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Construction of Generalized Incomplete Trojan-Type Designs

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Abstract: For crop research, near-Latin square and Latin square are important and very useful row-and-column designs, especially in greenhouse experiments, however, practical range of these designs are very scarce. Semi-Latin Square Designs (SLSD) do cover this range but have limited usability. Trojan Square Designs is classified into Optimal Semi-Latin Squares (OSLS) that generalizes the class of Latin Square Designs. By a discussion of Incomplete Trojan Square Designs (ITSD) attained by removing a single key row or column from a complete Trojan Square Design, body of already existing designs is drawn out to a greater extent. In this article, we introduce some constructions of Generalized Incomplete Trojan-Type Designs (GITTD) for$k$ = 2, 3, 4, 5, 6. Some of the designs are pair- wise balanced or variance balanced and the remaining are regular graphs.

Keywords: Cyclic Shifts, Latin Square, Trojan Square, Pair-wise Balanced, Semi-Latin Square, Variance Balanced.

1. INTRODUCTION

Row-and-Colum designs are proven to give valuable results and are of wide use to regulate non-treatment inconsistency in glasshouse experiments as well as in the field. Such designs are very useful in a rectangular array of the experimental units and when the two mutually orthogonal and independent sources of discrepancy run across along the columns and rows of the array. For horticultural and agricultural research, the simplest and more operative row-and-column design is considered the n x n Latin Square Designs (LSD).

Because of additive column and row effects, the effects of the treatment are orthogonal, which means that a model with additive column and row effects can be expected suitable, and thus, effect of treatment can be assessed with fully efficacy. In most of the practical experiments, Latin square are valuable, where treatments are lesser in numbers, but, due to the fact that number of treatments and replicates should always be equal, presented variety of designs usually limited to sizes from about 4×4 to 7×7.

By means of incomplete Latin square of size (n-1)×n or size n×(n-1) found by reducing one whole column or row from a Latin square of size n× n, the higher end of the size range can be lengthened (Yates 1936). However, by using augmented Latin square of size (n+1)×n or of size n×(n+1) assessed by repeating a whole column or row of a Latin square of size n×n, the lower end of the scale can be expanded (Pearce 1952).

Near-Orthogonal Latin Square and standard Latin Square Designs do not exist when the number of treatments is larger than the number of replicates hence a more general class of row-and-column design is obligatory. Thus Latin Squares offer valuable designs for different trials that need orthogonal double- blocking systems but rapidly converted in practicable as n increases. Trojan Square for nk treatments is the generalization of the Latin Square in which there are n rows and n columns. Where the point of intersection of each row and column a block of k plots, where each treatment is replicated. n-semi Latin designs exits for all values of n and k, which provides orthogonal double-blocking system, which based on several number of treatments and the treatments are replicated more number of times. In most of the crop research, Trojan Square Designs are used, which is the particular class of semi Latin square.

1.1- Construction of Generalized Incomplete Trojan- Type Designs

Trojan squares designs based on mutually orthogonal Latin squares and these orthogonal Latin squares can be constructed from semi Latin square (Darby and Gilbert, 1958). In the pair wise treatment comparison in the plots-within-blocks stratum, semi Latin squares have been proved that they are maximum efficient designs (Baily, 1992).

The analysis of Trojan squares designs based on simple analysis of variance technique (Edmondson, 1998) and generally they are balance design (Payne and Tobias, 1992). Trojan squares have been discussed by Edmondson (1998) in which if by removing a single row from a complete Trojan square, simple incomplete Trojan squares can be obtained, which has the practical value but cannot be used for all purposes adequately.

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By removing two or more complete rows from a complete Trojan squares, generalized Trojan square designs can be prepared. Now the drawback is that, simple incomplete Trojan square and generalized incomplete Trojan squares of the specific size are not equally effective. When a second set of treatment is superimposed an experimental units previously used for a first set of treatments which is discussed by Preece (1996), then Youden rectangles of size m×n is obtained for both sets of treatments.

