the fourth semester. Two more courses in algebra are introduced in the third year. All these courses are compulsory. Computer programming and numerical computing are introduced in the third and fourth semesters respectively. This provides a good background to adopt CAS for teaching algebra in the fourth semester and onwards. In the following we give some examples to demonstrate that how CAS techniques assist in visualizing various stages of an ongoing algebraic process. As regards Curricula for algebra and Mathematics some interesting references are Artin (Artin 1984) and Edger (Edger, 1986).

In the following we use MATLAB to demonstrate graphically that how the solutions of algebraic equations, system of linear equations and transcendental equations can be visualized to approach the exact values

2.1. Solution of algebraic equations

In the following we demonstrate that how it can be visualized to find the real root of a cubic equation. For this purpose consider the cubic equation

$$2x^3 + x - 2 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Let $f(x) = 2x^3 + x - 2$,

Then $x = x_o$ is a root of (1) if $f(x_o) = 0$

We start with an approximate solution $x_0 = 0.835122346$. Then, $f(x_0) = 1.2 \times 10^{-8}$. We can consider $f(x_0) \sim 0$. Using the interval [- 4, 4], the solution of the given equation can be exhibited in the following manner.

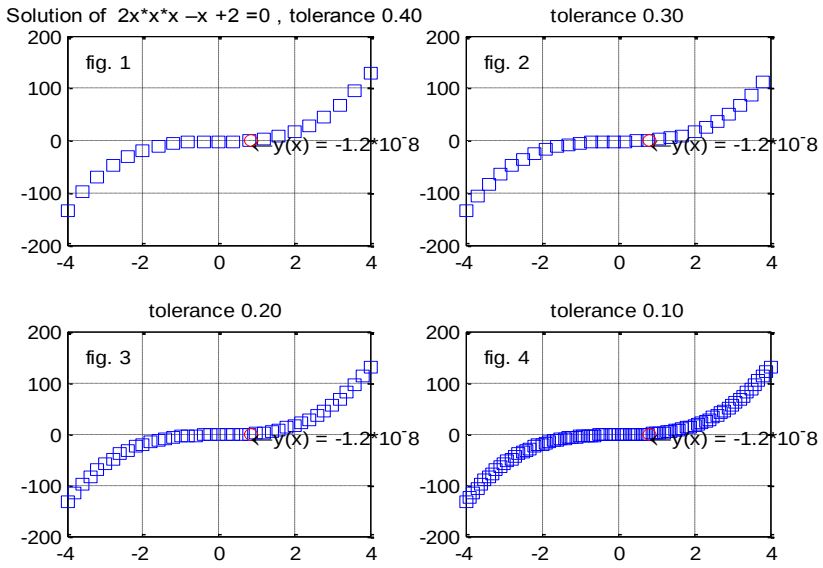


Fig. 2 demonstrates that for tolerance 0.30 in the interval (0, 2) we get more than six values of $f(x)$ which are very close to $f(x_0)$.

Fig. 1 demonstrates that for tolerance 0.40 in the interval (0, 2) we get only six values of $f(x)$ which are close to $f(x_0)$.

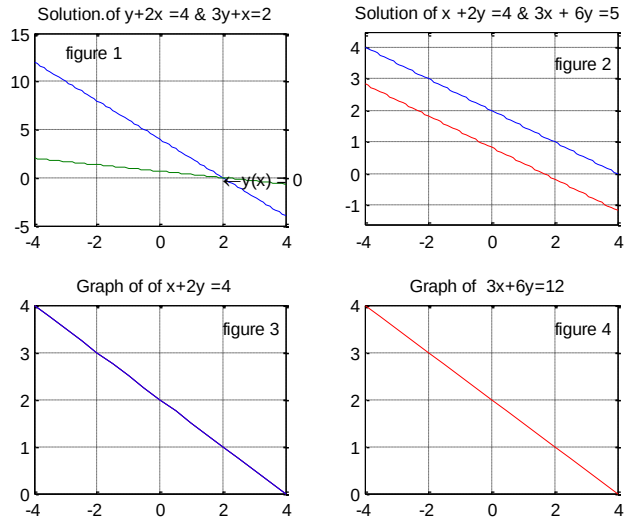
Fig. 3 demonstrates that for tolerance 0.20 in the interval (0, 2) the values of $f(x)$ which are close to $f(x_0)$ has further increased.

Fig. 4 demonstrates that for tolerance 0.10 in the interval (0, 2) the number of values of $f(x)$ which are close to $f(x_0)$ has become greater than the number of values appearing in Fig. 3.

In demonstrates that as the tolerance decreases the number of values of $f(x)$ increases and accumulate very close to $f(x_0)$. Hence x_0 is a solution.

2.2. Solution of a system of linear equations

We demonstrate here that how it can be visualized to determine the solution of a linear system of equations. We have considered three cases considering a system in which a unique solution exist, a system in which no solution exists and a system in which the system has infinite number of solutions.



