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Suitable Space Detecting Method for solving non-linear equations by using Numerical Differentiation

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Abstract: This paper demonstrates a new method for solving a non-linear equation by using Numerical Differentiation. Throughout the studyan iterative algorithm has been proposed to find the suitable space (where root is located) for non-linear equations and it has been discovered by numerical results that proposed algorithm needs lesser number of iterations than Bisection method and Newton Raphson method to get required accuracy. Therefore, it can be said that modified method is faster than Bisection method and Newton Raphson method. In order to compare the results, some examples have been taken related to algebraic, trigonometric and transcendental functions. The software MATLAB, C++ and MICROSOFT EXCEL are used for finding the roots and results of non-linear equations with their graphical representation. It has been discovered that modified method executes better in comparison with existing iterative method.

Keywords: Non-linear equations, Numerical Differentiation, Space location method, No. of iterations, Absolute Percentile error.

1. INTRODUCTION

Numerical analysis is very significant branch of mathematics which calculates numerical collection for resolving the problems of consecutive mathematics. If the function possesses a differential co-efficient where derivative is identified and not equal to zero, then the Newton Raphson method has an approval selection. This method is used for many complicated problems including the function of non-linear equation.

i.e. $F(x) = 0$

This method is very well known for its instant rate of convergence and very competent as compared to other methods. Nowadays, we can use calculators to calculate the non-linear functions faster. This method can be used in an effective manner to describe the internal value based on its menstruated permittivity. (Akram *et al.*, 2015), (Masoud *et al.*, 2012) used the Hybrid iteration method, New Hybrid iteration method and Mid-point Newton's method. It seems that their proposed modified Newton's method has high convergence and (Saeid., 2003) proposed a better another Adomain method for resolving non-linear equation $f(x) = 0$, with almost best convergence. (Tan. *et al.*, 2013) used the Newton Raphson method supported on Current Injection method only to re-compute the diagonal elements of its Jacobian Matrix and IEEE11 system had been applied to examine this method, and also analyzed with the conventional Newton Raphson method. Fuzzy modified Newton Raphson method was used by (Subash 2011) for solving non-linear equations and Triangular fuzzy number, non-linear equation, Newton Raphson method and Interpolation method. (RanbirSoram *et al.*, 2013) describedan estimator broadcast in Java which had been encrypted for measuring the cube root of numbers from 1 to 25 using Newton Raphson method, in the interval [1, 3]. (Cheong *et al.*, 2013) affected the conflicts between the uses of function of Scientific Calculator (Casio *fx-570ES*) and Self derivative function in solving non-linear equation by means of Newton Raphson method. (Ehiwario *et al.*, 2014) compared the pace of specific presentation among the Bisection method, Newton Raphson method and Secant method of root finding. The software, Mathematic 9.0 was utilized to find the root of the function. It has been discovered that the Bisection method converges at 52nd iteration while Newton Raphson method and Secant method converges at 8th and 6th iteration respectively. (Sangah *et al.*, 2016) described the contrast of existent Bracket out processes with modified Bracketing Algorithm for solving non-linear equations in single variable. In this paper a new bracketing formula has been germinated to discover the solution of non-linear equations. (Winai, 2016) suggested for solving polynomials of degrees higher than two. The unidentified coefficients in the decomposed polynomial of the higher degree were acquired by the n-D Newton Raphson method. (In-Won 1995) proposed that the curvature method is regenerated by the modified Newton method discoursed by Ralston in dealing with multiple roots. (Adeyemo *et al.*, 2013) demonstrated a fiction formula called Realistic Coded Genetics Algorithm Initialized Newton Raphson (GAIN) method for solving the preternatural non-linear equation qualifying harmonics in multilevel states. (Andres. *et al.*, 2016) used the formula for solving

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ordinary differential equation which has been evolving using implicit Runge Kutta method. The system of algebraic equations produced by the Runge Kutta method in each step of integration is solved with the help of Newton Raphson method. (Piotr, 2015) depicted numerical result of a rewarded concrete beam. The molding was executed with the rules of the Finite Element Method (FEM). In order to formalize the substance models, the solutions, acquired using the Newton Raphson method and (Cofarn *et al.*, 2011) exhibited a new formula that continues the famous Newton Raphson Algorithm through the comprehension of special regulation in the minimization procedure used to get the motion accumulation.

Moreover, this method demands only one iteration and the derivative evaluation per iteration. Discovering roots of non-linear equation with the help of this method furnishes good result with quick convergence speed. This method had been used by a vast number of users as they deed for equations like multinomial, rational, transcendental, and trigonometrically problems and so on. This method is also used in computers as it is a repetitive method in casual quality. And this method is used for finding the slope of functions at actual point and utilizes for the zero of tangent line. This is the way of iterating until the solution is establish. This method is much more effective than the Bisection method and other iterative methods. The glamour of this method locates in its skillfulness and veracity, a very high veracity is acquired very rapidly. Newton's method is formerly also known as Newton's iteration, because this method uses a repetitive process to converge one solution of a function.