Suitable (n×n)/2 Trojan designs can be constructed by removing suitable rows discussed by Preece (1966), if experimental units are regarded as plots-within-blocks. The designs which are discussed by Preece (1966) are used to estimate effects of a set of treatments adjusted for the residual effects of a first set of treatment, whereas estimation of all treatments effects simultaneously can be studied in Trojan designs. However, the criteria which is used for the construction of efficient incomplete Trojan design is not appropriate for efficient superimposed treatment designs.

2. HISTORICAL BACKGROUND

A long history of Semi-Latin Squares is discussed in the statistical literature given by Preece and Freeman (1983) but they are not commonly used in in practice. Yates (1935) criticized that in Semi Latin Squares, it is not sufficient for the multiple error structure of the design, simple column and row analysis is not appropriate. He pointed out that some of the treatments contrast are split out into two strata into two blocks strata and plots-within-blocks stratum in the situation of analysis. However, criticism has substantially been addressed that with the expansion of contemporary computer software, it becomes easier for the analysis of proper stratified incomplete block design. But the problem remains the same that for estimating treatment contrast, not all semi Latin square are equally efficient. Trojan square designs which is the specific class of Semi Latin Squares has efficient properties and that are the most appropriate designs in the field of glasshouse and crop research experiment.

Harshbarger and Davis (1952) discussed the Latinized Near Balance Rectangular Lattices but their designs were restricted on $k = 4n - 1$ and gave the alternate name Trojan square design. Later, Darby and Gilbert (1958) discussed the general case for $k<n$ and introduced the name Trojan Squares for designs with $k>2$. But, now all designs of the Latinized Rectangular Lattice type are commonly described as Trojan Squares for any $1<k<n$.4The combinatorial properties of Semi-Latin Squares and related designs were discussed by Preece and Freeman (1983). Several semi Latin squares and Trojan designs were constructed by Bailey (1988). The efficiency of semi Latin squares and General semi Latin squares are compared with the efficiency of Trojan square designs discussed by Bailey (1992). When pair wise comparison of treatment means is requisite and if all the treatments contrasts are of equivalent significance, the Trojan square designs is the ideal choice, which is also debated by the author. Dharma lingam (2002) made an application of Trojan square designs and applied them to get partial triallel crosses. Jaggi, and Sharma (2010) developed a method to construct the generalized incomplete Trojan-type design and discussed some properties of this class of designs. A class of generalized incomplete Trojan-type designs was used by Varghese and Jaggi (2011) proposed to find Partial Diallel Cross (PDC) and Partial Triallel Cross (PTC) plans with smaller number of crosses. Jaggi, and Varghese (2015) a presented a series of 3generalized incomplete Trojan-type designs for treatments of the form $v = sn + 1$ with 3flexibility in choosing the cell size (s) and the number of columns (n).

3. CONSTRUCTION OF TROJAN SQUARE FROM MUTUALLY ORTHOGONAL LATIN SQUARES

According to Darby and Gilbert (1958), Trojan squares are classified into semi-Latin squares as a special class, based on sets of mutually orthogonal superimposed Latin squares. Trojan squares also have been found to be very effective designs for the comparisons of pair-wise treatments in the plots-within-blocks stratum (Baily, 1992). Trojan squares of size (n×n)/k are based on k sets of n×n orthogonal Latin squares and have a natural factorial treatment structure. Trojan squares become Graeco-Latin squares when k=2 (Cochran and Cox, 1992; Wu and Hamada, 2000). A (4×4)/2 Trojan squares for eight treatments, represented as A, B, C, D, a, b, c and d, is given below:

Rows	**Columns**			
	i	**ii**	**iii**	**iv**
1	**Aa**	**Bb**	**Cc**	**Dd**
2	**Bc**	**Ad**	**Da**	**Cb**
3	**Cd**	**Dc**	**Ab**	**Ba**
4	**Db**	**Ca**	**Bd**	**Ac**

