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Performance Analysis of Linear Precoders in TDD Massive MIMO Systems

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Abstract: The channel estimation of the reverse link and forward link in the massive multiple-input-multiple-output (M-MIMO) systems is projected using reverse link pilot training and channel reciprocity of time-division duplex (TDD) protocol, respectively. Nevertheless, the utilization of non-orthogonal reverse link pilot sequences in the neighboring cells, owing to limited coherence time causes pilot contamination that results in inter-cell interference in forward link data transmission. To design a TDD M-MIMO system properly under a practical multi-cell environment, it is necessary to evaluate the achievable rate of such systems. The performance assessment of multi-cell TDD M-MIMO forward-link system with linear precoders is proposed. The reverse-link and forward-link channels are first estimated by the BSs using least square channel estimator. Later, the closed-form approximate achievable forward-link sum-rate expressions of M-MIMO system with matched-filter (MF) and zero-forcing (ZF) precoders are derived. Finally, the performances of the achievable forward-link sum-rate for MF and ZF precoders are compared with respect to cell size, number of BS antennas, pilot length, and transmission SNR. Numerical simulation outcomes support the precision of closed-form rate expressions.

Keywords: Massive MIMO; time-division duplex; pilot contamination; linear precoders

1. INTRODUCTION

The multimedia data traffic is increasing at an expeditious rate owing to the fast adoption of smartphones, tablets, and laptops. According to the forecast update of Cisco visual networking index, the amount of data transmitted by cellular communication networks worldwide will be 15.9 Exabyte per month by 2018. This is an increase of 81% to that of 2013. A straightforward solution to counter this increase is to purchase new spectrum. However, radio frequency spectrum is limited and very expensive. Massive MIMO (M-MIMO) systems seem to be the best solution to counter spectrum paucity, harsh wireless conditions and ever-increasing demand of good quality of service to users. M-MIMO is a break-through technology that has the capability to revolutionize the wireless communication systems. The extremely huge vector dimension of the signal at the M-MIMO base station (BS) fosters little complexity algorithms, such as, linear detectors based on maximal-ratio combining (MRC), zero-forcing (ZF) or MMSE for reverse-link and linear precoding/beamforming for instance matched-filter (MF) and ZF for the forward-link.Nevertheless, as number of antennas at the BS increases, linear precoding and detection methods such as MF and ZF become most favorable (Rusek *et al.,* 2013), (Ngo, *et al.* 2013a), (Hoydis, *et al.,* 2013), (Yang andMarzetta, 2013), (Jose, *et al.,* 2011).

To design a TDD M-MIMO system properly under a practical environment of multi-cell, it is necessary to evaluate achievable rate of such systems. There exist achievable rate bounds of reverse-link systems in literature, however common closed-form rate bounds for multi-cell TDD M-MIMO forward-link system are very few. Therefore, this paper studies a practical multi-cell TDD M-MIMO forward-link system and drives general closed-form rate expressions for both MF and ZF beamformers. Related works on achievable rate bounds (Hoydis *et al.,* 2013), (Yang andMarzetta, 2013), (Jose *et al.,* 2011), (Jubin *et al.,* 2011)take a specific receiver structure that considers channel mean as its effective channel for data detection in the forward-link scenario. In (Jubin *et al.,* 2011; Yang andMarzetta, 2013), single cell forward-link systems is addressed. Where as, authors in (Yang and Marzetta, 2013) compares the two multifarious well-known linear precoders, conjugate beam forming, and ZF, with respect to net forward-link sum-rate and radiated energy-efficiency in a simplified single-cell scenario where propagation is governed by independent Rayleigh fading, and where CSI acquisition and data transmission are both performed during a short coherence interval. Authors in (Ngo, *et al.,* 2013b) consider single cell forward-link transmission for MRC and ZF precoding with average transmit power normalization and forward-link pilot. Rate derivation for both reverse and forward-link of multi-cell TDD systems in (Hoydis *et al.,* 2013) is based on asymptotic rate analysis when numbers of BS antennas and users go to infinity, however the investigation presented in this paper is for BSs with finite number of antennas and finite number of users.

