



**Gross and Yield Period of Vibration of Low-Rise Reinforced Concrete Frames in Karachi**

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**Abstract:** Due to dearth of indigenous research on the structures in Pakistan, the practicing engineers are compelled to employ design and assessment provisions developed for the other parts of the world. To address this gap, the research presented herein focuses on characterization of the sample reinforced concrete structures using Eigen value and state-of-the-art adaptive pushover tools. Twelve existing three-dimensional reinforced concrete structures, comprising of four two-storeyed, four three-storeyed and four four-storeyed structures, are therefore modelled as bare as well as infilled frames and examined comprehensively to obtain vital parameters, namely gross period and yield period of vibration, required in the design and assessment procedures. Period-dependent relationships for gross and yield period of vibration are proposed for bare and infilled frames for the seismic design and assessment of buildings, and compared with seismic design provisions and other research studies.

**Keywords:** Pushover; period-height relationships; low-rise buildings; reinforced concrete; seismic design

**1. INTRODUCTION**

It is predominant practice in the construction industry in Pakistan to adopt US provisions American Concrete Institute (ACI). (2008) for the design and assessment of structural systems. Notably, Pakistan building code-seismic provisions Building (2007) was proposed in 2007 postKashmir earthquake in 2005 due to pressure on the engineering and research community owing to severe human, structural and financial losses in the seismic event, to fulfil the need for a legal document for the seismic design of buildings. Nevertheless, the proposed codified provisions essentially adopt Uniform Building Code (1997) due to complete lack of indigenous research on the structures designed and constructed in Pakistan. The research presented herein aims to address this gap particularly for the seismic design and assessment provisions. To this end, period of vibration of the building, which is one of the most imperative parameters required for the design and assessment, is evaluated for the existing structures using Eigenvalue and adaptive pushover analysis.

The period of vibration is vital parameter that reflects the mass and stiffness ratio of a building, and is used to evaluate the anticipated seismic demand of the structure. Although, the seismic provisions Building Code (1997) Code (2005) generally relate the parameter to the total height of a building, as represented by Eq. (1) shown below, it is depends on structural characteristics contributing to the seismic mass and stiffness of a structure. It is noted that the expression provided in Eq. (1) is adopted by both European as well as US seismic codes code-seismic

provisions Building (2007) Building Code (1997) Code (2005) NEHRP. (2003) for calculation of the period of vibration of the RC moment resisting frames. The period-height relationships have traditionally been obtained by regression analysis on the periods of the buildings measured from the seismic response data.

$$T = 0.073H^{0.75} \text{ (H in metres)} \quad (1)$$

This parameter has been investigated by numerous researchers using recorded data or analytical methods (Crowley *et al.*, 2010) (Goel and Chopra 1997) for instance, proposed Eq.(2) and (3) for lower and upper bound period of vibration respectively. (Crowley *et al.*, 2010) adopted Eigen value analysis to develop a period height relationship (refer Eq. (4)] for bare RC buildings in Europe.

$$T_{upper} = 0.0466H^{0.90} \text{ (H in metres)} \quad (2)$$

$$T_{lower} = 0.067H^{0.90} \text{ (H in metres)} \quad (3)$$

$$T = 0.0525H \text{ (H in metres)} \quad (4)$$

While the traditional force based design procedure requires gross period of vibration, alternative proposals such as yield period of vibration have been put forward to replace gross period to account for the period elongation resulting from stiffness reduction due to cracking of concrete (Crowley *et al.*, 2010) (Crowley *et al.*, 2010) proposed Eq.(5) and (6) for the yield period of structures for the bare and infilled frames with openings respectively for existing buildings in Europe. Similarly, (Kumar *et al.*, 2010) carried out the pushover analyses of the embedded beam structures from Turkish building stock to propose the regression model for bare

and infilled frames separately as shown in Eq.(7) and (8) respectively.

$$T_y = 0.0885H \text{ (H in metres)} \quad (5)$$

$$T_y = 0.055H \text{ (H in metres)} \quad (6)$$

$$T_y = 0.103H \text{ (H in metres)} \quad (7)$$

$$T_y = 0.045H \text{ (H in metres)} \quad (8)$$

To evaluate the gross and yield periods of buildings in Karachi, twelve low-rise structures of two to four storeys are examined. (Seismo 2014) is employed to develop three dimensional mathematical models, which are subsequently subjected to Eigenvalue analysis in both principal directions of the structures to evaluate elastic dynamic properties and adaptive pushover analysis to evaluate pushover curves and displacements at various nodes of the structures, which in turn are used to evaluate yield periods.