Trojan Squares were introduced by Harshbarger and Davis (1952) but then these were called as Latinized Near Balance Rectangles Lattice with $k = n - 1$. Later, Darby and Gilert (1958) presented the general case for $k < n$ and called them Trojan square designs. However, all designs of the Latinized Rectangles Lattice type are now comely described as Trojan Squares for any $2 \leq k < n$. Preece and Freeman (1983) discussed the combinatorial properties of SemiLatin Squares and related designs. Subsequently, Baily (1988, 1992) gave further methods of constructing a range of SemiLatin Squares and related designs and studied their efficiencies. She showed that Trojan Squares are the optimal choices of SemiLatin Squares for pair-wise comparisons of treatment means. Trojan Squares are normally the best choice of SemiLatin Squares for crop research (Edmondson, 1998). Bedford and Whitaker (2001) have given several methods of construction of SemiLatin Squares. Baily and Mood (2001) gave efficient SemiLatin rectangles useful for plant disease experiments. Dharmalingam (2002) used Trojan squares designs to obtain partial triallel crosses. Parsad (2006) proposed a method of constructing semi-Latin square with $v = 2n$ treatments in n rows, n columns and $k = 2$ by developing an initial column.

Complete Trojan Squares of size $(n \times n)/k$ have n^2 blocks each of size k and requiresnk treatments. However, due to non-existence of the design or limitation of resources, complete Trojan Squares cannot be used in many practical situations. Incomplete Trojan Squares are thus obtained by omitting a single row from complete Trojan Square which have been discussed by Edmondson (1998) and are of considerable practical utility but are not generalized sufficiently for all purposes. Subsequently, Edmondson (2002) generalized Incomplete Trojan Squares designs that are usefully denoted by $(m \times n)/k$, where m denotes the number of replicates of nk treatments and constructed designs are based on a set of k cyclic shifts Iqbal (1991).

In this article, generalized incomplete Trojan-Type designs have been introduced and a method of constructing these designs has been presented by cyclic shifts. Some general properties of these designs have also been discussed with the help of concurrence matrix.

4. CONSTRUCTION OF TROJAN INCOMPLETE BLOCK DESIGNS FROM BIBD AND REGULAR GRAPH DESIGNS

Trojan Incomplete Block Designs can be constructed from BIBD and RGD with different treatments in different columns. These designs are as efficient as the above-mentioned designs. All these designs are constructed on the basis of cyclic shifts.

CASE-I Whennumber of treatments is odd:

Example 1

$v = 7, \quad r = 4, \quad k = 4, \quad b = 7$

The Set of shift: (1 2 3)

0	1	2	3	4	5	6
1	2	3	4	5	6	0
3	4	5	6	0	1	2
6	0	1	2	3	4	5

$$\lambda = r\,(k-1)/v - 1 = 2$$

This design is row-wise complete (i.e. every row contains all 7 treatments)but balanced incomplete column-wise because $\lambda = 2$, which means that everytreatment pair occurs an equal number of times.

This is BIBD. The concurrence matrix of the design is (4 2 2 2 2 2 2).

$$NN' = \begin{bmatrix} 4 & 2 & 2 & 2 & 2 & 2 & 2 \\ 2 & 4 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 4 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 4 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 4 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 4 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 & 4 \end{bmatrix}$$

This concurrence can be made directly from the elements of the shift as (1 2 3). Write the elements of the shift in column wise as

$$\begin{array}{c|ccc} 1 & \begin{matrix}1\\(a_{1,1})\end{matrix} & \begin{matrix}2\\(a_{2,1})\end{matrix} & \begin{matrix}3\\(a_{1,2}+a_{1,3})\end{matrix} \\ 2 & & & \\ 3 & \begin{matrix}3\\(a_{3,1})\end{matrix} & \begin{matrix}5\\(a_{3,1}+a_{2,1})\end{matrix} & \begin{matrix}6\\(a_{3,2}+a_{1,1})\end{matrix} \end{array}$$

$$\begin{array}{cccc} 1 & 1 & 2 & \underline{3} \\ & 2 & & \\ 3 & 3 & 5 & \underline{6} \end{array}$$

