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Validation on Maturity Model of Feature Diagrams

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Abstract: Analysis of a model is always a hot area of research. The quality of a feature model plays a significant role in software production because it is being used in product development. In this paper, along with the verification of errors in feature models we are validating our previous work where we have presented algorithms for the detection of quality level for a given feature model. Here, we are presenting the linear formulas of the feature models that we use as input for the quality detection algorithms. Further, this version is validating our semantics based quality detection technique of feature models.

Keywords: Feature Model Analysis, Errors, Inconsistencies, dead features, invalid feature model.

1. INTRODUCTION

The large number of software's with almost same specifications lead us to Software Product Line (SPL) (Benavides, *et al.*,2007). In SPL approach, production is based on the reuse of existing components (Thomas, 2008). Programmers would be happy enough by getting the benefits of SPL by incorporating the commonalities and variabilities of software family. This leads us in the production of good quality software in a limited time period with reduced expenses (Naeem, 2012).

Feature models are modelled as hierarchical tree like combination of features which were coined in (Böckle, 2005). and was called FODA. The quality of a feature model has a vital role in designing a good quality products. Reflection of an effective and accurate representation of a system measures the quality of a feature model. The lesser is the presence of deficiencies in a feature modal, the more will be the quality of a feature modal (Clements,2001).

We have already contributed for the development of a quality detection mechanism and its semantics for feature modals in (Kang, *et al*, 1990), respectively. In this effort, we are presenting the linear logic based feature models verification for errors. This (Assad, *et al*, 2015), (Batory, *et al*, 2006) technique will also validate the semantics. The following sections of the paper are structured as: 2nd and 3rd sections provide important information and the validation of maturity model, respectively. The 4th section concludes the paper and reflects the future plans. (Kang, *et al*,1990).

2. BACKGROUND

Prof. Kang coined the term feature models in (FODA in 1990). Feature models are tree-like structures. The sets of permissible selection of features from a feature modal is called its instances. A correct instance of a feature model must contains all the mandatory features; at least one feature from an Or-group of features; exactly one feature from Alternative-group; and/or the optional features. This must be noted that a feature can only be selected only if its parent feature is selected. Thus $\{MP, Ca, Sc, Ba\}$ is the correct instance of a feature diagram shown in **(Fig-1)**.

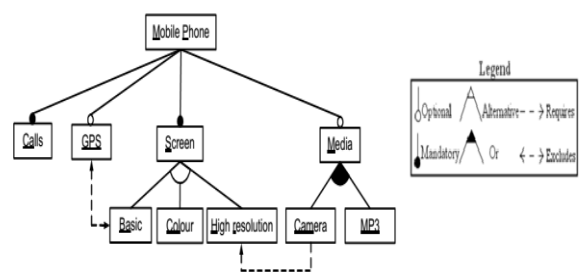

Fig. 1: A feature model of a mobile phone

FODA Maturity Model

We presented FODA Maturity Model which consist to multiple stages. The semantics of FODA maturity model are based on algorithms. Our algorithm are discussed in three levels:

Step 1. It consists of the code of logic.

Step 2. This contains the explanation of Level.

Step 3. This shows application of algorithm on the given example.

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For brevity, we have not added further details about the semantics. We refer the readers for further details about FODA Maturity Model and it semantics, respectively. **Linear Logic**

Jean-Yves Girard introduced Linear Logic (LL) in 1987 Unlike classical logic, more than one propositions of LL formula are not equivalent to single presence, i.e., $B \otimes B \neq B$, where B is an LL proposition. To explain LL semantics, let us use X and Yto represent the features:

1. Multiplicative Conjunction ($\otimes$). An LL formula$X \otimes Y$ shows the selection of X and Y features.
2. Additive Conjunction (&). A linear formula X&Y is representing choice X or Y.
3. Linear Implication($\multimap$). An LL formula $X \multimap Y$ means that a feature X must be selected if one needs to choose the feature Y.
4. Storage Operator(!). A linear expression ! A shows the choice of a feature A multiple times.