2. PROPOSED METHOD

Looking at the Newton Raphson method which converges the nonlinear equations another modification has been done to converge those equations more rapidly ever than before to save much of our time. This modification has been done by using Numerical Differentiation. If f is a continuous function and n times differentiable in an interval [x, x + h], there exist some point in the interval, then the Taylor Series is defined as:

$$f(x_q+h)=f(x_q)+hf'(x_q)+\frac{h^2}{2!}f''(x_q)+\cdots+\frac{h^{(n-1)}}{(n-1)!}f^{(n-1)}(x)+\frac{h^n}{n!}f^n(x_q+\lambda h).$$

If f is a so-called analytic function of which the derivatives of all orders exist.

$$\lim_{n\to\infty}\frac{h^n}{n!}f^n(x_q)=0$$

The above series can be written as follows:

$$f(x_q)+hf'(x_q)+\frac{h^2}{2!}f''(x_q)+\frac{h^3}{3!}f'''(x_q)+\ \ldots=0$$

By taking the first three terms of this series and equating them to zero and the remaining terms are said to be truncation error.

$$f(x_q)+hf'(x_q)+\frac{h^2}{2!}f''(x_q)=0 \qquad \ldots\ldots(a)$$

$$\frac{2f(x_q)+2hf'(x_q)+h^2f''(x_q)}{2}=0$$

$$2f(x_q)+2hf'(x_q)+h^2f''(x_q)=0$$

$$h^2f''(x_q)+2hf'(x_q)+2f(x_q)=0.....(b)$$

Now the above equation looks like a Quadratic Equation in terms of 'h', where

$$a=f''(x_q),b=2f'(x_q),c=2f(x_q)$$

Here we need to apply quadratic formula to obtain the value of 'h'. And when we will put that value of 'h' in existing Newton Raphson formula then it will converge more rapidly than the existing Newton Raphson formula.

As in existing Newton Raphson formula,

$$x_{q+1}=x_q+h$$

Where $\qquad h=-\frac{f(x_q)}{f'(x_q)}$

After applying quadratic formula on equation (b) we get,

$$h=\frac{-f'(x_q)+\sqrt{f'(x_q)^2-2f''(x_q)f(x_q)}}{f''(x_q)}\ldots\ldots(c)$$

Here we see that the second derivative occurs. Hence for removing this here we will apply the numerical differentiation for getting the best results. As we know that

$$f'(x_q)=\frac{1}{h}\Delta f_0$$

$$f''(x_q)=\frac{1}{h^2}\Delta^2 f_0$$

Put these formulas in eq. (c)… then we get…

$$h=\frac{\frac{-1}{h}\Delta f_0+\sqrt{\frac{1}{h^2}\Delta^2 f_0-2\times\frac{1}{h^2}\Delta^2 f_0\times f(x)}}{\frac{1}{h^2}\Delta^2 f_0}$$

$$h=\frac{\frac{-1}{h}\Delta f_0+\sqrt{\frac{1}{h^2}\Delta^2 f_0(1-2f(x)}}{(\frac{1}{h}\Delta f_0)^2}$$

$$h = \frac{\frac{-1}{h}\Delta f_0 + \frac{1}{h}\Delta f_0\sqrt{(1-2f(x))}}{(\frac{1}{h}\Delta f_0)^2}$$

$$h = \frac{\frac{1}{h}\Delta f_0[-1+\sqrt{(1-2f(x)))}}{(\frac{1}{h}\Delta f_0)^2}$$

$$h = \frac{-1+\sqrt{(1-2f(x))}}{\frac{1}{h}\Delta f_0}$$

$$h = \frac{-1+\sqrt{(1-2f(x))}}{f'(x)}$$

Now we put the above value of 'h' in following formula:

$$x_{n+1} = x_n + h$$

Where n= 0,1,2,3,…
then

$$x_{n+1} = x_n + \frac{-1+\sqrt{(1-2f(x)}}{f'(x)}$$

This is the modified method which is analyzed on some non-linear problems for getting results. Some quadratic, trigonometric, algebraic and transcendental problems are analyzed using modified (proposed) formula. And it seems that the modified method converges faster than the existing Newton Raphson method.

3. RESULT AND DISCUSSION

In order to explain the developed method, it is used on some examples containing algebraic, trigonometric and transcendental functions and compared with the Bisection Method (B.M), Regula-Falsi Method (R.F.M) and Newton Raphson Method (N.R.M) respectively. We find from problem 1, 3 and 4 that B.M, R.F.M and N.R.M are taking 23,5 and 4 iterations in problem 1 and 26,20 and 7 iterations in problem 3 and 27,100 and 4 iterations in problem 4 while, the proposed method is taking only 3 iterations in problem 1 and 4 iterations in problem 3 and 2 iterations in problem 4 respectively. So, by looking at the case of problems 1, 3 and 4 the proposed method is performing far better than the other three methods with respect to number of iterations and accuracy. Trigonometric problem has accuracy A.E% = 0.0000596046, A.E% = 0.00004 and A.E% = 0.00000119209 respectively in Problem 1 **(Table 1),** while the proposed method has taken accuracy A.E = 0.00000112 **(Table 1, Fig 1)**. Similarly, for the Algebraic function i-e 3rd problem has accuracy in other three methods A.E% = 0.00000119209, A.E% = 0.00001 and A.E% = 0.0000238419 respectively while proposed method has taken accuracy 0.0000059 **(Table 1, Fig 2)** and finally in the exponential function i-e 4th problem has taken accuracy A.E% = 0.0000298023, A.E% = 0.000091 and A.E% = 0.000140071 respectively, while the proposed method has taken accuracy A.E% = 0.00000437 **(Table 1, Fig 3)**. Table-1 and figures which distinctly exhibits that proposed method is not only taking lesser number of iterations but also gives less A.E% than other existing methods.