In the first row the underlined element 3 is the sum of 1 &2. In the third row the element 5 is the sum of 2&3. Similarly the underlined element 6 is the sum of 1,2&3. It is clear that the treatment 1 with its compliment 6 occurs 2 times. The treatment 2 with its compliment 5 occurs 2 times. The treatment 3 with itself occurs 2 times. So the concurrence is (2 2 2). Repeating the elements the concurrence is (2 2 2 2 2 2). In the design the replications are 4 so the complete concurrence is (4 2 2 2 2 2 2). The concurrence shows that there is no difference among the treatments.

Converting into Trojan-Type Design with block size 2 in columns 2

0 1 2 3 4 5 6   3 4 5 6 0 1 2
1 2 3 4 5 6 0   6 0 1 2 3 4 5

$\lambda_1=0 \ \lambda_2=1$

(0 2), (0 5) treatment pair does not occur. (0 1), (0 3), (0 4), (0 6) treatment pair occurs.

It is RGD.

0 1	3 6
1 2	4 0
2 3	5 1
3 4	6 2
4 5	0 3
5 6	1 2
6 0	2 5

The concurrence of the Trojan-type design can be made directly from the elements of the shift as:

Write the elements of the shift as1 2 3. The underlined elements of the shift are important. The treatment 1 is present. The treatment 2 is not present. The treatment 3 is present. The treatment 4 is not but its compliment 3 is present which is underlined. The treatment 5 is not present. The treatment 6 is not but its compliment 1 is present which is underlined. So the concurrence is (1 0 1 1 0 1). Since the replications are 4, therefore, the whole concurrence is (4 1 0 1 1 0 1). The concurrence shows that the difference among the treatments is zero or one.

Example 2

$v = 8\, r = 4\, k = 4\, b = 8$

The Set of shift: (1 2 2), λ=r (k-1)/v-1=4 (3)/7, This is not whole number which shows RG design.

Treatments with their compliments

1................7
2................6
3.................5
4

0 1 2 3 4 5 6 7

1 2 3 4 5 6 7 0

3 4 5 6 7 8 0 1

5 6 7 0 1 2 3 4

The concurrence of the design can be made directly from the shift as:

Write all the elements of the shift in the column

11 23
2
22 45

In the first row the underlined element 3 is the sum of 1 & 2. 4 is the sum of 2 and 2.

In the third row the underlined element 5 is the sum of 2, 2&1.

From the above the treatment 1 occurs only one time with no compliment. The treatment 2 occurs 2 times. The treatment 3 with its compliment 5 occurs 2 times. The treatment 4 with itself compliment. The treatment 5 with its compliment 2 occurs 2 times. The treatment 6 is not but its compliment 2 occurs 2 times. The treatment 7 is not but its compliment 1 occurs 1 time. So the concurrence is (1 2 2 2 2 2 1). In this design the replications are 4 so the complete concurrence is (4 1 2 2 2 2 2 1).

The concurrence shows that the maximum difference among the treatments is zero or one.

Converting into Trojan-Type Design with block size 2 in columns 2 for even number of Treatments

Number of rows=8**,** Number of columns=2

0 1	3 5
1 2	4 6
2 3	5 7
3 4	6 0
4 5	7 1
5 6	0 2
6 7	1 3
7 0	2 4

The above design is Trojan-type with following parameters:

v=8 k=2 r=4 b=16 c=2

The concurrence of the Trojan-type can be made directly from the shift as

Write the elements of the shift as

1 2 2

The underlined elements of the shift are 1 &2. By looking these two underlined elements concurrence can be made.