## 2. CHARACTERISTICS OF STRUCTURES

Architectural and structural drawings of all the reinforced concrete frames used in the study are available in Afreen (2014) The occupancy type of the two and three storeyed structures is residential; whereas, the four storeyed structures are commercial as well as residential. In these cases, buildings are susceptible to weak storey mechanism due to absence of infill at the ground floor typically assigned for commercial use. Median storey height of the buildings is found to be 3.2 m. Most frequently used cross-section of column of two, three and four storeyed structures is 152 mm x 457 mm, 152 mm x 457 mm and 203 mm x 610 mm respectively; whereas, most frequently used beam dimensions are 152 mm x 914 mm and 152 mm x 1067 mm. It is noted that in majority structures columns are reinforced with 1.0% steel with ties of No. 6 (6.35 mm) @ 152 mm c/c. Furthermore, median percentage of bottom and top reinforcement at mid-span of the beams is found to be 0.33% and 0.17% respectively; whereas, median percentage of reinforcement at ends of the beams at the top and bottom is found to be 0.23% and 0.23% respectively. The shear reinforcement of No. 6 (6.35 mm) @ 229 mm c/c and No. 6 (6.35 mm) @ 152 mm c/c is typically adopted at mid-span and ends respectively.

## 3. MATHEMATICAL MODELING

In total, twenty four three dimensional models are developed in (Seismo 2014) Beams and columns of the buildings are modelled using the in-elastic force based elements (Correia *et al.*, 2008) each element is divided into three to five integration sections depending on numerical stability and convergence of each model. Each section is further divided into 200 fibers for better distribution of stress and strains.

A uniaxial constant confinement model (Mander *et al.*, 1988) is used to represent nonlinear response of concrete. Specified compressive strength of 21 MPa, used in all buildings, is adopted for characterization of concrete. Tensile strength, strain at peak stress and specific weight of concrete are used as 3 MPa, 0.002 and 2300 kg/m<sup>3</sup> respectively; whereas confinement factor is computed for each cross section considering the relevant factors. Bilinear stress-strain model, defined by five parameters namely modulus of elasticity, yield strength, buckling strains, strain hardening parameter and specific weight, is employed for characterization of steel. Yield strength of steel is considered as 483 MPa using the study of (Rafi *et al.*, 2013) Modulus of elasticity strain hardening parameter, buckling strain and specific weight is taken as 200 GPa, 0.002361, 0.1849 and 7950 kg/m<sup>3</sup> respectively.

The infills are modelled using macro modelling approach Afreen (2014) To this end, the equivalent single strut modelled with strut width of 18% of the diagonal length of the infill, Modulus of elasticity as 2768 MPa with compressive strength of 1.96 MPa and the strain at peak stress is taken as 0.0015 employed in uniaxial constant confinement model by (Mander *et al.*, 1988) are used. The reduction factor proposed by (Asteris *et al.*, 2011) computed using Eq.(9), is used to account for presence of openings in the infill walls.

$$\lambda = 1 - 2\alpha_w^{0.54} + \alpha_w^{1.14} \quad (9)$$

In the above equation,  $\lambda$  is the reduction factor applied to the calculated width of the infill wall equivalent strut and  $\alpha_w$  is the opening percentage (area of opening to the area of infill wall).

A load combination of G+0.25L (PEER Report, 2010/05 Guidelines for Performance-Based (2010) is considered for the nonlinear static analysis, whereas G and L represent the dead and live loads on the structure respectively. Each bare and infill frame is subjected to Eigenvalue and displacement based pushover analyses in both principal directions (Antoniou and Pinho 2004) Initially, a nominal displacement is applied at each beam-column joint, and incremented gradually for as high global displacement as possible, as long as the solution is numerically stable. Displacement-based scaling strategy is selected with no spectral amplification consideration. The lateral loads are applied in the major and minor directions separately to assess the behaviour of the structures in both directions.

## 4. ANALYSIS AND DISCUSSION OF RESULTS

### A. EIGEN VALUE ANALYSIS

#### i. Gross Period

Eigen value analysis is an elastic response analysis which is performed to provide fundamental period,

effective modal mass participation and different modes shapes of the modelled structure. Period of vibration represents one of the key dynamic characteristics of a structure to help determine the seismic forces, as implemented in various codes (UBC International Conference of Building Officials. (1997) IBC I. (2006) EC8 Code (2005) In this study, gross period is calculated using Eigen value analysis of bare as well as in filled buildings, as shown in (Fig. 1). X and Y directions are assigned based on the long and short period of vibration for each bare structure without considering infills, thereby representing relatively weak and stiff directions respectively. From the results, it is noted that period of vibration is strongly dependent on the total height of the structure, as reflected in various studies and codified provisions (Asteris 2016) Furthermore, it is observed that, on average, period of vibration of weaker direction is approximately 35% higher than the stiffer direction for both bare as well as infilled frames. It is also evident that the presence of infill significantly alters the stiffness of the structures, which in turn, affects the period of the buildings. It is noted that, on average, the period of buildings with infills is approximately 70% of the period of the buildings without consideration of infills.