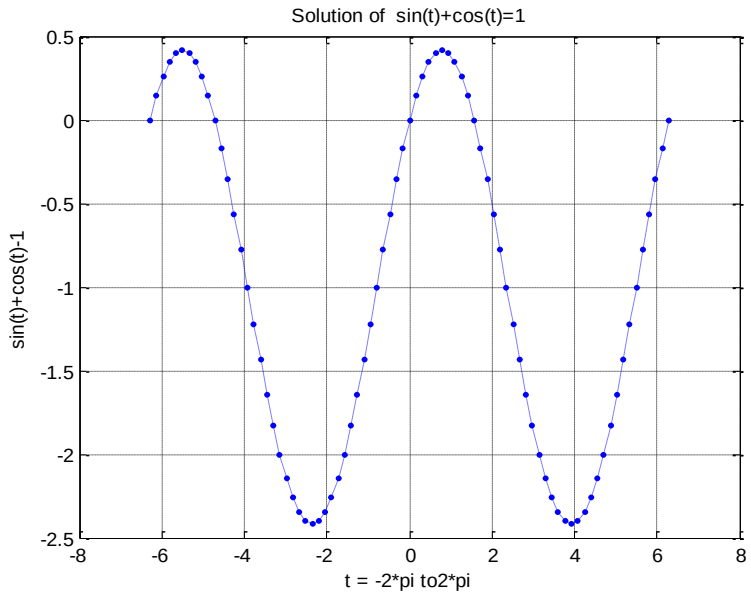
From figure 1 it is clear that both straight lines intersect at (2, 0) therefore the system of equations $y + 2x = 4$, $3y + x = 2$ has unique solution.

From figure 2 it is clear that both straight lines are parallel they will never meet, therefore the system of equations $2y + x = 4$, $y + 3x = 5$ will have no solution.

From figure 3 and figure 4 it is clear that the graphs of both straight lines are identical, therefore the system of equations $2y + x = 4$, $6y + 3x = 12$ reduces to a single equation $2y + x = 4$, if we take $x = c$ then $y = \frac{4-c}{2}$ and by assigning different values to c, we shall get corresponding values of y. So the system has infinite solutions.

2.3 Solution of a Transcendental Equation

Now we demonstrate that how one can visualize the behavior of the roots of a transcendental equation. For this purpose we consider the equation $\sin t + \cos t = 1$. Following is the graph of the given equation.



It is clear from the graph that the function $f(t)$ =sin t + cos t -1 has zeros in the interval $[-2\pi, 2\pi]$ in the neighborhoods of t=6, t=- 4, t=0, t=2, t=6.

In other words the zeros of $f(t)$ lie in the neighborhoods of these points. Consequently the equation sin t + cos t =1 has solution in these neighborhoods.

These examples show that how the visualization with the help of CAS techniques can support intuition. In addition, it creates additional interest in the routine lecture.

3. A Framework for Teaching Mathematics in General and Algebra in Particular

Teachers of Mathematics at all stages should care that their teaching must be object oriented. To initiate, in the following we propose a framework which describes some objectives of teaching algebra and suggest some of the hardware and software to be utilized.

3.1 Objectives of Teaching Algebra

1. Students should be able to appreciate the value of algebra as it is applied to other disciplines.

2. Students should be able to communicate correct algebra effectively both verbally and in writing in English as well as in other languages

3. Students should be able to use technology (packaged software/graphing calculators) to enhance learning of Algebra.

4. Students should be able to collect real-life data and analyze it using algebra.

5. Students should be able to draw appropriate conclusions orally and in writing.

3.2 Teaching Material

Along with the material to be covered in the regular syllabus, the teacher should communicate the following as well. It may be the choice of the teacher to select the proper time and stage.

1. An overview of current and contemporary texts used in Pakistan and abroad.

2. Application of Mathematics. In this regard to enhance the ability of the students towards the application of algebra additional stress must be given to the use of algebra in physical, life, social sciences and business.

3. Demonstration of the power of algebra as a modeling tool. For example stress may be given to linear and polynomial models.

4. Stress must be given to the use of algebra from the point of view of computing.

5. An introduction to the history of Mathematics in general and history of algebra in particular should be included in the curricula (Puig, L 2004).

It is to be mentioned that some of the topics described above may be the part of regular curriculum as elective subjects but a flavor should be provided to all the students getting the degree. Further, all efforts should

be made to keep the students as active participants and not the passive listeners. Positive reinforcement will always yield best results.

Along with the net supported specific aids to teach algebra some necessary software and hardware can be listed as follows.

1. Graphing packages

2. Packages for numeric computation and data analysis such as MATLAB, STATISTICA and SPSS etc.

3. Packages for numerical linear algebra techniques such as LINPACK etc.

4. Texas Instruments graphing and CAS calculators (TI-92, TI-Nspire CAS and up)

5. Use of MATHEMATICA to introduce CAS based teaching (Sham WT and Tigg J 1994).

This communication is a very brief overview of the changes which can be made to improve the quality of teaching algebra. Much can be added to the material present. The M. Sc. (two years program) for the time being is similar to the third year and fourth year BS (four year) program so the teaching of algebra at this stage can be followed in light of the above discussion.

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