The treatments 1 and 2 occurs (1 1). The treatments 3 and 4 are not present. (1 1 0 0). The treatment 5 is not present. (1 1 0 0 0). The treatment 6 is not present but its compliment 2 which is underlined is present. (1 1 0 0 0 1). The treatment 7 is not present but its compliment 1 which is underlined is present. (1 1 0 0 0 1 1). Since the replications are 4 so the complete concurrence is (4 1 1 0 0 0 1 1).

From the concurrence, it can be concluded that the difference among the treatments is 0 or 1.

Example 3

V=7 k=6 b=7 r=6

The set of shift: (1,1,2,2,4)

In the construction it is essential that sum of the elements of the shift not equal to

7,14,21,28,35,42,49,…

Treatment with their compliments

1...........6

2...........5

3...........4

Every treatment occurs equal time i.e. 6 times. $\lambda = r(k-1)/v-1, \lambda = 6(5)/6 = 5$, it means that every treatment pair occurs 5 times.

0 1 2 3 4 5 6
1 2 3 4 5 6 0
2 3 4 5 6 0 1
4 5 6 0 1 2 3
6 0 1 2 3 4 5
3 4 5 6 0 1 2

The concurrence matrix can be directly from the shift as:

Write all the elements of the shift in the column

1	1	1	2		
1					
2	2	3	4		
2	2	4	5	6	
4	4	6	1	2	3

From the above, it is clear that the treatment 1 with its compliment 6 occurs 5 times. The treatment 2 with its compliment 5 occurs 5 times. The treatment 3 with its compliment 4 occurs 5 times. So the concurrence is (5 5 5). Repeating the elements it becomes (5 5 5 5 5 5). There are 6 replications in the design so the complete concurrence is (6 5 5 5 5 5 5). Concurrence shows that the difference among the treatments is zero i.e. the design is BIB design.

Converting into Trojan-Type Design

0 1 2 3 4 5 6	2 3 4 5 6 0 1	6 0 1 2 3 4 5
1 2 3 4 5 6 0	4 5 6 0 1 2 3	3 4 5 6 0 1 2
0 1	2 4	6 3
1 2	3 5	0 4
2 3	4 6	1 5
3 4	5 0 2 6	
4 5	6 1	3 0
5 6	0 2	4 1
6 0	1 3	5 2

Trojan-Type Design has parameters:

$v = 7\ k = 2\ r = 6\ b = 21\ c = 3$

Trojan-Type Design is efficient as nested balanced design and also balanced design.

The concurrence of the Trojan-type design can be made directly from the elements of the shift as:

Only take the underlined elements of the shift
1 2 4
The treatments 1 and 2 are present(1 1). The treatment 3 is not but its compliment 4 is present(1 1 1). The treatment 4 is present (1 1 1 1). The treatment 5 is not but its compliment 2 is present (1 1 1 1 1). The treatment 6 is not but its compliment 1 is present
(1 1 1 1 1 1). Since there are 6 replications so the complete concurrence is (6 1 1 1 1 1 1).

From the concurrence it can be concluded that the difference among the treatments are 0.

It is important to note that from the concurrence that every treatment pairs occurs 1 time while in the existing design the every treatment pairs occurs 5 times. The concurrence of the existing design and Trojan-type design are also different.

The above Nested design can be constructed by different shifts such as (1 1 1 1 1) and also by (2 2 2 2 2)but the concurrence will be different.

When resources are available and maximum efficiency is required then block size can be increased. It is noticeable that when block size increased then cyclic shift also changed. Cyclic shift is an important key in the construction of Trojan-Type Design. Already available existing designs can be converted into Trojan-type designs with k=3 4 5……….

Example 4

v=7 r=6 k=6 b=7

The Set of Shift: (2 1 3 2 4)

λ=r(k-1)/v-1=5, v=Number of treatments, r=Number of replications, k=Block size, b=Number of blocks in the whole design.

