



Robustness and Clustering Analysis of PTCL Routers Network

A. W. MAHESAR, K. BROHI, Y. A. MALKANI*, H. A. NIZAMANI*

IICT, University of Sindh, Jamshoro, Pakistan

Received 1st November 2017 and Revised 14th October 2018

Abstract: Many complex systems as the networked structures have shown robustness with highly clustered behavior in different domains. Various topological and structural analysis metrics are being used to pinpoint the main nodes and robustness of the network but the closeness centrality and clustering plays very significant role in evolution and communication of nodes in growing networks. Regrettably, both network analysis metrics have been used primarily either on the basis of degree of nodes or with much focused from perspective of weight (strength) of links. In this research, the behavior of routers network is analyzed by using global structures perspective (closeness centrality and clustering) based on the dataset of PTCL (Pakistan Telecommunication Limited) router network in Pakistan. The results suggest that router network is highly clustered with few close nodes and due to this is very robust under random attack (link percolation) with weighted analysis.

Keywords: Robustness, Clustering, Network Analysis, Technological Networks

1. INTRODUCTION

Networks formation has been observed in many diverse fields of life owing to the accessibility of enormous quantity of data gathered and analyzed with the help of high processing and storage capabilities of modern processing systems. Unfortunately, in spite of the availability of a huge amount of data, there are still many microscopic phenomenon to be needed to fully understand the complexity level and dynamic behavior of these networks. The complex network modeling as well as analysis are challenging owing to the heterogeneity in terms of vertices and links diversity with changeable attributes (routers, switches, bandwidth, delay, etc.). Therefore, heterogeneous capacities, character and intensities are very important to understand these networks. Similarly, many complex systems have been analyzed from entirely different domains to understand their resilience towards breakdown under random attack. The most suitable method to represent any network is through graph theory. In graph theory, vertices represent the nodes and edges represent the links in the network. For example, the complex network of Internet is a network of domain or routers. The domain or routers are nodes and their physical connections are the links or edges in between them. Similarly, air transportation network is the network of airports, in which airports are nodes and routes are the links in this complex network.

This paper is structured as follows. In section II, we discuss the two network analysis metrics namely closeness centrality and clustering coefficient both

global and local with shortest distance. In section III, we introduce the dataset of PTCL router network. Section IV; discuss the analysis and results of this network approach based on centrality and vulnerability under random attack and finally section V concludes the paper with future work.

2. NETWORK ANALYSIS METRICS

In this section we discuss the network analysis metrics and their background to understand the network analysis of PTCL router network.

2.1 Shortest routes in complex networks

Newman and Brandes they both anticipated to reverse edges weight in case of measuring the closeness value of vertices because this need the shortest routes in complex networks. Their projected use of Dijkstra's algorithm, is properly defined in equation 1

$$d^w(i, j) = \min\left(\frac{1}{w_{ih}} + \dots + \frac{1}{w_{jh}}\right) \quad (1)$$

here d^w represent the distance with weight of links through vertices i and j .

The Opsahl generalization is based on the tuning parameter α , before applying Dijkstra's algorithm to find shortest path. This parameter confirms that the number of intermediate vertices as well as weight of vertices is used in finding the shortest paths in weighted networks with weighted vertices. Therefore, his generalization is given in equation 2(2)

$$d^{w\alpha}(i, j) = \min\left(\frac{1}{(w_{ih})^\alpha} + \dots + \frac{1}{(w_{jh})^\alpha}\right)$$

**Corresponding Author Email: waheed.mahessar@usindh.edu.pk, kamran@usindh.edu.pk, vasir.malkani@usindh.edu.pk, hn@usindh.edu.pk

*IMCS University of Sindh, Jamshoro, Pakistan

here $d^{w\alpha}$ represent the distance with weight of links in between vertices i and j and α is used as tuning parameter in equation 2.

2.2 Closeness centrality

The term closeness is defined as, the inverse of the sum of all shortest paths from a main node to all the other nodes in the network Freeman defined closeness as given in 3

$$C_c(i) = [\sum_j^N d(i, j)]^{-1} \quad (3)$$

Here, $C_c(i)$ represent closeness centrality of particular vertex i in the network and $d(i, j)$ is the binary distance between vertices i and j .

Whereas (Opsahl *et al.*, 2010) generalizations depends on the calculation of shortest paths. The (Opsahl *et al.*, 2010) defined closeness based on equation 4

$$C_c^w(i) = [\sum_j^N d^w(i, j)]^{-1} \quad (4)$$

Where $C_c^{w\alpha}$ represent closeness centrality of weighted vertices i , and α works as tuning parameter where 0 is used to find the binary shortest distance. On the other hands, if the value is 1 it will use Dijkstra's shortest path algorithm.