Sequent Calculus

A sequent written in Gentzen's style consists of two LL formulae separated by $\vdash$. If Γ and Δ are the multisets of the countable formulae then $\Gamma \vdash \Delta$ means that the $\otimes$ of sequences of Γ produces the & of formulae in Δ

$$\frac{\text{Hypothesis1} \qquad \text{Hypothesis2}}{\text{Conclusion}} \text{(Rule)}$$

In above inference rule, Rule is applied to Hypothesis1, Hyothesis2 to produce Conclusion. The inference system of LL has the following rule which we used in this paper:

$$\frac{}{A \vdash A} \text{(id)}$$

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \otimes B \vdash \Delta} (L\otimes) \qquad \frac{\Gamma \vdash A, \Delta \qquad \Gamma' \vdash B, \Delta}{\Gamma, \Gamma' \vdash A \otimes B, \Delta} (R\otimes)$$

$$\frac{\Gamma \vdash A, \Delta \qquad \Gamma \vdash B, \Delta}{\Gamma \vdash A \& B, \Delta} (R\&) \qquad \frac{\Gamma, B_1 \vdash \Delta}{\Gamma, A_1 \& B_2 \vdash \Delta} (L\&)$$

$$\frac{\Gamma \vdash A, \Delta \qquad \Gamma', B \vdash \Delta'}{\Gamma, A \rightarrow B, \Gamma' \vdash \Delta, \Delta'} (L\rightarrow) \qquad \frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \rightarrow B, \Delta} (R\rightarrow)$$

$$\frac{\Gamma \vdash \Delta}{\Gamma, !B \vdash \Delta} (W!) \qquad \frac{\Gamma, !B, !B \vdash \Delta}{\Gamma, !B \vdash \Delta} (C!)$$

$$\frac{\Gamma, B \vdash \Delta}{\Gamma, !B \vdash \Delta} (D!) \qquad \frac{!\Gamma \vdash B}{!\Gamma \vdash !B} (R!)$$

Validation of Maturity Model

For the validation of our approach of(Böckle, and Linden, 2005), we validate each stage by using LF(fd) $\vdash if_i$, where LF(fd) and *if* refer to the LL formula of a feature model (fd) the derivable instance of fd, respectively. Let us now validate Level (stage) 1:

Level 1: Instance-able

This level contain void feature models. For example, let us consider a feature model depicted below:

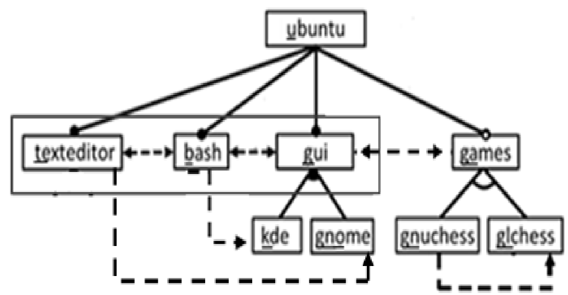

Fig. 2: Void feature model

By using the rules shown in (Mendonça, 2009), we encode the above feature model in LL formula (LF(fd)) as:

$$!u \otimes (!u \rightarrow !t \otimes !b \otimes (!t^{\perp} \otimes !b^{\perp}) \otimes (!g \otimes (!g \rightarrow (!k^{\perp} \otimes !gno) \& (!k \otimes !(gno)^{\perp}) \& (!k \otimes !gno))) \otimes (!b \otimes !k) \otimes (!t \otimes !gno) \otimes (!b^{\perp} \otimes !g^{\perp} \otimes !k^{\perp} \otimes !(gno)^{\perp}) \otimes ((!(ga)^{\perp}((\otimes !gn)^{\perp}) \otimes !(gl)^{\perp}) \& (!ga \otimes (!ga \rightarrow (!(gn)^{\perp} \otimes !gl) \& (!gn \otimes !(gl)^{\perp}))) \otimes (!gn \otimes !gl)) \otimes (!g^{\perp} \otimes !(ga)^{\perp} \otimes !(gn)^{\perp} \otimes !(gl)^{\perp}))$$

