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Determination of Approximated Root of Nonlinear Equation by Interpolation Technique

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Abstract: This beautiful universe is full of mathematical and engineering problems involving nonlinear equations $f(x) = 0$. The main theme of this paper is to develop an Algorithm to enhance the speed and convergence of bracketing methods for determining the root of nonlinear equations. For this cause, Newton forward interpolation difference formula and Bisection Method are recruit. Developed interpolation technique method guarantees that it converges towards thereal root faster than Bisection Method and RegulaFalsi Method. Few numerical examples are also conferred in this paper to inspect the efficiency of developed method and compared with other existing methods. It is examined from the results that the performance of developed method is better than the existing methods such as Bisection Method and RegulaFalsi Method.

Keywords: Bracketing Methods, Nonlinear Equations, Convergence, Newton Forward Interpolation and Error.

1. INTRODUCTION

To solve nonlinear equation a lot of numerical methods are available such as Bisection, RegulaFalsi, Newton Raphson method and Aitken's method. Bisection is the most famous method. In this method interval is broken into two equal parts. This method always converges to the root. Drawback of this method is that it converges very slowly. RegulaFalsi is an alternative method. It is more efficient than Bisection. It converges to root faster than Bisection method. There are many cases where its performance is very poor. In such cases, Bisection method works better than it. Newton Raphson's method is world famous method. Bisection and RegulaFalsi two points are joined and a chord is made on the graph of $y = f(x)$, while Newton Raphson uses to tangent at one point on the graph $y = f(x)$therefore it requires only one point instead of two initial guesses. It converges to root faster than Bisection and Regula-falsi. Drawback of this method is that at any point of computation if $f'(x) = 0$, that is, the tangent is parallel to the x-axis then this method will not work at all. Furthermore, if the initial point lies away from the real root, the convergence will be slow. Moreover, Simple Iteration Method (Fixed Point Method) is familiar method. It involves rearranging or transforming the equation $f(x) = 0$ in the form $x = g(x)$.But the restriction on this transformation is that there must be a one to one relationship with solutions to $x = g(x)$for root being stored. Next is Secant Method. It requires two initial points but sign variation is not important because it does not belong to family of bracketing methods. A small changing of Newton's method for functions whose derivatives are difficult to evaluate for such conditions the derivative can be approximated by a backward finite divided difference. Secant Method has same pitfall as Newton's Method and that convergence is not guaranteed for all x_0. Method titled as A new method by Taylor expansion was proposed by (Allame*et al*., 2015). Problems containing partial and non-partial transcendental terms can be solved by this method.On Modified Method by mid-point" (Allame & Azad, 2012) plays a vital role to solve nonlinear equations. It depends upon the famous mid-point Newton's Method. A powerful method named as New Second Order Iteration Method is introduced by (Kang *et al*., 2013). Functional Equations can be solved by this method. Derivative free iterative Methods were intended by (Shah & Noor, 2015). Its result is at least better than Stephenson Method. Various Newton Type iterative methods were created by (Kumar, *et al.*, 2013). Those methods were Ninth and seventh order convergent. A new method is proposed by (Solanki, 2014) with the help of Bisection method. As Bisection guaranteed to converge to a root of $f(x)$ if $f(x)$ is continuous function in an interval [a,b] this method also guaranteed to converge. By using decomposition technique a family of iterative methods is introduced by (Shah & Noor, 2015) contains many familiar iterative methods as exceptional case. To increase the speed of fixed point method (Ali *et al*. , 2015)introduced Modified Two-Step Fixed Point Iterative Method. Its order of convergence is 2. It converges faster than Fixed Point Method. A new third order iterative method was established by (Kang, *et al*., 2014). 1.4422 is its efficiency index which is also efficiency index of Halley's and Householder methods. To solve the nonlinear functional equations a new fixed point method has been discovered by (Ahmad *et al*., 2015). Its convergence and performance is better than fixed point method and its order of convergence is two. Modified Bracketing Algorithm was purposed by(Sangah *et al*., 2016). It is faster than other bracketing method such as Bisection Method and Regula-falsi Method. A multi-step derivative free method was introduced by (Soomro *et al*., 2016). It is derivative free method. It depends on Newton Raphson Method and Taylor Series. It has no any pitfall. Hybrid closed algorithm was developed by (Siyal*et al*., 2016). It performs better than existing closed methods in terms of iteration and convergence and get better efficiency index. Stand form of Van Der Wall equation can also be solved by this developed method. It is equally applicable to all class of problems.

2. PROPOSED METHOD:

The new method will be developed by using,

1. **Newton Forward Interpolation difference formula**

$$\boldsymbol{y_c = y_0 + p\Delta y_0 + \frac{1}{2!}p(p-1)\Delta^2 y_0 + \cdots}$$

Where

$$\boldsymbol{p = \frac{x - x_0}{h}}$$

2. **Bisection Method**

$$\boldsymbol{x = \frac{1}{2}(c + d)}$$

Where c is lower limit of chosen class interval and d is upper limit of class interval.