Considering the dependence of period of buildings to the total height, the periods obtained from Eigen value analysis are plotted against the total height of structures, as shown in (Fig. 1 and 2). respectively for the bare and in filled buildings. A linear regression equation is proposed using the data set obtained for models without infill and with infill, as shown in Eq. (10) and (11) respectively. The coefficient of determination ( $R^2$ ) for bare and infilled models is found to be 0.4 and 0.41 respectively. Low coefficient of determination reflects significant uncertainty associated with gross period height relationship.

$$T = 0.059H \text{ (H in m)} \quad (10)$$

$$T = 0.0389H \text{ (H in m)} \quad (11)$$

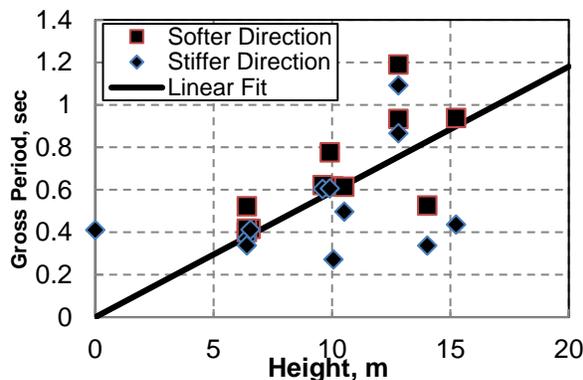


Fig. 1. Observed gross period of bare structures for softer and stiffer direction with the proposed equation

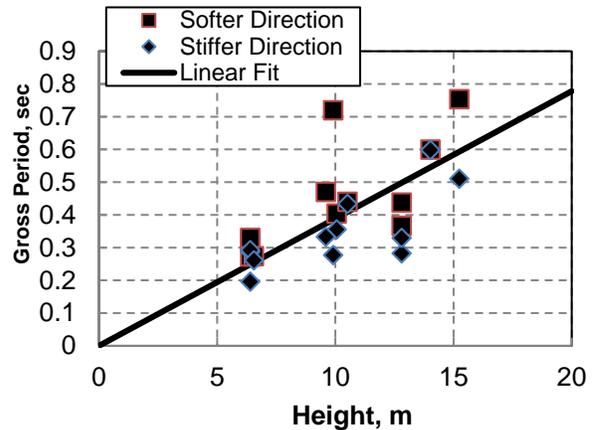


Fig. 2 Observed gross period of infilled structures for softer and stiffer direction with the proposed equation

The expressions evaluated in this study for the bare and infilled structures are compared in (Fig. 3) with the period-height relationships provided by UBC International Conference of Building Officials. (1997) EC8 Code. (2005) and NEHRP (2003) as shown in Eq. (1), upper bound and lower bound period height relationships by Goel and Chopra (Goel and Chopra 1997) provided in Eq.(2) and (3) respectively, and the relationship proposed by Crowley and Pinho (2006) as expressed in Eq.(4). It can be deduced from the comparison that the period relationship proposed in this study for structures without modelling infill walls significantly over predicts the gross period of buildings, thereby indicating importance of modelling infill walls. It is, however, noted that the relationship proposed herein for the structures incorporating the associated effects of infill walls shows very good match with the relationships proposed in the codified provisions and lower bound estimates proposed by (Goel and Chopra 1997) in general. Furthermore, it is observed that the relationship proposed by Crowley and Pinho (2006) is very similar to the relationship proposed for the infilled structures in this work.

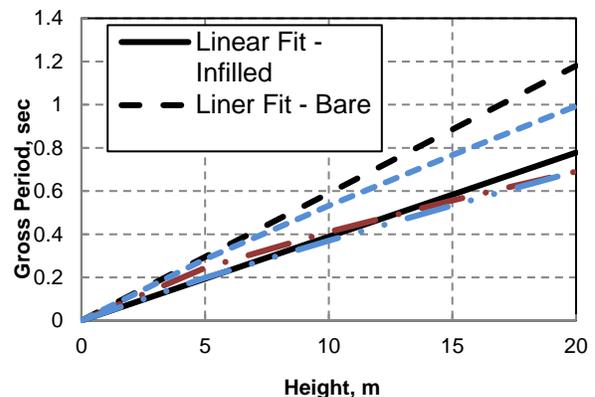


Fig. 3. Comparison of the proposed models with the codified provisions

**B. STATIC ADAPTIVE PUSHOVER ANALYSIS**

**i. Yield Period**

The seismic design methodology, in general, ensures that a structure undergoes non-linearity for energy dissipation. Resultantly, a structure subjected to an earthquake excitation will typically undergo cracking of concrete and yielding of reinforcement causing elongation of the period of vibration of the structure. To address this issue, various researchers have proposed different definition of periods to account this phenomenon.