Only formula that can be derived from the above formula is

$$!u \otimes !t^{\perp} \otimes !b^{\perp} \otimes !g^{\perp} \otimes !k^{\perp} \otimes !gno^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}$$

This formula clearly states that only selectable feature is *!u* all the other features cannot be selected. This discussion can also be validated formally by using linear logic as:

$\overline{\qquad\qquad\qquad\qquad\qquad\qquad}$ (id) $\qquad\qquad$ $\overline{\qquad}$ (id)

$!g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp} \vdash !g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}$ $\qquad\qquad$ $!gn \vdash !gn$

$\overline{\qquad\qquad\qquad\qquad\qquad\qquad}$ (R⊗)

$!gn, !g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp} \vdash !g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp} \otimes !gn$

$\overline{\qquad\qquad\qquad\qquad\qquad\qquad}$ (W!, L⊗, L⊥, W!, L⊗)

$!gl, !gn, !g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp} \vdash !g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp} \otimes !gn$

$\overline{\qquad}$ (id) $\qquad$ $\overline{\qquad\qquad\qquad\qquad\qquad\qquad}$ (L&)

$!ga \vdash !ga$ $\qquad$ $(!gn^{\perp} \otimes !gl) \& (!gn \otimes !gl^{\perp}) \otimes (!gl \otimes !gn) \otimes (!g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) \vdash g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}$

$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$ (L→, L⊗)

$(!ga \otimes (!ga \rightarrow (!gn^{\perp} \otimes !gl) \& (!gn \otimes !gl^{\perp}))) \otimes (!gl \otimes !gn) \otimes (!g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) \vdash !g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}$

$\overline{\qquad\qquad\qquad\qquad\qquad\qquad}$ (L&) $\qquad$ $\overline{\qquad\qquad}$ (id)

$((!ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) \& (!ga \otimes (!ga \otimes (!ga \rightarrow (!gn^{\perp} \otimes !gl) \& (!gn \otimes !gl^{\perp})))) \otimes (!gn \otimes !gl)) \otimes (!g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) \vdash !g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}$ $\qquad$ $!b^{\perp} \otimes !k^{\perp} \otimes !gno^{\perp} \vdash !b^{\perp} \otimes !k^{\perp} \otimes !gno^{\perp}$

$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$ (R⊗, W!, W!, L⊗, W!, W!, L⊗)

$(!b \otimes !k) \otimes (!gno \otimes !t) \otimes (!b^{\perp} \otimes !g^{\perp} \otimes !k^{\perp} \otimes !gno^{\perp}) \otimes ((!ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) \& (!ga \otimes (!ga \rightarrow (!gn^{\perp} \otimes !gl) \& (!gn \otimes !gl^{\perp}))) \otimes (!gn \otimes !gl)) \otimes (!g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) \vdash !g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp} \otimes !b^{\perp} \otimes !k^{\perp} \otimes !gno^{\perp}$

$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$ (L⊗, W!, W!,L⊗)

$(!k^{\perp} \otimes !gno), (!b \otimes !k) \otimes (!gno \otimes !t) \otimes (!b^{\perp} \otimes !g \otimes !k \otimes !gno^{\perp}) \otimes ((!ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) \& (!ga \otimes (!ga \rightarrow (!gn^{\perp} \otimes !gl) \& (!gn \otimes !gl^{\perp}))) \otimes (!gn \otimes !gl)) \otimes (!g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) \vdash !g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp} \otimes !b^{\perp} \otimes !k^{\perp} \otimes !gno^{\perp}$

$\overline{\qquad}$ (id) $\qquad$ $\overline{\qquad\qquad\qquad\qquad\qquad\qquad}$ (L &)

$!g \vdash !g$ $\qquad$ $(!k^{\perp} \otimes !gno) \& (!k \otimes !gno^{\perp}) \& (!k \otimes !gno), (!b \otimes !k) \otimes (!gno \otimes !t) \otimes (!b^{\perp} \otimes !g^{\perp} \otimes !k^{\perp} \otimes !gno^{\perp}) \otimes ((!ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) \& (!ga \otimes (!ga \rightarrow (!gn^{\perp} \otimes !gl) \& (!gn \otimes !gl^{\perp}))) \otimes (!gn \otimes !gl)) \otimes (!g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) \vdash !g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp} \otimes !b^{\perp} \otimes !k \otimes !gno^{\perp}$