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Furthermore, (Hoydis *et al.,* 2013) forward-linkutilizes average total transmit power normalization. Multi-cell TDD forward-link scenarios were considered in (Jose *et al.,* 2011) and it proposes a new multi-cell MMSE-based precoding scheme that mitigates pilot contamination dilemma. Besides being linear, this precoding scheme has a straightforward closed-form expression, which outcomes from an instinctive optimization. Recently, (Khansefid and Minn, 2015) studied M-MIMO forward-link system with or without pilot-aided coherent detection considering scenario with pilot contamination, and derived closed-form estimated achievable forward-link rate expressions for MRC and ZF precoders

A grave impairment that limits the performance of M-MIMO systems as mentioned in (Marzetta, 2010), (Lu, *et al.,* 2014) is pilot contamination. M-MIMO is considered a Random Matrix Theory and Wireless Communication senabler technology for future broadband networks and it has many advantages over existing systems (Larsson, *et al.,* 2014). However, it is only possible with M-MIMO systems without pilot contamination. A review of available techniques is detailed in (Elijah, *et al.,* 2016). Consequently, pilot contamination will eventually be eliminated once and for all from M-MIMO systems. The important question is the performance of M-MIMO systems under practical environments. Can M-MIMO deliver what it is supposed to offer? As achievable rate expressions or their bounds play an important metrics in evaluating performance of various M-MIMO scenarios. Therefore, a performance assessment of multi-cell TDD M-MIMO forward-link system with linear precoders is proposed in this paper. The reverse-link and forward-link channels are first estimated by the BSs using least square channel estimator. Later, the closed-form approximate achievable downlink sum-rate expressions of M-MIMO system with MF and ZF precoders are derived. The performance analysis presented in this paper is for BSs with finite number of antennas, finite number of users, and employs multi-cell environment.

The notations used in this paper are as under. The **boldface** variables signify the *matrices* and *vectors*. Transpose and the Hermitian transpose are indicated by**(.)$^{\mathbf{T}}$** and **(.)$^{\mathbf{H}}$**, respectively. A **diag{d}** symbolizes a diagonal matrix with diagonal entries equal to components of vector **d** and '*' indicates element-wise multiplication. The trace and inverse operations are represented by **tr{.}** and **(.)$^{\mathbf{-1}}$**, respectively. Two-norm, expectation, and variance are symbolized as **||.||**, **E[.]**and **var{.}**, respectively.

The paper is organized as follows.Section 2 presents the system model and design. Section 3 describes the forward link data transmission, whereas the simulation results of the model are shown in Section 4 along with the results and discussion. Finally, we conclude the work in Section 5

2. SYSTEM MODEL

A cellular network consisting of C hexagonal cells with unity frequency reuse (UFR) is considered. Each cell has a single BS with L antennas at its center and U single-antenna mobile stations (MSs) that share the same bandwidth. Furthermore, r_c and r_n are the cell radius (from center to vertex) and central disk radius, respectively.

2.1 Propagation Model

Orthogonal Frequency Division Multiplexing (OFDM) is employed and flat-fading channel model is considered for each subcarrier of OFDM. The channel vector, for a given subcarrier, connecting l-th BS antenna of q-th cell and u-th (MS) of r-th cell is,

$$g_{lqur} = \sqrt{\beta_{qur}}\, h_{lqur},\ u = 1, 2, ..., U, \quad \text{and } q, r = 1, 2, ..., C \tag{1.}$$

where $h_{lqur} \sim CN(0,1)$ channel vectors model small-scale fading and are considered to be statistically independent across MSs. Whereas, β_{qur} channel vectors model large-scale fading that includes both geometric attenuation and shadow fading as

$$\beta_{qur} = \frac{\kappa_{qur}}{\left(d_{qur} / r_n\right)^{\nu}} \tag{2.}$$

where d_{qur} is the distance among u-th MS of r-th cell and BS of q-th cell, ν represents decay exponent, and κ_{qur} is long-normal random variable with standard deviation σ_{shadow}, as elucidated in (Khansefid and Minn, 2015).

Moreover, TDD operation and perfect synchronization between BSs and MSs for each symbol are also considered.**(Fig.1)**shows the used TDD protocol, where channel vectors remainconstantfor T symbols. Furthermore, frequencyblock fading model is also considered. In frequency block fading model, channel vectors remainconstant for N_{block} consecutive subcarriers, i.e.coherence bandwidth.

