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Development of New Technique to Solve Degeneracy in Linear Programming

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Abstract: This Paper Leads To A Technique To Solve Degeneracy Occurring In Simplex Method In Linear Programming Problems By Presenting A New Algorithm To Choose The Particular Leaving Variable. This Method Involves Lesser Time Than The Existing Simplex Method To Get The Exact Required Results Of All The Non-Basic Variables Whereas, The Existing Simplex Method Doesn't Always Show All The Required Results. Proposed Technique Ue/Algorithm Is Better Choice To Avoid The Confusions Of Taking Arbitrary Values To Choose Leaving Variables And Hence The Proposed Technique Is Robust To Solve Degeneracy Linear Programming Problems

Keywords: Simplex Method, Occurrence of Degeneracy, Alternative Solution, Modified Simplex Algorithm, Optimum Solution.

1. INTRODUCTION

Linear Programming (LP) is also known as Linear Optimization, which is a breakthrough to achieve the best outcomes, such as minimum cost and maximum profit. An algorithm for solving Linear Programming problems was developed by George Dantzig, which helped to solve the problems, by constructing a feasible solution until an optimum is reached. Simplex method or Dantzig's simplex algorithm is actually the algorithm for linear programming in Mathematical optimization. According to the journal "Computing in Science and Engineering" simplex algorithm is one of the top 10 algorithms of the twentieth century. The name of this algorithm had been suggested by Motzkin who derived it from the concept of a simplex. For maximizing a linear function of several variables under several constraints, simplex algorithm is a standard method.

The standard form of simplex algorithm is:

Maximize $\mathbf{Z = c_1x_1 + c_2x_2 + c_3x_3 + \ldots + c_nx_n}$

Subject to conditions

$\mathbf{a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n\ (\leq, =, \geq)\ b_1}$

$\mathbf{a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n\ (\leq, =, \geq)\ b_2}$

$\mathbf{a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots + a_{mn}x_n\ (\leq, =, \geq)\ b_n}$

and non-negativity restriction is $\mathbf{x_j \geq 0,\ j = 1, 2, 3, \ldots, n}$

when c_j, b_i and a_{ij} (i = 1, 2, 3,…, n) are constants (model parameters) and x_j are decision variables and $m \leq n$.

Sometimes, in Simplex method there occurs same ratio in solution column and in such cases the question arises for leaving variables. In these types of cases, tie between leaving basic variables occurs. The tie can be broken arbitrarily; it is said The Degeneracy in Simplex method. In a basic feasible solution, when one algorithm quite slower. So, the main purpose of my research is it develops an algorithm to solve the degeneracy and have an optimal solution. (Etoa 2016) gave a new pivot rule to solve Linear Programming problems by simplex method more efficiently as it solves the cycling problem in original simplex method when the size of problem is very large. (Bourab *et al.*, 2015) introduce a primal Algorithm LFP which solves Linear Program based on a linear fractional pricing problem. In this algorithm, the dual variables are optimized to find the largest possible minimum reduced cost value at every iteration. (Grover, *et al.,* 2014) present the discussions about the tactics to define the introduction of Interior-Point Methods for students having various backgrounds even if they are not having Mathematics majors. (Nelder and Mead 2015) describe an algorithm for the minimization of function of an variables. The method seems to be effective and computationally compact. A procedure is given for the estimation of Hessain matrix. (Ping-Qi 2008) proposes an algorithm answerable to real world LP problems, which are often degenerate or even highly degenerate. Compared to the simplex algorithm this algorithm would solve them with potentially improved stability. (Michael 2001) examines linear programming history looking at simplex, ellipsoid, interior-point and other methods. He concludes on the future that linear programming has a history of reinventing itself and hopes that in next fifty years we are going to have too much excitement. (Ping-Qi 1997) introduces an idea to highly degenerate problems in linear programming when basis is not allowed to be a square matrix and this inflexibility results too many zero steps in solving the real-world LP problems by Simplex Method. In this paper an attempt of allowing the deficient basis is

2. PROBLEM STATEMENT AND METHODOLOGY

Let Max: $Z=ax_1+bx_2$

Subject to: $a_1x_1+b_1x_2 \leq c_1$

$a_2x_1+b_2x_2 \leq c_2$

Thento convert $\leq$ sign into equal sign we use slack variables (adding S_1, S_2, S_3, …….)

So Max: $Z-ax_1-bx_2+0(S_1+S_2)=0$

Subject to: $a_1x_1+b_1x_2+S_1+0S_2= c_1$

$a_2x_1+b_2x_2+0S_1+S2= c_2$

	x_1	x_2	S_1	S_2	Solution
Z	-a	-b	0	0	0
S_1	a_1	b_1	1	0	c_1
S_2	a_2	b_2	0	1	c_2

Z, S_1, S_2 are basic variables

X_1, X_2 are non-basic variables

For S_1 the entries are a_1, b_1, 1, 0

For S_2 the entries are a_2, b_2, 0, 1

If -b is the most negative number then we choose x_2 as the entering variable, then:

	x_1	x_2	S_1	S_2	Solution
Z	-a	***-b***	0	0	0
S_1	a_1	$\boldsymbol{b_1}$	1	0	$c_1/b_1 = r$
S_2	a_2	$\boldsymbol{b_2}$	0	1	$c_2/b_2 = r$

Where r is a constant.

attempted. (Ping-Qi Pan 1997) develops a method to provide a stable alternative setting for the dual simplex method. Computational results by using NETLIB are encouraging. (Hall, and Mc Kinnon 1996) introduce a class of LP examples which cause the simplex method to cycle indefinitely. The structure of these examples repeats after two iterations. Furthermore, EXPAND anti-cycling procedure of Gill is also not guaranteed to prevent cyclin.