$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$ (L →, L⊗)

$(!g \otimes (!g \rightarrow (!k^{\perp} \otimes !gno) \& (!k \otimes !gno^{\perp}) \& (!k \otimes !gno))), (!b \otimes !k) \otimes (!gno \otimes !t) \otimes (!b^{\perp} \otimes !g^{\perp} \otimes !k^{\perp} \otimes !gno^{\perp}) \otimes ((!ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) \& (!ga \otimes (!ga \rightarrow (!gn^{\perp} \otimes !gl) \& (!gn \otimes !gl^{\perp}))) \otimes (!gn \otimes !gl)) \otimes (!g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) !g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl \otimes !b^{\perp} \otimes !k^{\perp} \otimes !gno^{\perp}$

$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$ (L⊗) $\qquad$ $\overline{\quad}$ (id)

$(!g \otimes (!g \rightarrow (!k^{\perp} \otimes !gno) \& (!k \otimes !gno^{\perp}) \& (!k \otimes !gno))) \otimes (!b \otimes !k) \otimes (!gno \otimes !t) \otimes (!b^{\perp} \otimes !g^{\perp} \otimes !k^{\perp} \otimes !gno^{\perp}) \otimes ((!ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) \& (!ga \otimes (!ga \otimes (!ga \rightarrow (!gn^{\perp} \otimes !gl) \& (!gn \otimes !gl^{\perp})))) \otimes (!gn \otimes !gl)) \otimes (!g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) \vdash !g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp} \otimes !b^{\perp} \otimes !k^{\perp} \otimes !gno^{\perp}$ $\qquad$ $!t^{\perp} \vdash !t^{\perp}$

$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$ (R⊗, W!, L⊗,W!,W!)

$!b, !t, (!b^{\perp} \otimes !t^{\perp}) \otimes (!g \otimes (!g \rightarrow (!k^{\perp} \otimes !gno) \& (!k \otimes !gno^{\perp}) \& (!k \otimes !gno)))) \otimes (!b \otimes !k) \otimes (!gno \otimes !t) \otimes (!b^{\perp} \otimes !g^{\perp} \otimes !^{\perp} \otimes !gno^{\perp}) \otimes ((!ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) \& (!ga \otimes (!ga \rightarrow (!gn^{\perp} \otimes !gl) \& (!gn \otimes !gl^{\perp})))) \otimes (!gn \otimes !gl)) \otimes (!g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) \vdash !g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp} \otimes !b^{\perp} \otimes !k^{\perp} \otimes !gno^{\perp} \otimes !t^{\perp}$

$\overline{\quad}$ (id) $\qquad$ $\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$ (L⊗)

$!u \vdash !u$ $\qquad$ $(!b \otimes !t) \otimes (!b^{\perp} \otimes !t^{\perp}) \otimes (!g \otimes (!g \rightarrow (!k^{\perp} \otimes !gno) \& (!k \otimes !gno^{\perp}) \& (!k \otimes !gno))) \otimes (!b \otimes !k) \otimes (!gno \otimes !t) \otimes (!b^{\perp} \otimes !g^{\perp} \otimes !k^{\perp} \otimes !gno^{\perp}) \otimes ((!ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) \& (!ga \otimes (!ga \rightarrow (!gn^{\perp} \otimes !gl) \& (!gn \otimes !gl^{\perp}))) \otimes (!gn \otimes !gl)) \otimes (!g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) \vdash !g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp} \otimes !b^{\perp} \otimes !k^{\perp} \otimes !gno^{\perp} \otimes !t^{\perp}$

$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$ (L→) $\qquad$ $\overline{\qquad}$ (id)