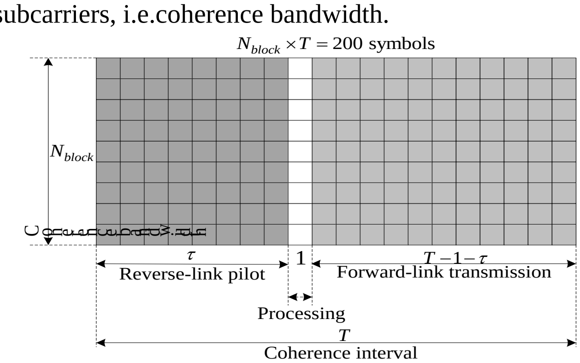

Fig. 1: Employed TDD protocol

2.2 Channel Estimation

Consider, $\sqrt{\tau}\chi_{uc}$ (normalized such that $\chi_{uc}^{\mathrm{H}}\chi_{uc}=1$) is the reverse-link pilot sequence of length τ transmitted by *u*-th MS in *c*-th cell and it is given as $\chi_{uc}=[\chi_{uc}^{[1]} \quad \chi_{uc}^{[2]} \quad \chi_{uc}^{[3]} \quad \ldots \quad \chi_{uc}^{[\tau]}]^{\mathrm{T}}$, where $\chi_{uc}^{[b]}$ is an element of the pilot sequence. Furthermore, all the available pilot sequences are mutually orthogonal within a cell; however reuse of pilot sequences across the neighboring cells sources pilot contamination. Consequently, the *l*-th antenna of *q*-th BS receives signal (Jose *et al.*, 2011), (Yin, *et al.*, 2013), (Zhang *et al.*, 2014),

$$\mathbf{y}_{lq}=\sum_{u=1}^{U}\sqrt{\rho_{re}\tau\beta_{quq}}\,h_{lquq}\chi_{uq}+\sum_{c=1}^{C}\sum_{u=1}^{U}\sqrt{\rho_{re}\tau\beta_{quc}}\,h_{lquc}\chi_{uc}+\mathbf{v}_{lq} \tag{3.}$$

where ρ_{re} is the average power of MS and $\mathbf{v}_{lq}$ is additive white Gaussian noise (AWGN) with independent identically distributed (i.i.d), zero mean and unit variance (CN (0, 1)) random variable entries. Equation (3) can also be constituted for all the signals received at all antenna elements of the *q*-th BS, as

$$\mathbf{Y}_q=\underbrace{\sqrt{\rho_{re}\tau\mathbf{D}_{qq}}\mathbf{H}_{qq}\mathbf{Z}}_{\text{Desired Term}}+\underbrace{\sum_{c=1}^{C}\sqrt{\rho_{re}\tau\mathbf{D}_{qc}}\mathbf{H}_{qc}\mathbf{Z}}_{\text{Inter-cell Interference}}+\mathbf{V}_q \tag{4.}$$

where $\mathbf{Y}_q=[\mathbf{y}_{1q} \quad \mathbf{y}_{2q} \quad \ldots \quad \mathbf{y}_{Lq}]_{\tau\times L}$, $\mathbf{V}_q=[\mathbf{v}_{1q} \quad \mathbf{v}_{2q} \quad \ldots \quad \mathbf{v}_{Lq}]_{\tau\times L}$, $\mathbf{Z}=[\chi_1 \quad \chi_2 \quad \ldots \quad \chi_U]_{\tau\times U}$ satisfying constraint $\mathbf{Z}^{\mathrm{H}}\mathbf{Z}=\mathbf{I}$, as all the BSs have the same set of reverse-link pilot sequences, therefore we dropped BS index, $\mathbf{D}_{qc}=\mathrm{diag}\{[\beta_{q1c} \quad \beta_{q2c} \quad \ldots \quad \beta_{qUc}]\}$, and $\mathbf{H}_{qc}=\begin{bmatrix} h_{1q1c} & \cdots & h_{Lq1c} \\ \vdots & \ddots & \vdots \\ h_{1qUc} & \cdots & h_{LqUc}\end{bmatrix}$.

The *q*-th BS, after receiving the signal $\mathbf{Y}_q$, estimates channel vectors $\hat{\mathbf{G}}_{qq}$ for all users placed in same cell using LSestimate that results in (Zhang *et al.*, 2014)

$$\hat{\mathbf{G}}_{qq}=\mathbf{Y}_q\mathbf{Z}^{\mathrm{H}}(\mathbf{Z}\mathbf{Z}^{\mathrm{H}})^{-1}=\mathbf{Y}_q\mathbf{Z}^{\mathrm{H}} \tag{5.}$$

$$\hat{\mathbf{G}}_{qq}=\underbrace{\sqrt{\rho_{re}\tau}\mathbf{G}_{qq}}_{\text{Desired Signal}}+\underbrace{\sum_{c\neq q,c=1}^{C}\sqrt{\rho_{re}\tau}\mathbf{G}_{qq}}_{\substack{\text{Interference signal owing to}\\\text{pilot contamination}}}+\mathbf{V}_q\mathbf{Z}^{\mathrm{H}} \tag{6.}$$

The channel estimation (CE) clearly suffers from pilot contamination, which consequently will result in forward-link interference owing to the contaminated estimation of the channel vectors.