As we know that in simplex method we take the smallest ratio from the solution column to choose leaving variable from basic variables but in degeneracy there occurs same ratio, as:

and , shown in table above

Here we are free to take an arbitrary value but that makes confusion and wastes our time if we fail to have the values of all the non-basic variables.

So, for such situations of the problems being degenerate in simplex method, the new technique (modified simplex algorithm) has been developed.

In this method (algorithm)we will take that element as the leaving one:

- ✓ whose entries' addition in its row is smaller.

Mathematically: If $\sum S_1<\sum S_2$ then S_1 will be chosen as the leaving element

for this we shall find $\sum S_1$ i-e row sum of S_1 as:

$\sum S_1 = a_1+b_1+1+0$

we will find $\sum S_2$ i-e row of S_2 as:

$\sum S_2 = a_2+b_2+0+1$

And then the same procedure of simplex method will be repeated.

Examples

Example#1: An Engineering University plans to hire staff members for two departments: Computer Science and Mathematics. There is total availability of 3 Assistant Professors, 5 Lecturers and 4 lower staff members. Department of Computer Science requires 3 Assistant Professors, 5 Lecturers and 1 lower staff member and has 4 lack available money, while department of Mathematics requires 1 Assistant Professor, 3Lecturers and 2 lower staff members and has 2 lack available money. Determine how many staff members will the University hire keeping within its resources constraints so that it maximizes the profit.

Resources	Computer Science	Mathematics	Availability
Assistant Professors	3	1	3
Lecturers	5	3	5
Lower Staff Members	1	2	4
Profit/Available money	400000	200000	

Let x_1 is no. of staff members in Computer Science Department hired by University & X_2 is no. of staff members in Mathematics department hired by University

Max: $Z=400000x_1+200000x_2$

Subject to: $3x_1+x_2\leq 3$

$5x_1+3x_2\leq 5$

$x_1+2x_2\leq 4$, x_1, $x_2\geq 0$

Results: $Z = 400000$, $x_1 = 1$, $x_2 = 0$ (by proposed method)

Status: Verified

Hence 1 staff member can be hired by Computer Science department and no staff member can be hired by Mathematics department keeping within its resources constraints so that it maximizes the profit.

Example#2:Max: $Z=2x_1+x_2$

Subject to: $3x_1+x_2\leq 3$

$4x_1+3x_2\leq 6$

$3x_1+2x_2\leq 3$, x_1, $x_2\geq$

Results: $Z=2$, $x_1=1$, $x_2=0$

Status: Verified

Example#3:Max: $Z=45x_1+80x_2$

Subject to: $x_1+4x_2\leq 80$

$2x_1+5x_2\leq 100$, x_1, $x_2\geq 0$

Results: $Z=1600$, $x=0$, $x_2=20$

Status: Verified

3. RESULTS AND DISCUSSION

Example	Simplex Method	Modified Algorithm	Discussion
Max: $Z=400000x_1+200000x_2$ Subject to: $3x_1+x_2\leq 3$ $5x_1+3x_2\leq 5$ $x_1+2x_2\leq 4$ x_1, $x_2\geq 0$	Results: $Z=400000$, $x_1=1$ and we consider $x_2=0$ Status: Verified	Results: $Z = 400000$, $x_1 = 1$, $x_2 = 0$ Status: Verified	In proposed method, the confusion of taking an arbitrary value has been removed by taking a fixed value. And we can also get all the required results by the process of proposed method.
Max: $Z=2x_1+x_2$ Subject to $3x_1+x_2\leq 3$ $4x_1+3x_2\leq 6$ $3x_1+2x_2\leq 3$ x_1, $x_2\geq$	Results: $Z=2$, $x_1=1$ and we consider $x_2=0$ Status: Verified	Results: $Z=2$, $x_1=1$, $x_2=0$ Status: Verified	
Max: $Z=45x_1+80x_2$ Subject to $x_1+4x_2\leq 80$ $2x_1+5x_2\leq 100$ x_1, $x_2\geq 0$	Results: $Z=1600$, $x_2=20$ and we consider $x_1=0$ Status: Verified	Results: $Z=1600$, $x=0$, $x_2=20$ Status: Verified	

4. CONCLUSION

The proposed research is limited to the exact solutions of maximization in Linear programming problems in Simplex Method and its degeneracy. In the problems of simplex method, when hardship (tie between leaving basic variables) occurs, we need to take arbitrary elements as the leaving elements and that makes confusion. In order to remove that confusion a technique is introduced through which we can take, a fixed element as in defined algorithm and save our time. So, by taking any other value we will need to consider the missing value equal to 0 as we don't get it by the process but by the proposed method we get all the values of non-basic variables by the process.

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