$!u, (!u \rightarrow (!b \otimes !t) \otimes (!b^{\perp} \otimes !t^{\perp}) \otimes (!g \otimes (!g \rightarrow (!k^{\perp} \otimes !gno) \& (!k \otimes !gno^{\perp}) \& (!k \otimes !gno))) \otimes (!b \otimes !k) \otimes !gno \otimes !t) \otimes (!b^{\perp} \otimes !g^{\perp} \otimes !k^{\perp} \otimes !gno^{\perp}) \otimes ((!ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) \& (!ga \otimes (!ga \rightarrow (!gn^{\perp} \otimes !gl) \& (!gn \otimes !gl^{\perp}))) \otimes (!gn \otimes !gl)) \otimes (!g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp})) \vdash !g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp} \otimes !b^{\perp} \otimes !k^{\perp} \otimes !gno^{\perp} \otimes !t^{\perp}$ $\qquad$ $!u \vdash !u$

$\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$ (R⊗, C!, L⊗)

$!u \otimes (u \rightarrow (!b \otimes !t) \otimes (!b^{\perp} \otimes !t^{\perp}) \otimes (!g \otimes (!g \rightarrow (!k^{\perp} \otimes !gno) \& (!k \otimes !gno^{\perp}) \& (!k \otimes !gno))) \otimes (!b \otimes !k) \otimes (!gno \otimes !t) \otimes (!b^{\perp} \otimes !g^{\perp} \otimes !k^{\perp} \otimes !gno^{\perp}) \otimes ((!ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp}) \& (!ga \otimes (!ga \rightarrow (!gn^{\perp} \otimes !gl) \& (!gn \otimes !gl^{\perp}))) \otimes (!gn \otimes !gl)) \otimes (!g \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp})))) \vdash !g^{\perp} \otimes !ga^{\perp} \otimes !gn^{\perp} \otimes !gl^{\perp} \otimes !b^{\perp} \otimes !k^{\perp} \otimes !gno^{\perp} \otimes !t^{\perp} \otimes !u$

Level 2: Acceptable

basic constraints of feature models already discussed in Section 2.

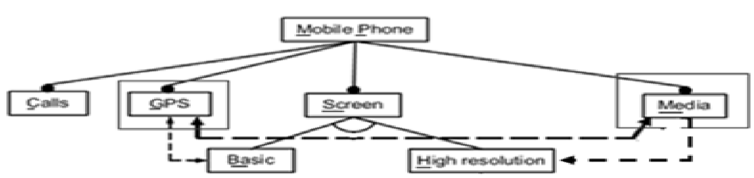

Fig. 3: Invalid product feature model

Let us now validate by using the encoding provided in [13], LL formula LF(fd) of **Fig3** is given as:

$$!MP \otimes (!MP \rightarrow !C \otimes !G \otimes (!Sc \otimes (!Sc \rightarrow (!Ba \otimes !H^{\perp}) \& (!Ba^{\perp} \otimes !H)))) \otimes (!G^{\perp} \otimes !Ba^{\perp}) \otimes !Me \otimes (!Me \otimes !H) \otimes (!G^{\perp} \otimes !Me^{\perp}))))$$

LF(fd) above deriving the linear formula

$$!MP \otimes !C \otimes !G^{\perp} \otimes !Sc \otimes !Ba^{\perp} \otimes !H \otimes !Me^{\perp}$$

validates our discussion about this level. Let us now check the validation of this instance from LF(fd):

$$\dfrac{}{!G^{\perp}\otimes !Me^{\perp} \vdash !G^{\perp}\otimes !Me^{\perp}}\,(id) \qquad \dfrac{}{!H \vdash !H}\,(id)$$

$$(!Me\otimes !H) \otimes (!G^{\perp}\otimes !Me^{\perp}) \vdash !G^{\perp}\otimes !Me^{\perp}\otimes !H \quad (R\otimes,\ W!,\ L\otimes)$$

$$!Me\otimes(!Me\otimes !H)\otimes(!G^{\perp}\otimes !Me^{\perp}) \vdash !G^{\perp}\otimes !Me^{\perp}\otimes !H \quad (W!,\ L\otimes) \qquad \dfrac{}{!Ba^{\perp} \vdash !Ba^{\perp}}\,(id)$$