3. FORWARD-LINK DATATRANSMISSION

Now, if pilot contamination cannot be eliminated from M-MIMO systems, then all its advantages over existing wireless communication systems will be in vain (Elijah *et al.*, 2016). Therefore, an extensive research has been carried out to address this stern pilot contamination problem and there exist many pilot contamination elimination schemes in the literature. As a result, pilot contamination will eventually be eliminated once and for all from M-MIMO systems.Therefore, LS channel estimation of the channel vectors $\mathbf{G}_{qq}$ considered hereafter is free from pilot contamination, that is

$$\hat{\mathbf{G}}_{qq}=\sqrt{\rho_{re}\tau}\mathbf{G}_{qq}+\mathbf{V}_q\mathbf{Z}^{\mathrm{H}} \tag{7.}$$

The estimated reverse-link CE will be processed to acquire the forward-link CE for forward-link data transmission through the pre-coding matrix. Let the data symbols transmitted by the BS of the *q*-th cell to its MSs are $\mathbf{s}_q=[s_{1q} \; s_{2q} \; \ldots \; s_{Uq}]^{\mathrm{T}}$ and the $L\times U$ precoding matrix is $\mathbf{P}_q=\Re(\hat{\mathbf{G}}_{qq})$ where $\Re(.)$ denotes a particular precoding method employed in the system. Hence, the forward-link signal transmitted byBS of*q*-th cell is $\mathbf{P}_q\mathbf{m}_q$ and we consider this forward-link signal satisfy $\mathrm{E}\left[\mathbf{s}_q\right]=0$, $\mathrm{E}\left[\mathbf{s}_q\mathbf{s}_q^{\mathrm{H}}\right]=\mathbf{I}$ and $\mathrm{tr}\left\{\mathbf{P}_q^{\mathrm{H}}\mathbf{P}_q\right\}=1$. The satisfaction of these conditions means that average power constraint at BS is fulfilled(Yang and Marzetta, 2013), (Jose *et al.*, 2011), (Khansefid and Minn, 2015), (Marzetta, 2010), (Zhang *et al.*, 2014).

The noisy forward-link signal vector received by MSs of *q*-th cell is

$$\mathbf{x}_q=\sqrt{\rho_{fl}}\mathbf{G}_{qq}\mathbf{P}_q\mathbf{s}_q+\mathbf{O}_q\,,\,(U\times 1\text{ vector}) \tag{8.}$$

where ρ_{fl} is average power of BS and $\mathbf{O}_q$ is i.i.d AWGN with CN (0, 1). Therefore, signal received by *u*-th MS is

$$\begin{aligned} x_{uq} &= \sum_{u=1}^{U} \sqrt{\rho_{fl}\beta_{quq}} \left[g_{1quq} \; \dots \; g_{Lquq} \right] \mathbf{p}_{uq} s_{uq} + o_{uq} \\ &= \sum_{u=1}^{U} \sqrt{\rho_{fl}} \mathbf{g}_{uq} \mathbf{p}_{uq} s_{uq} + o_{uq} \end{aligned} \quad (9.)$$

where $\mathbf{p}_{uq}$ is u-th column of precoding matrix $\mathbf{P}_q$ and o_{uq} is u-th element of $\mathbf{O}_q$. According to(J. Jose *et al.*, 2011), (9) can be re-written into a knownchannel system as

$$x_{uq} = i_{uq} s_{uq} + o_{uq} \quad (10.)$$

where s_{uq} , x_{uq} , $i_{uq} = \sum_{u=1}^{U} \sqrt{\rho_{fl}} \mathbf{g}_{uq} \mathbf{p}_{uq}$, and o_{uq} are the input, output, known channel and additive noise, respectively. Then according to (Yang and Marzetta, 2013), the achievable rate of the point-to-point Communication Channel given by (10) is

$$R_{uq} = \log_2 \left(1 + \frac{|i_{uq}|^2}{\mathrm{var}\{o_{uq}\}} \right) \quad (11.)$$

