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A Pivot Rule for Maximization Degeneracy Problems of Simplex Method for Linear Programming Problems

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Abstract: The simplex algorithm is a powerful technique, widely used to find the optimal solution of linear programming problems. Pivot rule is the most important first step of the simplex algorithm in the maximization degeneracy problems of LP for selecting the entering variable/work column (to obtain the smallest ratios) and after than leaving variable for pivot equation. The selection of an effective pivot rule leads the smallest number of iterations of the optimal solution of L.P. The selecting of most negative number (for entering variable) in the maximization problems is known as G.B Dantzig's pivot rule. The purpose of this paper is to model an algorithm that improves in the entering and leaving variable for the maximization degeneracy problems of LP. In this article, we have introduced a powerful technique for pivot rule which reduces the number of iterations to obtain the optimal solution as compare to Dantzig's pivot rule. This article gives better concept of selection the entering variable as well leaving variable. This article also gives a helpful imminent into the unique and constructive performance of the proposed method by coverage computational experiments.

Keywords: Maximization Degeneracy Problems of Simplex

1. INTRODUCTION

The Simplex Method is used to find the optimal solution in linear programming problems although it involves hundreds of variables and constraints. It arises in many practical applications, such as inventory control, employment scheduling, and personal assignment (Soomro *et al*, 2014). In 1946, simplex method was developed by George Dantzig to use problems in L.P and convenience a more emetic procedure to propagate and proof candidate extremely key to a linear program. Moreover the simplex method manages to classify even if no result actually occur. We have noticed that this method is rapacious as it choices for the best choice at every iterations with no requirement for instruction from prior iterations. In the application of feasibility of degeneracy problems of simplex method connect with the minimum ratio and may be broken in arbitrarily. In this type of problems at least one basic variable must be zero in next iteration and new solution said to be degeneracy problems.

The rule to select the efficient entering variable is the first step of simplex method of maximization degeneracy problems for LP. The efficient entering variable leads to the better optimal solution and also reduces the number of iterations. In some problems of degeneracy the original G.B Dantzig's rule is not fit because of large number of iterations for getting the optimal solution. To get the optimal solution in the short time, the wide research happing in the period of time. (Yan *et al*, 2005) have studied on the weak Pareto of solution and their structure in the multi objective linear programming (MOLP) is very difficult task. They introduced an algorithm which solved all these difficulties very easily. The developed algorithm escapes the degeneracy problems and numerate finite number of weight, all those which are related to a weighted sum problems. (Pan, 2008) studied on the real world linear programming and highly degenerate problems by applying the sparse LU components of defective basis. Later, (Etoa, 2016), worked on the Dantzig's pivot rule of simplex algorithm, to reduce the number of iterations of large size problems. Further, (Elhallaoui *et al*, 2011) studies of degeneracy problems in linear programming. Their Modified Method solve degeneracy problems by taking basic solution with p positive-valued variables, the objective value can always be enhanced in a maximum of m-p+1 pivots, where p is the number of constraints. Also (Vaidya *et al*, 2014) worked on simplex algorithm and two phase simplex method. Their modified algorithm obtained results in very lest iterations as well lesser time as compared to existing simplex algorithm and two phase method. The new algorithm also equally worked for cyclic problems. Further, (Suliman *et al*, 2013) have worked on Quadratic Fractional programming problems (QFPP) for find the max Z value of QFPP.

In this paper, we have introduced a new pivot rule called it maximization degeneracy problems of simplex for LP. This rule is very efficient for such problems of degeneracy in **(Table-1)** which similar ratios arises. Due to degeneracy, the problems will be more complicated and takes large number of iterations to

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Table- No.1

Iteration	Basis	X_1	X_2	X_3	S_1	S_2	S_3	Solution	Ratio
1	Z	c_1	c_2	c_3	0	0	0	b_1	
	S_1	c_4	c_5	c_6	1	0	0	b_2	r_1
	S_2	c_7	c_8	c_9	0	1	0	b_3	r
	S_3	c_{10}	c_{11}	c_{12}	0	0	1	b_4	r

obtain the optimal solution. In such type of problems, this proposed algorithm plays a vital rule to obtain the optimal solution. By small changing in the entering variable the problems resolves the issues of degeneracy/similar ratios and we obtain the optimal solution in small number of iterations. Finally we compare our results with the original G.B Dantzig's solution.

The problem of degeneracy is a big issue in the maximization problems of simplex method of linear programming problem. Due to degeneracy the problems will be more complicated and takes lots of iterations for its solution. General algorithm is as follows:

Suppose that X_1 is a greatest negative number and called a work column/entering variable and "r" will be there same ratios. It is too difficult to select the ratio for leaving variable in case of same ratio as shown in (Table1)

In this paper we have introduce a technique that solve such type of problems by selecting the smallest negative number instead of a greatest negative number from the z-equation, the problem of same ratios will be solved.

