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Super Linear Iterated Method for Solving Non-Linear Equations

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Abstract: In this paper a super linear iterated method has been suggested for solving non-linear equations. The proposed super linear method is very much effective and convenient for solving non-linear equations, and it is a derivative free two-point method. The proposed iterated method is derived from Newton Raphson Method and Taylor Series. We have observed in numerical outcome is that the super line a rmethod is rapidly converge with the assessment of Bisection Method, Regula-Falsi Method and Secant Method. Its hypothetical out comes and efficacy is inveterate by Numerical problems. Throughout the study, it has been perceived that the developed super linear algorithm is a decent attainment for estimating a single root of nonlinear equations.

Keywords: Non-Linear Equations, two-point methods, number of iterations, rate of convergence, accuracy

1 INTRODUCTION

Numerous applications in Science and Engineering such as (Iwetan, 2012), (Biswa, 2012) and (Golbabai, 2007)contains the nonlinear functions of Algebraic and Transcendental nature in the equation of the form

$$f(x) = 0 \qquad --- (A)$$

For solving (A), lots of method shadesignated and analyzed in literatures for solving nonlinear equations. Most of the cases analytical methods have failed to find the exact root. Due to that reason numerical methods have played a vital role for solving of nonlinear equations. Numerical analysis is a very important branch Mathematics that deals with the study of algorithms that use numerical approximation in mathematical analysis. The utmost elementary numerical technique is bisection technique with a rather slow convergence (Solanki, 2014). Alternative elementary root solving technique is a Falsi position method. Mutually techniques are a linear order of convergence, buttheregula-falsimethod suffers due to the slow convergence in some cases. On the other hand, the secant technique likewise Regula Falsi technique for estimating a single root of nonlinear problems. Secant technique is healthier than Newton's technique when equations well-defined to real line (Argyros, 2007). Relative work of many technique shad deliberated in which is settled that secant technique by (Ehiwario, 2014). Consequently, many modifications in bisection method, regula-falsi method and secant method have been proposed in literature (Siyal, 2017), (Tanakan, 2013) and (Alberto, 2015), these methods are two-point methods and very essential methods for solving non-linear equations. Furthermore, to increase reliability varioustwo-point numerical methods have been suggested by using quadrature formula and finite difference (Hafiz, 2012) and (Noor, 2010). In recently investigation, without Derivative Techniques have been developedby using Taylor series and difference operatorfor Solving nonlinear equations by (Soomro, 2016) and (Qureshi, 2017).In the bright of overhead investigation, this paper has been discussed a super linear method. The proposed iterated method is fast converging to approaching the root and free from pitfall. The proposed method has given a theoretical analysis of the behavior and reports some numerical experience with the process.

2. SUPER LINEAR METHOD

The proposed super linear iterated method is developed by using Taylor's series,

$$f(x_n + h) = f(x_n) + hf\,'(x_n)\,h\,f\,''(x_n)/2! \\ + hf\,'''(x_n)/3! + \ldots\ldots$$

Truncating the Taylor's series up to two terms, such as

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$$f(x_n + h) = f(x_n) + hf'(x_n)$$

By expanding truncated Taylor's series in points of $(x_{n-1} + h, x_n)$ and (x_{n-1}, x_n), we have

$$f(x_{n-1} + h) = f(x_n) + hf'(x_n) \qquad (1)$$

and

$$f(x_{n-1}) = f(x_n) + hf'(x_n) \qquad (2)$$

subtracted (3) by (2), we get

$$f'(x_n) = \frac{f(x_{n-1} + h) - f(x_{n-1})}{h} \qquad (3)$$

(3) can also be written as

$$(x_n) = \frac{f(x_{n-1} + f(x_{n-1})) - f(x_{n-1})}{f(x_{n-1})} \qquad (4)$$

Using (4) in well known Newton Raphson Method, then the new method is formulated as,

$$x_{n+1} = x_n - \frac{f(x_{n-1})}{f(x_{n-1} + f(x_{n-1})) - f(x_{n-1})} f(x_n)$$

Hence this is the super linear iterated method.

3. RATE OF CONVERGENCE

This segment has been shown that the new iterative method has super linear Convergence.