2.3 Clustering

Clustering measure the degree of transitivity. This measure has been analyzed by many network analysts from different research perspective This measure finds the number of present triplets in complex networks. This is the reason clustering is recognized as triadic closure in majority of literature .To see this type of behaviour two different versions (global and local) clustering can be used in the network.

2.3.1 Global clustering coefficient

This metric is defined as given in equation 5

$$G_c = \frac{3 \times \Delta}{\text{triplets}} = \frac{\text{closed triplets}}{\text{triplets}} = \frac{\tau_\Delta}{\tau} \quad (5)$$

In above metric weight of links is not included. Therefore, weighted global clustering coefficient defined by (Opsahl, and Panzarasa, 2009), with different ways such as geometric mean, minimum, maximum and arithmetic mean. The weighted network as sample with diverse weight of links is shown in (Fig.1). The different effects of using diverse procedures for computing triplet value omega are given in (Table I). The values of global clustering coefficient for weighted network of Fig. 1 based on these 4 methods are given below.

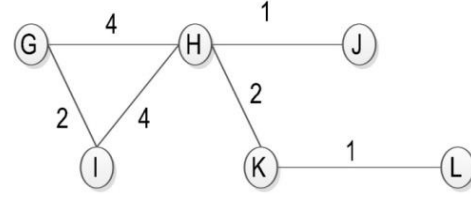


Fig. 1: The network of weighted links

Table I: Different methods of defining triplet values (Source (Opsahl, *et al.*, 2010)).

Method	Triplet value ω of	
Maximum	$\text{Max}\{4,4\} = 4$	$\text{Max}\{3,5\} = 5$
Minimum	$\text{Min}\{4,4\} = 4$	$\text{Min}\{3,5\} = 3$
Arithmetic mean	$(4+4)/2 = 4$	$(3+5)/2 = 4$
Geometric mean	$\sqrt{4 \times 4} = 4$	$\sqrt{3 \times 5} = 3.8$

3. PTCL router network dataset

There are 134 routers and these are connected with 216 links with different band widths. In this one-mode network, routers on different geographical area are modeled as nodes and physical links provide interconnectivity between them. From the given dataset there are 6 core routers each having 3 (2*10 G) and 1(1*10 G) links. In this research, we have assigned different bandwidths scales based on the speed provided by links ranging from 1 to 4. The dataset shows the longitudinal nature where initially the number of nodes was 75 and the network has grown up to 134 routers.

3.1 Closeness analysis

The Table i-I given below highlight the result of ranks of routers in this network when the bandwidth of links is not considered and hence operate as binary network as ($\alpha=0.0$). The second column shows the closeness centralities values when the actual bandwidth is counted on the basis of scale by excluding the number of links. The node numbers, names and their closeness centralities rank wise are shown in columns of (Table 2). We have shown the top fifteen routers and their closeness centrality scores. The closeness analysis shows that if the bandwidths of links are not considered then those routers with high number of links are central in the network. On the other hands, when the links are analyzed with bandwidth then routers with maximum bandwidth support are top in the ranks.

Table 2: The Closeness centrality of the Top fifteen nodes in the network when alpha (α) = 0.0 and 2.0.

Bi	Arithmetic	Geometric	Maximum	Minimum
0.35	0.79438	0.60415	0.82746	0.73074

Table 3: The random links removal as correlation of ranking of nodes in true and observed networks

Num ber of Node	The effect on closeness centrality when (alpha=0.0)		The effect on closeness centrality when (alpha=2.0)	
	Node Closeness		Node Closeness	
1	Router 48	0.086744361	Router 05	0.645885165
2	Router 102	0.076744361	Router 132	0.487755105
3	Router 05	0.062706767	Router 69	0.468421053
4	Router 47	0.072706767	Router 63	0.398421053
5	Router 121	0.052706767	Router 64	0.388421053
6	Router 23	0.062518797	Router 65	0.378421053
7	Router 79	0.073518797	Router 66	0.379421053
8	Router 125	0.088518797	Router 67	0.369421053
9	Router 52	0.076669173	Router 68	0.367421053
10	Router 69	0.077669173	Router 102	0.365480624
11	Router 70	0.089669173	Router 70	0.334168752
12	Router 62	0.710150376	Router 48	0.334140541
13	Router 19	0.052150376	Router 23	0.30575188
14	Router 32	0.065263579	Router 47	0.275510797
15	Router 03	0.045112782	Router 79	0.277728571

Table 4: Global Clustering coefficient between Routers

Alpha	(Net1) 3 % links random removal5% links random Removal	
α	Closeness	Closeness
0.0	0.50	0.51
0.5	0.70	0.64
1.0	0.81	0.76