0 1 2 3 4 5 6
2 3 4 5 6 0 1
3 4 5 6 0 1 2
6 0 1 2 3 4 5
1 2 3 4 5 6 0
5 6 0 1 2 3 4
$A = (7,14,21,28,35,42,49,56\ldots)$

The treatments in the shift are allocated in such a way that their preceding sum is not equal to the sequence A. Therefore at each step sum of the treatments must be kept in mind. In block size 3 the underlined treatments are important because concurrence is made. The other treatments may be anyone but their sum is not equal to A.Treatments with their compliments

1..........................6

2..........................5

3..........................4

The concurrence can be made directly from the shift as

2	2	1	3		
1					
3	3	4	6		
2	2	5	6	1	
4	4	6	2	3	5

In the first column the elements of the shift are written in the column wise.

In the table, in first column i=2,j=1,k=3,l=2,m=4.

In R_1 i, j, i+j

In R_3 k, j+k, j+k+i

In R_4 l, l+k, l+k+j, l+k+j+i

In R_5 m, m+l, m+l+k, m+l+k+j, m+l+k+j+i

If any sum exceeds the number of treatments then mode is taken from the number of treatments.

In the above table, consider the columns from 2 to 6.

The element 1 with its compliment 6 occurs 5 times. The treatment 2 with its compliments 5 occurs 5 times. The treatment 3 with its compliments occurs 5 times. So the concurrence is (5 5 5) and by repeating (5 5 5 5 5 5). In the above design replications are 6 so the concurrence is (6 5 5 5 5 5 5).

Making the whole NN´ matrix then the diagonal values are the number of replications and off-diagonal values are constant which is 5 equal to λ. It means that every treatment pairs occurs an equal number of times.

The above design is row-wise complete because every row contains 7 treatments but column-wise incomplete each column contain 6 treatment. Another important feature of the design is that, if one row is added then it becomes Latin-square design of 7*7. For the addition of a row 3 elements are added in the shift which destroys the shift and also the design. The design will be unbalanced.

Converting into Trojan-Type Design

0 1 2 3 4 5 6	6 0 1 2 3 4 5
2 3 4 5 6 0 1	1 2 3 4 5 6 0
3 4 5 6 0 1 2	5 6 0 1 2 3 4

In the above design every treatment pairs occurs 2 times .i.e. λ=2

C-1	C-2
0 2 3	6 1 5
1 3 4	0 2 6
2 4 5	1 3 0
3 5 6	2 4 1
4 6 0	3 5 2
5 0 1	4 6 3
6 1 2	5 0 4

In C-1 the shaded set shows that treatment 0 occurs with all other treatments. In C-2 also treatment 0 occurs with all other treatments.

The concurrence of the Trojan-Type Design is made as:

Taking first block (0 2 3) in C-1, the treatments which are present in the bracket take values 1 and reaming 0 in the first column. The first column of the N matrix may be shown as:

(1 0 1 1 0 0 0). Taking second block (1 3 4) the treatments present in the bracket takes the values 1 and remaining 0 in the second column. The second column of the N matrix may be shown as (0 1 0 1 1 0 0) and so on. Similarly in C-2, taking first block (6 1 5) the treatments present in the bracket takes 1 and remaining 0 in the first column of N matrix. The first column of the N matrix is (0 1 0 0 0 1 1). Taking second block (0 2 6) the treatments present in the bracket takes 1 and remaining 0 in the second column of N matrix. The second column of the N matrix is (1 0 1 0 0 0 1) and so on.

The concurrence of the Trojan-type design can be made directly from the elements of the shift.

The procedure of making concurrence in block size 3 is different from the block size 2.