Proof:

By using Taylor series for estimating $f(x_n), f(x_{n-1} + f(x_{n-1}))$ and $f(x_{n-1})$ with ignoring higher order term, that is

$$f(x_n) = f(a) + e_n f`(a) + e_n{}^2 \frac{f``(a)}{2} \quad --- \text{ (i)}$$

$$f(x_{n-1} + f(x_{n-1})) = f(a) + (e_{n-1} + f(x_{n-1})) f`(a) + (e_{n-1} + f(x_{n-1}))^2 \frac{f``(a)}{2} \quad --- \text{ (ii)}$$

$$f(x_{n-1}) = f(a) + e_{n-1} f`(a) + e_{n-1}{}^2 \frac{f``(a)}{2} --- \text{ (iii)}$$

(Note that $f(a) = 0$ and $c = \frac{f``(a)}{2f`(a)}$

By using (ii) & (iii), we obtain

$$f(x_{n-1} + f(x_{n-1})) - f(x_{n-1}) = f(x_{n-1}) f`(a)[1 + (2e_{n-1} + f(x_{n-1}))c] \quad --- \text{ (iv)}$$

Now using (ii) & (iv),

$$\frac{f(x_{n-1} + f(x_{n-1})) - f(x_{n-1})}{f(x_{n-1})} = f`(a)[1 + (2e_{n-1} + e_{n-1} f`(a))c] \quad (v)$$

Substitute in (i) and (v) in New Method, we get

$$e_{n+1} = e_n - \frac{e_n f`(a)(1 + e_n c)}{f`(a)[1 + (2e_{n-1} + e_{n-1} f`(a))c]}$$

$$e_{n+1} = e_n - e_n (1 + e_n c)(1 + (2e_{n-1} + e_{n-1} f`(a))c)^{-1}$$

$$e_{n+1} = e_n - e_n (1 + e_n c)(1 - e_{n-1}(2 + f`(a))c)$$

$$e_{n+1} = e_n - e_n (1 - e_{n-1}(2 + f`(a))c + e_n c)$$

$$e_{n+1} = c e_n e_{n-1}(2 + f`(a)) + c e^2{}_n \quad --- \text{ (vi)}$$

Thus(vi) at leastlead,

$$e_{n+2} = c e_n[e_{n-1}(2 + f`(a)) + e_n] \quad --- (vii)$$

Todetermine`p`by using lemma

$$e_{n+1} = C e^p{}_n \text{Similarly, } e_n = C e^p{}_{n-1} \quad -- \text{ (viii)}$$

From equation (vii), we get

$$\frac{e_{n+1}}{e_n[e_{n-1}(2 + f`(a)) + e_n]} = \boldsymbol{c}$$

By using(viii),

$$\frac{C e^p{}_n}{C e^p{}_{n-1}[e_{n-1}(2 + f`(a)) + C e^p{}_{n-1}]} = \boldsymbol{c}$$

$$\frac{C^p e_{n-1}{}^{p^2}}{e_{n-1}{}^{p+1}[(2 + f`(a)) + C e_{n-1}{}^{p-1}]} = \boldsymbol{c}$$

$$\frac{\boldsymbol{C^p} e_{n-1}{}^{\boldsymbol{p^2-p-1}}}{(2 + f`(a))} = \boldsymbol{c} \quad --- \ (\boldsymbol{ix})$$

The Relation (ix) can be only satisfied if,

$$p^2 - p - 1 = 0$$

Which has the solution 1.618, this has proven that the Proposed iterated method has super linear convergence of order 1.618.

4. NUMERICAL RESULTS

In this section, the developed method is applied on few examples of nonlinear equations. From the numerical result of table-1, it has been observed that the super linear method is reducing the number of iterations which is less than the number of iteration of two-point methods and likewise accuracy perspective. C++ programming used to justify the results of proposed Method. The new method is compared with Bisection Method, Regula-Falsi Method and Secant Method, such as in Table-1
Method, such as in **(Table-1)**.

Table-1

FUNCTIONs	METHODS	ITERATIONS	x_n	$\lvert f(x_n)\rvert$
Sinx-x+1 (1,2)	Bisection Method	19	1.93456	0.0000092049
	Regula-Falsi Method	4		0.0000179110
	Secant Method	4		0.0000105607
	New Method	4		0.0000092049
e^x-3x^2 (0,1)	Bisection Method	20	0.910008	0.0002452918
	Regula-Falsi Method	6		0.0001857727
	Secant Method	5		0.0001262549
	New Method	5		0.0001262549
2x-lnx-7 (4,5)	Bisection Method	18	4.21991	0.0000061978
	Regula-Falsi Method	3		0.0000061978
	Secant Method	3		0.0000061978
	New Method	3		0.0000062178
x^3+9x-1 (0,1)	Bisection Method	22	0.110959	0.0000061530
	Regula-Falsi Method	6		0.0000119210
	Secant Method	4		0.0000119210
	New Method	4		0.0002411140
e^{-x}-cosx (4,5)	Bisection Method	18	4.72129	0.0000073053
	Regula-Falsi Method	4		0.0000027833
	Secant Method	4		0.0000027833
	New Method	4		0.0001677143

5. CONCLUSION

The present study has presented a super linear iterated method for resolving nonlinear problems. The proposed iterative method is keeping1.618 order of convergence. The results shown here that the new super linear method has per formed supercilious through the other well-known and commonly useful two-point methods. Hence forth, the super linear method seems to be very easy and virtuous achievement for solving nonlinear equations with the assessment of existing two-point methods.

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