2, <u>METHODOLOGY</u>

A standard form of L.P problems as given

Maximize $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$

Subjected to $a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2$

……………………………..

$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_m$

And $x_{n \geq} 0$ where n = 1, 2 …,

In standard form all variables must be non-negative and all constraints will be in equality form. Slack variables (S_1, S_2 … S_n) are add for constraints and "less than or equal to" sign change into equality sign (=).

Algorithm for existing method

Step1. Initial Basic Feasible Solution

In basic feasible solution can be finding by setting (n-m) by putting non-basic variables equal to zero. Where n is number of variables and m is number of constraints.

Step2. Optimality Test

In case of maximization, for the optimal solution all numbers in Z-equation should be positive or zero. Other way repeats the step-3.

Step3. To promote the result

(i) Entering variable

For maximization, we select least negative entering variable from Z-equation and call it as work column in which a number appears.

(ii) Leaving variable

For leaving variable we find ratio by dividing all numbers of cost column by all positive numbers of work column and locate smallest ratio as a pivot equation.

a) In case, all numbers are negative in work column, then the problem has no solution.

b) In case, same ratios we prefer to the dominating slack variable.

(iii) **Pivot equation and new basic feasible solution**

By using elementary row operations, we compute pivot equation and other equations.

Number #1 To obtain the pivot equation

New Pivot equation = Old pivot equation ÷ Pivot element

Number #2 To obtain the all new equations including Z-equation.

New equation = Old equation – (its entering column coefficient) × (New pivot equation)

Tabular Form of Standard L.P Model is given **(Table-2)**

Table No.2

Basis	x_1 $x_2,\ldots x_n$ S_1	$S_2,\ldots S_n$	Solution	Ratio
Z	$-c_1$ $-c_2,\ldots c_n$ 0	$0,\ldots 0$	0	
S_1	a_{11} $a_{12},\ldots a_{1n}$ 1	$0,\ldots 0$	b_1	b_1/a_{11}
S_2	a_{21} $a_{22},\ldots a_{2n}$ 0	$1,\ldots 0$	b_2	b_2/a_{21}
.	. .		.	.
.	. .		.	.
.	. .		.	.
S_n	a_{m1} $a_{m2},\ldots a_{mnm}$ 0	$0,S_n$	b_m	b_m/a_{mn}

Let $-c_1$ is the smallest negative number.

Step2. Optimality Test

There are still negative numbers in Z-equation. So, the solution is not optimal, go to step-3.

Step3. To promote the result

i) Determination of entering variable

The least negative number in Z-equation is $-c_1$. So, x_1 is an entering variable.

ii) Determination of leaving variable

From the column of ratios in **(Table 3),** Let the smallest ratio is b_1/a_{11}. So, S_1 is a leaving variable.

Table No.3

Basis	x_1 $x_2,\ldots x_n S_1$ $S_2,\ldots S_n$	Solution
Z		
x_1 S_2 . Sn	1 $a_{12}/a_{11},\ldots a_{1n}/a_{11}$ $1/a_{11}$ 0,…0	b_1/a_{11}

Note: (In case of similar ratios in maximization of Simplex Method of L.P.P, we prefer to the dominating slack variable).

iii) Pivot equation and new basic feasible solution for pivot equation divide S_1 row by pivot element a_{11} by using Guass Jorden method, we get the **(Table 4).**

New equation = Old equation – (Its entering column coefficient) × (New pivot equation).

Table No.4

Basis	x_1 $x_2,\ldots x_n$ S_1 $S_2,\ldots S_n$	Solution
Z	0 $c_1a_{12}/a_{11}-c_2,\ldots,c_1a_{1n}/a_{11}-c_m$ c_1/a_{11} 0,…0	c_1b_1/a_{11}
x_1 S_2 . S_n	1 $a_{12}/a_{11},\ldots a_{1n}/a_{11}$ $1/a_{11}$ 0,…0 0 $a_{22}-a_{21}a_{12}/a_{11},\ldots a_{2n}-a_{21}a_{1n}/a_{11}$ $-a_{21}/a_{11}$ 1,0,…0 0 $a_{m2}-a_{m1}a_{12}/a_{11},\ldots a_{mn}-a_{m1}a_{1n}/a_{11}$ $-a_{m1}/a_{11}$ 0,1,0,...Sn	b_1/a_{11} $b_2-a_{21}b_1/a_{11}$ $b_m-a_{m1}b_1/a_{11}$

This process will be repeated unless all numbers in Z-equation become non-negative and the solution is called optimal solution.