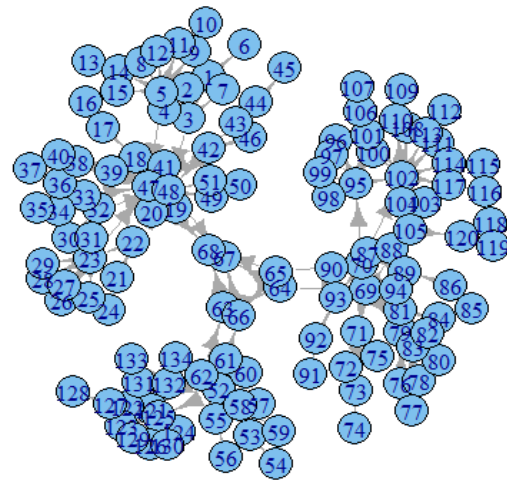
In (Table 3), the closeness centrality measure is calculated after randomly removing 3% links and 5% links. The experiment is repeated thirty times and the average values of tuning parameter are calculated. When α is 0.0, 0.5 and 1.0 the average values of centrality measure are 0.50, 0.70 and 0.81 respectively. When we compare 3% with 5% concerning the closeness, when $\alpha=1$, in 5% random removal, value is 0.51, 0.64 and 0.76. It shows that network becomes more vulnerable as more links are removed. Whereas with fewer random removal of links when alpha is 0.0, 0.5 and 1.0 has less effect and the network shows robustness under random failures.

3.2 Clustering analysis

In (Table 4), we have calculated the global clustering coefficient in router network using the five methods. These are Binary, Arithmetic mean, Geometric mean, Maximum and Minimum method.

The overall gc coefficient value of this technological system represent that this weighted systems (network) is densely clustered with routers as

nodes and bandwidths as links weight. As there are many links which have very less weight and due to this if we see the result of Maximum method, it shows highest value as the 3-paths are computed based on highest values in the network. Therefore, for global clustering coefficient the maximum method is appropriate to use in this network which indicates that the weight distribution is very inhomogeneous. This is the reason that the random links removal has less effect on this network due to the high clustering coefficient values in the network. The visualization of PTCL router network is shown in (Fig. 2).

**Fig. 2: PTCL router network visualization in graph package of R-project**

4.

CONCLUSION

Many different domains have been analyzed as complex networks. In this Paper, we analyzed the PTCL routers network from the perspective of closeness and clustering of routers. The results have shown that few routers have high closeness in terms of bandwidth in the network. Further, there are many triangles between all these routers which show that they all are highly and thickly connected. Both these measures shows that the random link removal have not any significant effect in the information flowing capability of this network and this network is very robust in nature. In future directions many network analysis metrics can be used to fine grained the behavior of this technological network.

REFERENCES:

Barrat, A., M. Barthelemy, R. Pastor-Satorras, and A. Vespignani. (2007) "The architecture of complex weighted networks." *Proceedings of the National Academy of Sciences of the United States of America* 101, no. 11: 3747-3752.

Chang, S. and L. Li, (2014). Reliability analysis of highway and transportation network with paths failure. In *Advanced Research and Technology in Industry*

- Applications (WARTIA), IEEE Workshop on, 50-53. IEEE.
- Chen, G., Z. Fan, and X. Li, (2005). Modelling the complex Internet topology. In *Complex Dynamics in Communication Networks* 213-234. Springer Berlin Heidelberg.
- Fellman, P. V., J. P. Clemens, R. Wright, J. V. Post, and M. Dadmun, (2015). Disrupting terrorist networks: a dynamic fitness landscape approach. In *Conflict and Complexity* 165-178. Springer New York.
- Goh, K. I., M. E. Cusick, D. Valle, B. Childs, M. Vidal, and A. L. Barabasi, (2007). The human disease network. *Proceedings of the National Academy of Sciences*, 104(21), 8685-8690.
- Leventhal, G. E., A. L. Hill, M. A. Nowak, and S. Bonhoeffer, (2015). Evolution and emergence of infectious diseases in theoretical and real-world networks. *Nature communications*, 6.
- Newman, M. E. (2004). Analysis of weighted networks. *Physical Review E*, 70(5):056131.
- Opsahl, T., F. Agneessens, and J. Skvoretz, (2010). Node centrality in weighted networks: Generalizing degree and shortest paths 32(3):245– 251
- Opsahl, T. and P. Panzarasa, (2009). Clustering in weighted networks. *networks*, 31(2):155–163.
- Pastor-Satorras, R. and A. Vespignani. (2007) *Evolution and structure of the Internet: A statistical physics approach*. Cambridge University Press.
- Snijders, T. A. (2001). The statistical evaluation of dynamics. *Sociological methodology*, 31(1):361–395.
- Snijders, T. A., P. E. Pattison, G. L. Robins, and M. S. Handcock, (2006). New specifications for exponential random graph models. *Sociological methodology*, 36(1):99–153
- Wasserman, S. and K. Faust, (1994). *analysis: Methods and applications*, vol. 8. Cambridge University Press.