Function	Methods	No: of Iteration	X	A E %
Sinx-x/2 (-3, 2`)	B Method R F Method N R Method Modified method	23 5 4 3	**1.89549**	0.0000596046 0.00004 0.000000119209 0.000000119209
x-cosx (0.5`, 2)	B Method R F Method N R Method Modified method	25 9 8 3	**0.739085**	0.0000596046 0.000001 0.0000565052 0.00000223041
x^3 -2x+2 (-2`, 5)	B Method R F Method N R Method Modified method	26 20 7 4	**1.76929**	0.0000119209 0.00001 0.0000238419 0.00000599623
x^2 – e^x-3x+2 (-2, 0.5`)	B Method R F Method N R Method Modified method	27 100 4 2	**0.25753**	0.0000298023 0.000091 0.000140071 0.00000437796

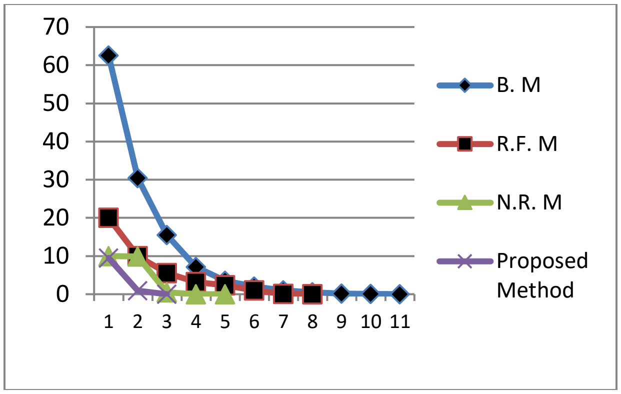

Fig.1. Comparison of accuracy analysis for the case of problem 1

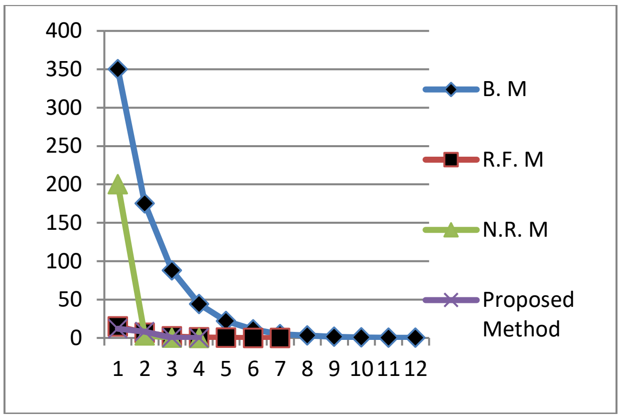

Fig.2. Comparison of accuracy analysis for the case of problem 3

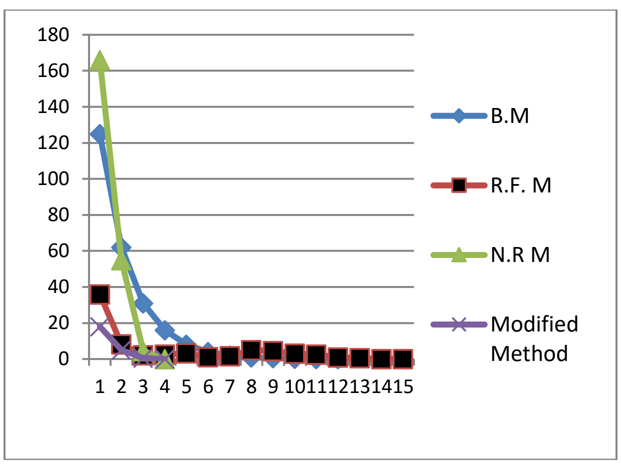

Fig.3. Comparison of accuracy analysis for the case of problem 4

4. CONCLUSION

In this study, based on our results and discussion we now decide that the proposed method is formally the most efficient than other three existing methods we have considered here in the study. It has been detected that solving a non-linear equation $F(x) = 0$ by proposed method, number of iterations is decreased with better accuracy **(Table-1)** in comparison with other existing methods including Bisection method, Regula-Falsi method and Newton Raphson method. Hence it is concluded that the proposed method is executing better than other existing methods which have already discussed in the study.

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