Let $y(x)$ is a nonlinear equation and it is interested to find its approximated root. First of all, we will find interval in which root lies by incremental search method

$$\boldsymbol{f(a).f(b)} < 0$$

let $[a, b]$ is required interval. We will find midpoint of that interval suppose m is midpoint of interval.

$$\boldsymbol{m = \frac{a+b}{2}}$$

Now we will find value of given real valued function at points$a, b\ and\ m$.Let $y_a, y_m and\ y_b$ are values of given nonlinear equation. Make Newton forward interpolation table of these values, apply newton forward interpolation on the three values $y_l, y_m and\ y_u$ then apply formula of Newton forward interpolation

$$\boldsymbol{y_c = y_0 + p\Delta y_0 + \frac{1}{2!}p(p-1)\Delta^2 y_0 + \cdots}$$

And put $y_c = 0$.

$$\boldsymbol{0 = y_0 + p\Delta y_0 + \frac{1}{2!}p(p-1)\Delta^2 y_0 + \cdots}$$

After putting $y_c = 0$ it will become quadratic equation in variable p

$$\boldsymbol{\Delta^2 y_0 p^2 + (2\Delta y_0 - \Delta^2 y_0)p + 2y_0 = 0}$$

Find general roots of 'p' by applying quadratic formula. Once we find the roots of 'p'then according to newton forward interpolation we know that

$$\boldsymbol{p = \frac{x - x_0}{h}}$$

After that we will put roots of p in $p = \frac{x-x_0}{h}$ then we will simplify the formula for the x.In the end we will get a formula

$$\boldsymbol{x = x_0 + \frac{h\left(-B \pm \sqrt{B^2 - 4AC}\right)}{2A}}$$

Which will give us the approximated root of given function. Repeat process until we get the best approximate root. In the end compare results with existing methods.

3. RESULT AND DISCUSSION

For analyzing Interpolation Technique Method (ITM), some problems involving nonlinear equation(Algebraic and transcendental) have been solved by Interpolation Technique Method (ITM). Its results are also compared with Bisection Method and RegulaFalsi Methods, observe Table-1. It has been observed from Table that in the case of Algebraic Function Bisection Method and RegulaFalsi Method take 19 and 20 number of iteration while Interpolation Technique Method takes only 4 number of iterations. So, in case of Algebraic Function ITM performs better than BM and RFM It gives better accuracy, function value and Relative Error percent are -3×10^{-7} and 1×10^{-4} respectively. In exponential case two problems have been solved in first problem BM and RFM take 20 and 6 number of iteration while ITM takes 5 iterations. Functional Value and Relative Error percent are $-7 \times 10^{-6}$and 7×10^{-2}. In second problem BM and RFM take 17 and 11 iterations while ITM takes 3iterations. Functional Value is 8×10^{-8} and RE% is 0.0108. So, in the case of Exponential Function ITM performs better far than other two methods. In case of logarithm function BM and RFM take 15 and 4 iterations while ITM takes 3 iterations and give better accuracy as compare to other two methods. Functional value and RE% are $-2.6 \times 10^{-8}$and 8.1×10^{-5}, so in logarithm case ITM performs better than BN and RFM. In the last, Trigonometric function has been taken. In this case BM and RFM take 13 and 3 iterations while ITM takes 2 iterations and gives better accuracy in order of functional value and RE%. Functional value and RE% are 5×10^{-10}and5.7×10^{-4}. The table clearly

Table-1

Functions	Methods	Iterations	x	$f(x)$	R.E%
$f(x) = x^5 - 8x^4 + 44x^2 - 91x^2 + 85x - 26$	Bisection	18	0.557026	0.000008	0.000685
	R.F.M	20	0.557038	0.000235	0.001562
	I.T.M	4	0.557025	-0.000000304	0.0001796
$f(x) = 10xe^{-x^2} - 1$	Bisection	20	0.101027	0.000007	0.00188
	R.F.M	6	0.101026	0.000001	0.004373
	I.T.M	5	0.101025	-0.000007	0.007952
$f(x) = cosx - xe^x$	Bisection	17	0.517754	0.000011	0.001474
	R.F.M	11	0.517754	0.000009	0.001243
	I.T.M	3	0.5177573	0.000000084	0.0108
$f(x) = xlogx - 1.2$	Bisection	15	2.740631	-0.000013	0.0100
	R.F.M	4	2.740646	-0.0000001	0.007059
	I.T.M	3	2.740646	-0.000000026	0.000081
$f(x) = sin\sqrt{x} + cos(1 + \sqrt{x})$	Bisection	13	3.445435	0.000004	0.003543
	R.F.M	3	3.445475	-0.000003	0.015763
	I.T.M	2	3.44545798	0.0000000005	0.00057

Shows that ITM takes less number of iterations and gives better accuracy than BM and RFM.

4. CONCLUSION

From the above results it is concluded that Interpolation Technique Method is better than Bisection Method and RegulaFalsi Method. No any pitfall has been observed in the developed technique and it has also good privilege that, it always converges faster than other bracketing methods.

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