3. RESULTS AND DISCUSSION FOR NUMERICAL EXAMPLES

Example: 1

Maximization $Z = 2x_1 + x_2$

Subject to

$$4x_1 + 3x_2 \le 12$$
$$4x_1 + x_2 \le 8$$
$$4x_1 - x_2 \le 8$$

Table -5

Iterations	Basis	X_1	X_2	S_1	S_2	S_3	Solutions	Ratios
1	Z	-2	-1	0	0	0	0	
	S_1	4	3	1	0	0	12	3
	S_2	4	1	0	1	0	8	2
	S_3	4		0	0	1	8	2

In the Z-equation, -2 is the greatest negative number **(Table 5)** so, X_1 will be work column/entering variable. To obtain the ratios divide all the elements of X_1 column to the elements of the solution column. We found the similar ratios as 2 in the column of ratio and that is known as degeneracy problems. To avoid from degeneracy problem we prefer to the smallest negative number in the Z-equation. Computation is given in **(Table 6).** We prefer to the X_1 and select least negative number as an entering column which gives the different ratios for leaving variable and that avoids the situation of degeneracy problems.

Table No.6

Iterations	Basis	X_1	X_2	S_1	S_2	S_3	Solutions	Ratios
1	Z	-2	-1	0	0	0	0	
	S_1	4	3	1	0	0	12	4
	S_2	4	1	0	1	0	8	8
	S_3	4	-1	0	0	1	8	-8

Example: 2

Maximization $Z = 0.75x_1 - 20x_2 + 0.5x_3 - 6x_4$

Subject to

$0.25x_1 - 8x_2 - x_3 + 9x_4 \le 0$

$0.5x_1 - 12x_2 - 0.5x_3 + 3x_4 \le 0$ $x_3 \le 1$

Where x_1, x_2, x_3 and $x_4 \ge 0$

Table No.7

Iterations	Basis	X_1	X_2	X_3	X_4	S_1	S_2	S_3	Solution	Ratio
1	Z	-3/4	20	-1/2	6	0	0	0	0	
	S_1	¼	-8	-1	9	1	0	0	0	0
	S_2	½	-12	-1/2	3	0	1	0	0	0
	S_3	0	0	1	0	0	0	1	1	Not exist

In this example 2, by using the G.B. Dantzig rule (taking as greatest negative number) shows in **(Table 7)** from the z-equation we notice that the given problem is a type of degeneracy or cyclic problem. Optimal solution of the problems obtained by using modified technique shows in **(Table 8).**

Table No.8

Iterations	Basis	X_1	X_2	X_3	X_4	S_1	S_2	S_3	Solution	Ratio
1	Z	-3/4	20	-1/2	6	0	0	0	0	
	S_1	¼	-8	-1	9	1	0	0	0	0
	S_2	½	-12	-1/2	3	0	1	0	0	0
	S_3	0	0	1	0	0	0	1	1	1

Example: 3

Maximization $Z = 20x_1 + 6x_2 + 8x_3$

Subject to

$8x_1 + 2x_2 + 3x_3 \leq 250$

$4x_1 + 3x_2 \leq 150$

$2x_1 + x_3 \leq 50$

Where x_1, x_2, and $x_3 \geq 0$

Table No. 9

Itera tion	Ba sis	X_1	X_2	X_3	S_1	S_2	S_3	Soluti on	Ratio
1	Z	-20	-6	-8	0	0	0	0	
	S_1	8	2	3	1	0	0	250	31.25
	S_2	4	3	0	0	1	0	150	37.5
	S_3	2	0	1	0	0	1	50	25

In the **(Table 9),** X_1 is called the work column by taking-20 is the negative greatest number of the z-equation. Then all elements of work column will divide the all elements of the solution and we get smallest ratio as 25 which make the pivot equation by dividing the all elements of S_3 equation by the 2. This is simple simplex method of linear programming problems. (**Table-10).**

Iterat ion	Ba sis	X_1	X_2	X_3	S_1	S_2	S_3	Solu tion	
1	Z	-20	-6	-8	0	0	0	0	
	S_1	8	2	3	1	0	0	250	125
	S_2	4	3	0	0	1	0	150	50
	S_3	2	0	1	0	0	1	50	Not exist

If we give preference to the least negative number in the z-equation then X_2 will be the work column. Divide all numbers of solution by the numbers of work column, gives 50 as a least number which make the pivot equation by dividing all numbers of S_2 by arbitrary number as 3.

Table No.11

S. no	Size of Problems		Optimal solution by Modified Algorithm		Optimal solution by Dantzig's Algorithm	
	M	N	No. of iterations	Optimal sol.	No. of iterations	Optimal Sol.
1	3	5	3	5	5	5
2	3	7	3	1.25	7	1.25
3	3	6	3	700	4	700

4. CONCLUSION

In this article, we have introduced a new pivot rule for maximization degeneracy problems. This modified technique helps to resolve the issue of degeneracy or cyclic problems as well as to select the entering variable (work column) and leaving variable (pivot equation). From the above **(Table 11),** we have noticed that modified pivot rule is faster and reduce the number of iterations as compare to existing pivot rule of simplex algorithm. Both techniques give same optimal solution. According to our observation the powerful pivot rule works very fast on the large size problems of simplex method of linear programming problems. Moreover, the pivot rule is equally efficient for simplex algorithm and their types of degeneracy as well as cyclic linear programming problems.

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