$$!Ba^{\perp}, !Me\otimes (!Me\otimes !H) \otimes (!G^{\perp}\otimes !Me^{\perp}) \vdash !G^{\perp}\otimes !Me^{\perp}\otimes !H\otimes !Ba^{\perp} \quad (R\otimes)$$

$$!G^{\perp}, !Ba^{\perp}, !Me\otimes (!Me\otimes !H) \otimes (!G^{\perp}\otimes !Me^{\perp}) \vdash !G^{\perp}\otimes !Me^{\perp}\otimes !H\otimes !Ba^{\perp} \quad (W!)$$

$$(!G^{\perp}\otimes !Ba^{\perp})\otimes !Me\otimes (!Me\otimes !H) \otimes (!G^{\perp}\otimes !Me^{\perp}) \vdash !G^{\perp}\otimes !Me^{\perp}\otimes !H\otimes !Ba^{\perp} \quad (L\otimes)$$

$$!H, (!G^{\perp}\otimes !Ba^{\perp})\otimes !Me\otimes(!Me\otimes !H)\otimes(!G^{\perp}\otimes !Me^{\perp}) !G^{\perp}\otimes !Me^{\perp}\otimes !H\otimes !Ba^{\perp} \quad (W!)$$

$$!Ba^{\perp}, !H, (!G^{\perp}\otimes !Ba^{\perp})\otimes !Me\otimes (!Me\otimes !H) \otimes (!G^{\perp}\otimes !Me^{\perp}) \vdash !G^{\perp}\otimes !Me^{\perp}\otimes !H\otimes !Ba^{\perp} \quad (W!)$$

$$(!H\otimes !Ba^{\perp})\otimes (!G^{\perp}\otimes !Ba^{\perp})\otimes !Me\otimes (!Me\otimes !H) \otimes (!G^{\perp}\otimes !Me^{\perp}) \vdash !G^{\perp}\otimes !Me^{\perp}\otimes !H\otimes !Ba^{\perp} \quad (L\otimes)$$

$$\dfrac{}{!Sc \vdash !Sc}\,(id) \qquad (!Ba\otimes !H^{\perp})\&(!Ba^{\perp}\otimes !H)\otimes(!G^{\perp}\otimes !Ba^{\perp})\otimes !Me\otimes(!Me\otimes !H)\otimes(!G^{\perp}\otimes !Me^{\perp}) \vdash !G^{\perp}\otimes !Me^{\perp}\otimes !H\otimes !Ba^{\perp} \quad (L\&)$$

$$\dfrac{}{!Sc \vdash !Sc}\,(id) \qquad !Sc, (!Sc\to(!Ba\otimes !H^{\perp})\&(!Ba^{\perp}\otimes !H))\otimes(!G^{\perp}\otimes !Ba^{\perp})\otimes !Me\otimes(!Me\otimes !H)\otimes(!G^{\perp}\otimes !Me^{\perp}) \vdash !G^{\perp}\otimes !Me^{\perp}\otimes !H\otimes !Ba^{\perp} \quad (L\to)$$

$$!Sc\otimes(!Sc\to(!Ba\otimes !H^{\perp})\&(!Ba^{\perp}\otimes !H)))\otimes(!G^{\perp}\otimes !Ba^{\perp})\otimes !Me\otimes(!Me\otimes !H)\otimes(!G^{\perp}\otimes !Me^{\perp}) \vdash !G^{\perp}\otimes !Me^{\perp}\otimes !H\otimes !Ba^{\perp}\otimes !Sc \quad (R\otimes,\ C!,\ L\otimes)$$

$$!G\otimes(!Sc\otimes(!Sc\to(!Ba\otimes !H^{\perp})\&(!Ba^{\perp}\otimes !H)))\otimes(!G^{\perp}\otimes !Ba^{\perp})\otimes !Me\otimes(!Me\otimes !H)\otimes(!G^{\perp}\otimes !Me^{\perp}) \vdash !G^{\perp}\otimes !Me^{\perp}\otimes !H\otimes !Ba^{\perp}\otimes !Sc \quad (W!,\ L\otimes) \qquad \dfrac{}{!C \vdash !C}\,(id)$$