3.1 Closed form achievable forward-linkdata rate of ZF precoder

For ZF, the linear precoder is $\mathbf{P} = c_1 \hat{\mathbf{G}}^{\mathrm{H}} (\hat{\mathbf{G}} \hat{\mathbf{G}}^{\mathrm{H}})^{-1}$ where c_1 is a constant scalar. Therefore, from (9), we have

$$\mathbf{x}_q = c_1 \sqrt{\rho_{fl}} \mathbf{G}_{qq} \hat{\mathbf{G}}_{qq}^{\mathrm{H}} (\hat{\mathbf{G}}_{qq} \hat{\mathbf{G}}_{qq}^{\mathrm{H}})^{-1} \mathbf{s}_q + \mathbf{O}_q \quad (12.)$$

$$\mathbf{x}_q = c_1 \sqrt{\rho_{fl}} \mathbf{s}_q + \mathbf{O}_q + c_1 \sqrt{\rho_{fl}} \tilde{\mathbf{G}}_{qq} \hat{\mathbf{G}}_{qq}^{\mathrm{H}} (\hat{\mathbf{G}}_{qq} \hat{\mathbf{G}}_{qq}^{\mathrm{H}})^{-1} \mathbf{s}_q \quad (13.)$$

Hence, the signal received by the u-th MSis

$$x_{uq} = c_1 \sqrt{\rho_{fl}} s_{uq} + o_{uq} + c_1 \sqrt{\rho_{fl}} \tilde{\mathbf{g}}_{uq} \hat{\mathbf{G}}_{qq}^{\mathrm{H}} (\hat{\mathbf{G}}_{qq} \hat{\mathbf{G}}_{qq}^{\mathrm{H}})^{-1} \mathbf{s}_q \quad (14.)$$

It can be seen from (7), that $\tilde{\mathbf{g}}_{uq} = 0$. Therefore (14) becomes

$$x_{uq} = c_1 \sqrt{\rho_{fl}} s_{uq} + o_{uq} \quad (15.)$$

Then according to (11), the achievable forward-link data rate of (15) is

$$R_u^{\mathrm{zf}} = \log_2 \left(1 + c_1^2 \rho_{fl} \right) \quad (16.)$$

In order to acquire an explicit expression for achievabledownlink data rate, constant scalar c_1 needs to be computed. The constant scalar can be given as

$$c_1 = \left(\sqrt{\mathrm{E}\left[\mathrm{tr}\left\{ \left(\hat{\mathbf{G}}_{qq} \hat{\mathbf{G}}_{qq}^{\mathrm{H}} \right)^{-1} \right\} \right]} \right)^{-1} \quad (17.)$$

From (7) $\hat{\mathbf{G}}_{qq} = \sqrt{\rho_{re}\tau} \mathbf{G}_{qq}$, therefore $\left(\hat{\mathbf{G}}_{qq} \hat{\mathbf{G}}_{qq}^{\mathrm{H}} \right)^{-1} = \left(\rho_{re} \tau \mathbf{G}_{qq} \mathbf{G}_{qq}^{\mathrm{H}} \right)^{-1} = \frac{1}{\rho_{re}\tau} \left(\mathbf{G}_{qq} \mathbf{G}_{qq}^{\mathrm{H}} \right)^{-1}$ .Whereas, $\mathbf{G}_{qq} = \mathbf{D}_{qq}^{1/2} \mathbf{H}_{qq}$, therefore $\left(\hat{\mathbf{G}}_{qq} \hat{\mathbf{G}}_{qq}^{\mathrm{H}} \right)^{-1} == \frac{1}{\rho_{re} \tau \mathbf{D}_{qq}} \left(\mathbf{H}_{qq} \mathbf{H}_{qq}^{\mathrm{H}} \right)^{-1}$.