The focus of the study presented herein is the yield period of vibration. Yieldperiod is computed at the first yield or 75% of ultimate capacity of the structure obtained from the nonlinear analysis. Yield period is calculated by converting a Multi-degree of Freedom (MDOF) system to equivalent Single-degree of Freedom (SDOF) system having effective mass ( $M_{eff}$ ) and effective height ( $H_e$ ). The effective mass is computes as expressed in Eq. (12):

$$M_{eff} = \frac{(\sum_{i=1}^N m_i \phi_{i1})^2}{\sum_{i=1}^N (m_i \phi_{i1})^2} \quad (12)$$

Where  $m_i$  is the seismic mass at  $i^{th}$  storey and  $\phi_{i1}$  is the displacement at each  $i^{th}$ storey corresponding to the first mode of vibration.

Yield stiffness ( $K_y$ ) is calculated at 75% of ultimate base shear obtained from the pushover analysis. This is used to calculate yield time period of the structure as shown in Eq. (13)

$$T_y = 2\pi \sqrt{\frac{M_{eff}}{K_y}} \quad (13)$$

A linear regression equation is proposed using data set obtained by bare and infill frame structure for the yield period as shown in (Fig. 4 and 5). The proposed relationships are provided in Eq.(14) and (15). The coefficient of determination for the relationships proposed for bare and infilled structures is found to be 0.4 and 0.5 respectively.

$$T = 0.102H \text{ (H in m)} \quad (14)$$

$$T = 0.0501H \text{ (H in m)} \quad (15)$$

The relationship proposed herein is compared with the relationships proposed by Crowley and Pinho (2014) as shown in Eq.(5) and Eq.(6), and Kumar *et al.*, (2010) as shown in Eq. (7) and (8). The comparisons show that the relationship proposed herein are in very good agreement with the relationships for the yield period proposed by the other researchers, as demonstrated in (Fig 4 and 5) for bare and infilled structures respectively.

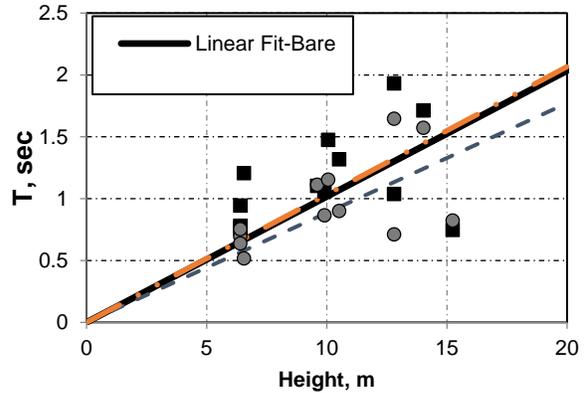


Fig.4 Observed yield period of bare structures for softer and stiffer direction with the proposed equation and codified provisions

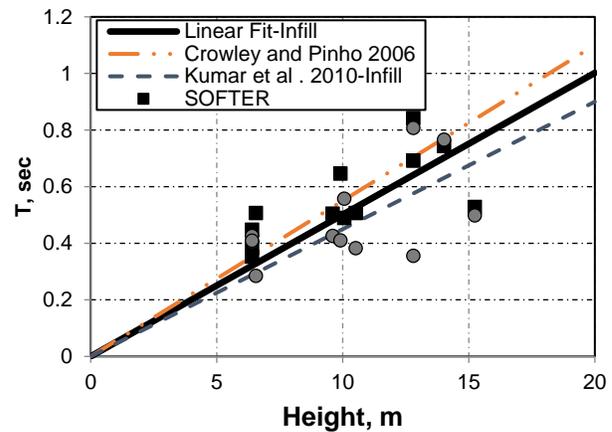


Fig. 5 Observed yield period of infilled structures for softer and stiffer direction with the proposed equation and codified provisions

**5. CONCLUSIONS**

A comprehensive assessment of twelve existing reinforced concrete structures is carried out by modelling them as bare structures without infill as well as infilled structures considering the effects of openings and geometrical properties. Gross period-height relationships for the bare and infilled structures are proposed herein for the low-rise structures constructed in Pakistan. From the study, it is observed that the gross period of vibration is reduced by 30% due to stiffness enhancement by infill. Similarly, yield period-height relationships are developed for the bare and infilled structures. It is noted that, on average, the yield period of infilled structures is approximately half of the corresponding bare structure.

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