Procedure of Concurrence from the Cyclic Shift

From the shift, the underlined elements are:

$$\begin{array}{c|ccc} 2 & 2 & 1 & \underline{3} \\ 1 & & & \end{array}$$

$$\begin{array}{c|ccc} 2 & 2 & 4 & \underline{6} \\ 4 & & & \end{array}$$

From the above two sets, the underlined treatments 3 and 6 comes by the addition of 2 and 1, 2 and 4 respectively. Combining two sets, it can be shown that treatment 1 with its compliment. Treatment 2 Treatment 3 with its compliment occurs 2 times. So the concurrence is (2 2 2)

repeating (2 2 2 2 2 2) and the diagonal will be 6 which is the number of replication and the concurrence becomes (6 2 2 2 2 2 2).

The whole concurrence is:

$$NN' = \begin{bmatrix} 6 & 2 & 2 & 2 & 2 & 2 & 2 \\ 2 & 6 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 6 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 6 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 6 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 6 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 & 6 \end{bmatrix}$$

From the concurrence, it is clear that the treatment pairs difference is 0 which shows the balanced.

It is important to note that the concurrence for both designs i.e. existing design and the Trojan-type designs are different. In the existing design every treatment pairs occurs 5 times but in Trojan-type design every treatment pairs occurs 2 times.

Table 1: Conversion of Different Existing Designs into Trojan Incomplete Block Design.

No.	v	R	K_1	B_1	k_2	b_2	c	Existing Designs	Sets of Shifts
1	5	4	4	5	2	10	2	Resolvable	(1 1 2)
2	6	4	4	6	2	12	2	Regular Graph within Regular Graph	(1 1 2)
3	7	6	6	7	2	14	3	Balanced within balanced	(1 1 2 2 4)
4	8	6	6	8	2	24	3	Regular Graph within Regular Graph	(1 2 2 5 5)
5	10	10	8	10	2	40	4	Balanced within balanced	(1 1 2 1 3 5 4)
6	8	12	6	16	2	48	6	Regular Graph Design	(1 2 7 4 6)+(3 6 4 5 2)
7	9	8	8	9	2	36	4	BIBD	(1 2 3 8 2 1 5)
8	9	8	4	18	2	36	4	BIBD	(1 1 4)+(3 2 6)
9	11	10	10	11	2	55	5	BIBD	(1 1 2 1 3 1 5 3 4)
10	12	10	12	12	2	60	5	BIBD	(1 1 3 10 5 8 2 1 4)

Table 2: Odd Number of Treatment

No.	Parameters				Set of Shifts	Nature of Design	Trojan Type Design				
	V	r	k_1	b_1			v	r	k2	b2	c
1	9	6	6	9	(1,2,3,1,4)	RG design nested within RG design	9	6	3	18	2
2	11	6	6	11	(1,2,3,4,5)	RG design nested within balanced design	11	6	3	22	2
3	11	9	9	11	(1,3,10,2,5,7,1,2)	RG design	11	9	33	3	3
4	13	12	12	13	(1,3,12,2,5,5,5,2,12,3,1)	BIBD	13	12	26	3	2
					(1,3,12,2,5,5,5,2,12,3,1)	BIBD	13	12	29	3	3
					(1,3,12,2,5,5,5,2,12,3,1)	BIBD	13	12	52	3	4
5	15	12	12	12	(1,3,14,2,6,11,1,4,9,3,5)	BIBD	15	12	60	3	4

Table 3: Even Number of Treatments

No.	**Parameters**				**Set of Shifts**	**Nature of Design**	**Trojan Type Design**				
	v	**r**	**k_1**	**b_1**			**v**	**r**	**k_2**	**b_2**	**c**
1	8	6	6	8	(1,2,7,3,1)	RG design	8	6	3	16	2
2	10	6	6	10	(1,3,8,5,2)	RG design	10	6	3	20	2
3	10	18	6	30	(1,2,4,5,6)+(1,2,4,5,3)+ (4,2,5,7,9)	Balanced design nested within balanced design	10	18	3	60	6
4	12	9	9	12	(1,3,11,2,3,10,1,3)	RG design	12	9	3	36	3
5	14	12	12	14	(1,3,13,2,5,10,5,1,9,2,4)	RG design	14	12	3	48	4

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