Furthermore, the only channel vector that model large-scale fading for the u-th MS is β_{quq} , therefore $\left(\hat{\mathbf{G}}_{qq} \hat{\mathbf{G}}_{qq}^{\mathrm{H}} \right)^{-1} = \frac{1}{\rho_{re} \tau \beta_{quq}} \left(\mathbf{H}_{qq} \mathbf{H}_{qq}^{\mathrm{H}} \right)^{-1}$ .Whereas $\mathbf{H}_{qq} \mathbf{H}_{qq}^{\mathrm{H}}$ is a complex Wishart matrix. Therefore, from (Yang and Marzetta, 2013) and (Tulino and Verdú, 2004), we see

$$\mathrm{E}\left[\mathrm{tr}\left\{ \left(\mathbf{H}_{qq} \mathbf{H}_{qq}^{\mathrm{H}} \right)^{-1} \right\} \right] = \frac{U}{L-U} \quad (18.)$$

this implies

$$c_1^2 = \frac{\rho_{re} \tau \beta_{quq} \left(L - U \right)}{U} \quad (19.)$$

Finally, combining equations (17) and (19) and dropping user subscript

$$R^{\mathrm{zf}} = \log_2 \left(1 + \frac{\rho_{fl} \rho_{re} \tau \beta_{qq} \left(L - U \right)}{U} \right) \quad (20.)$$

and the achievable forward-link sum rate for the U MS is

$$R_{\mathrm{sum}}^{\mathrm{zf}} = U \log_2 \left(1 + \frac{\rho_{fl} \rho_{re} \tau \beta_{qq} \left(L - U \right)}{U} \right) \quad (21.)$$

3.2 Closed form achievable forward-linkdata rate of MF precoder

For MF, the linear precoder is $\mathbf{P}_{qq} = \frac{\hat{\mathbf{G}}_{qq}^{\mathrm{H}}}{\left\| \hat{\mathbf{G}}_{qq} \right\|}$.

Therefore, from (8), we have

$$\mathbf{x}_q = \sqrt{\rho_{fl}} \mathbf{G}_{qq} \mathbf{P}_{qq} \mathbf{s}_q + \mathbf{O}_q \quad (22.)$$

Hence, the signal received by the u-th MSis

$$x_{uq} = i_{uq} s_{uq} + o_{uq} \quad (23.)$$

where $i_{uq} = \sum_{u=1}^{U} \sqrt{\rho_{fl}} \mathbf{g}_{uq} \mathbf{p}_{uq}$. Then, (23) can be written as

$$x_{uq} = \mathrm{E}\left[i_{uq}\right] s_{uq} + \left(i_{uq} - \mathrm{E}\left[i_{uq}\right]\right) s_{uq} + o_{uq} \qquad (24.)$$

Hence, the following rate is achievable(J. Jose *et al.*, 2011)

$$R_u^{\mathrm{mf}} = \log_2 \left(1 + \frac{\left|\mathrm{E}\left[i_{uq}\right]\right|^2}{1 + \mathrm{var}\left\{i_{uq}\right\}} \right) \qquad (25.)$$

An explicit expression for achievable data rate of (25) can be acquired using the method suggested in (Jose *et al.*,2011). Then, the valuesof $\left|\mathrm{E}\left[i_{uq}\right]\right|^2$ and $\mathrm{var}\left\{i_{uq}\right\}$ with LS estimation are

$$\left|\mathrm{E}\left[i_{uq}\right]\right|^2 = \rho_{fl} \rho_{re} \tau \beta_{qq} \mathrm{E}^2\left[\theta\right] \qquad (26.)$$

$$\mathrm{var}\left\{i_{uq}\right\} = \rho_{fl} \rho_{re} \tau \beta_{qq} \,\mathrm{var}\{\theta\} \qquad (27.)$$

where $\theta = \sqrt{\sum_{l=0}^{L} |k_l|^2}$ and $\{k_l\}$ is i.i.d with CN (0,1). Then, achievable sum rate of U MSs with MF precoding is,

$$R_{\mathrm{sum}}^{\mathrm{mf}} = U \log_2 \left(1 + \frac{\rho_{fl} \rho_{re} \tau \beta_{qq} \mathrm{E}^2\left[\theta\right]}{1 + \rho_{fl} \rho_{re} \tau \beta_{qq} \,\mathrm{var}\{\theta\}} \right) \qquad (28.)$$

where $\mathrm{E}\left[\theta\right] = \frac{\Gamma\left(L + \frac{1}{2}\right)}{\Gamma\left(L\right)}$ and $\mathrm{var}\{\theta\} = L - \mathrm{E}^2\left[\theta\right]$. Here $\Gamma(.)$ is a gamma function.

4. NUMERICAL SIMULATION RESULTS AND DISCUSSIONS

A typical multi-cell TDD M-MIMO scenario with simulation parameters listed in **(Table 1)** has been utilized for simulation.**(Fig.2)** and **(Fig.3)** show the relation amid cell radius r_c andforward-link SINR when a number of BS antennas per cell L is limited. It was reported in (Marzetta, 2010) that SIR remaininvariable with respect tosize of the cell radius when L is infinite. However, in a regime with finite L and with dissimilar cell sizes and power controls,SINR is roughly inversely proportional to the cell radius. This result consents with instinct that small cell sizes will improve system performance for smaller MS-BS distances. **(Fig.2)** and **(Fig.3)** illustratethe cumulative distribution function (CDF) outcomes of downlinkSINR based on (22) and (29), when celledge reference SNRis low (−8 dB) and high (12 dB), matching to noise-limited and the inter ference-limite dsections, respectively. Outcomes are offered for three dissimilar cell radii, i.e., r_c = 700, 1100, 1500 m and MSs are randomly and uniformly placed. Plots in Figs. 2 and 3 consent fine with above instincts, i.e., shorter the cell size, larger the forward-link SINR and vice versa.

(Fig.4) illustratesimpact of increasing number of BSantennas on downlink sum rates of ZF precoding as well as MF precoding, respectively. As can be seen, ZF performs better than the MF as it has smaller denominator than the MF. It can also be seen that asnumber of BS antennas growsbeyond 40sum-rate starts getting asymptotic. An increase of 30 bits/s/Hz and 20 bits/s/Hz for ZF and MFprecoders, respectively, asnumber of antennas rises to 40. Nevertheless, beyond 40 number of BS antennasincrease in sum-rate for both ZF and MF is steady.

Table 1: Simulation Parameters

Parameter	Symbol	Value (Unit)
Number of cells	C	7
Number of MSs per cell	U	8
Number of uplink pilot sequences	τ	8
Coherence interval	T	20 symbols
Number of BS antennas per cell	L	100
Cell radius (from center to vertex)	r_c	1000 m
Central disk radius (protection distance)	r_n	100 m
Decay exponent	ν	3.8
Standard deviation	σ_{shadow}	8 dB
Average power of MS	ρ_{re}	10 dB
Average power of BS	ρ_{fl}	15 dB

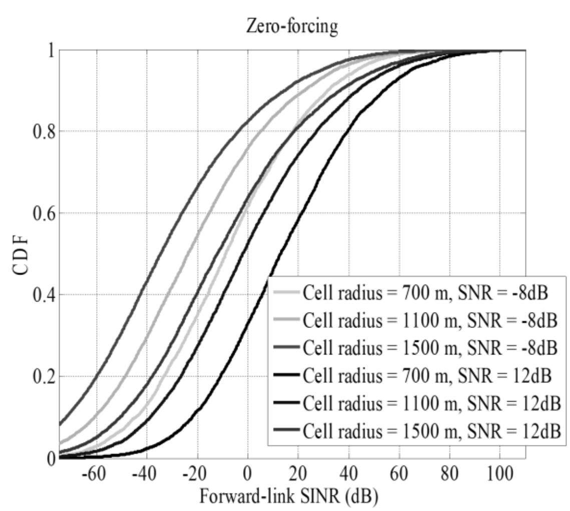

Fig.2 : Forward-link SINR CDF of ZF for different cell radii r_c

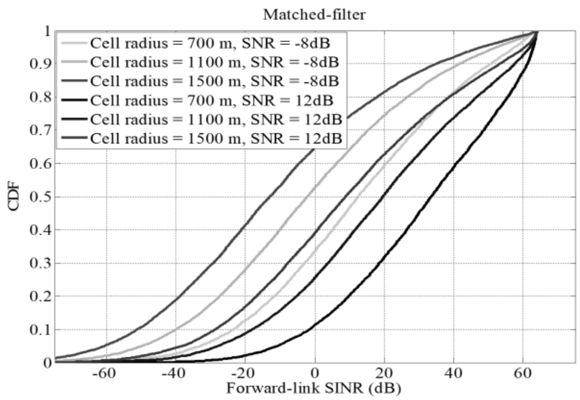

Fig. 3: Forward-link SINR CDF of MF for different cell radii r_c

(Fig.5) shows the impact of different lengths of uplink pilots τ onnet sum-rate. As can be seen, smaller the length of τ, higher the net sum rate per user.When the τ is sixteen that implies only 3 symbols are available for forward-link data transmission, which yields just 1 bps/Hz/user of the forward-link net sum-tare. The larger τ can give us a higher number of users per cell but poor sum-rate performance. Similarly, smaller τ can give us better sum-rate performance; however, with a small τ low number of users per cell can be served. Therefore, an optimal size of τ should be chosen for an optimal number of users per cell and sum-rate performance, respectively.

(Fig. 6) illustrates sum rate against forward-link transmission SNR for$L$ = 100. Significant increase in the sum-rate performance of ZF precoder can be seen as the SNR increases. This meansan interference free operating condition with ZF precoder and recommends a high SNRregion foroperation of M-MIMO systems in the multi-cell environments. Whereas, the sum-rate of MF precoder remains almost constant as the SNR increases from low to high. This implies interferencelimited operating condition with MF precoder and suggests either region of SNR for the operation of the M-MIMO systems in the multi-cell environments. However, this interference is not due to the pilot contamination.

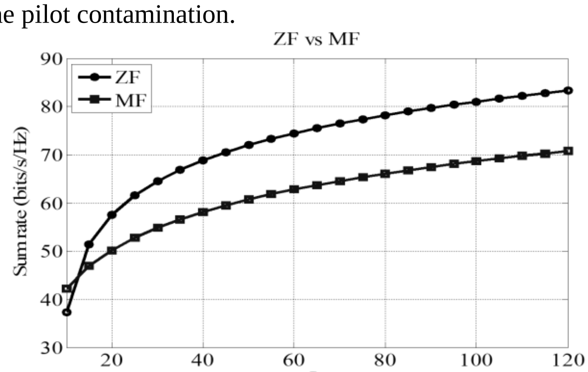

Fig. 4: Effect of number of BS antennas on forward-link sum rate

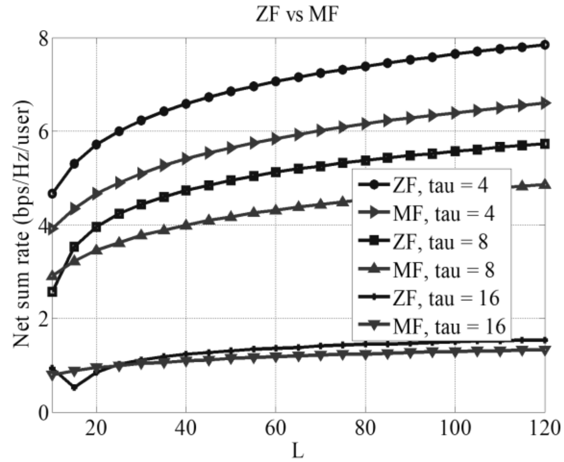

Fig. 5: The effect of the pilot length on the forward-linknet sum rate per user

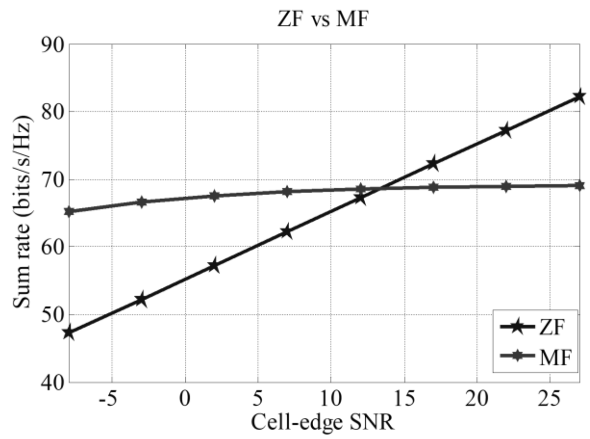

Fig. 6: The effect of the cell-edge SNR on the forward-link sum rate

5. CONCLUSION

The performance of multi-cell TDD M-MIMO forward-link system without pilot contamination for MF and ZF precoders at the BS is studied in this paper. Achievable forward-link closed-form expressions for both MF and ZF precoders have been derived. Simulation results show that M-MIMO forward-link system can be best deployed in a low SNR region as sum-rate performance saturates for both MF and ZF precoders in a high SNR region. It is also shown that M-MIMO systems can attain higher sum-rates, which show the advantage of M-MIMO systems. Furthermore, based on these achievable forward-link sum-rates and SINR expressions of ZF and MF precoders, the effect of cell size, BS antennas, pilot length and the transmission powers on the system performance have been investigated. It can be observed that performance of ZF precoder is better than the MF precoder.Hence, our closed-form sum-rate performance analysis offers insights and direction for an efficient multi-cell TDD M-MIMO system design.

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