$$\dfrac{}{!MP \vdash !MP}\,(id) \qquad !C\otimes !G\otimes(!Sc\otimes(!Sc\to(!Ba\otimes !H^{\perp})\&(!Ba^{\perp}\otimes !H))))\otimes(!G^{\perp}\otimes !Ba^{\perp})\otimes !Me\otimes(!Me\otimes !H)\otimes(!G^{\perp}\otimes !Me^{\perp}) \vdash !G^{\perp}\otimes !Me^{\perp}\otimes !H\otimes !Ba^{\perp}\otimes !Sc\otimes !C \quad (R\otimes,\ L\otimes)$$

$$(!MP\otimes(!MP\to C\otimes !G\otimes(Sc\otimes(Sc\otimes(Sc\to(!Ba\otimes !H^{\perp})\&(!Ba^{\perp}\otimes !H)))))\otimes(!G^{\perp}\otimes !Ba^{\perp})\otimes !Me\otimes(!Me\otimes !H)\otimes(!G^{\perp}\otimes !Me^{\perp}))) \vdash !G^{\perp}\otimes !Me^{\perp}\otimes !H\otimes !Ba^{\perp}\otimes !Sc\otimes !C \quad (L\to,\ L\otimes)$$

Level 3: Managed

This level contains the feature models that suffer from dead feature, false variable feature, and conditionally dead feature errors. For example, please consider the feature models depicted in **(Fig. 4).**

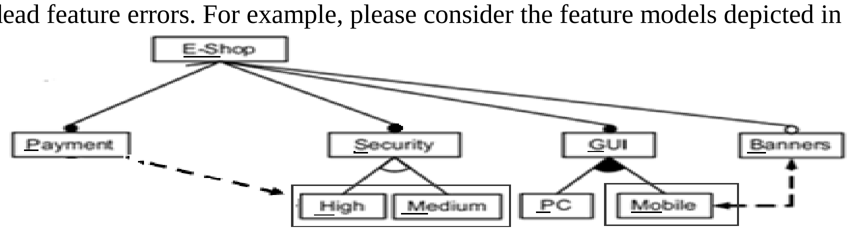

Fig. 4:Defected feature model

By using the encoding rules mentioned in we interpret the feature model of above diagram in LL and show the required derivation along with its instance as:

$$(!ES \otimes (!ES \to !P \otimes (!S \otimes (!S \to (!H \otimes !M^{\perp})\&(!H^{\perp} \otimes !M)))) \otimes (!P \otimes !H) \otimes (!G \otimes (!G \to (!PC \otimes !Mo^{\perp})\&(!PC^{\perp} \otimes !Mo)\&(!PC \otimes !Mo)))) \otimes (!B^{\perp}\&!B) \otimes (!Mo^{\perp} \otimes !B^{\perp})))) \vdash !ES\otimes !P\otimes !S\otimes !H\otimes !M^{\perp}\otimes !G\otimes !PC\otimes !Mo^{\perp}\otimes !B$$

The proof tree of required derivation is obtained by using the formal inference system of linear logic as:

(id)

$!Mo^{\bot} \vdash !Mo^{\bot}$

(W!)

$!B^{\bot}, !Mo^{\bot} \vdash !Mo^{\bot}$

(L⊗) (id)

$!B^{\bot}\otimes !Mo^{\bot} \vdash !Mo^{\bot}$ $\quad !B \vdash !B$

(R⊗, L&)

$(!B^{\bot} \,\&\, !B), !B^{\bot}\otimes !Mo^{\bot} \vdash !Mo^{\bot}\otimes !B$

(L⊗) (id)

$(!B^{\bot}\&!B)\otimes(!B^{\bot}\otimes!Mo^{\bot}) \vdash !Mo^{\bot}\otimes!B$ $\quad !PC\vdash!PC$

(R⊗, W!)

$(!Mo\otimes!PC),(!B^{\bot}\&!B)\otimes(!B^{\bot}\otimes!Mo^{\bot}) \vdash !Mo^{\bot}\otimes!B\otimes!PC$

(id) (L&)

$!G\vdash!G$ $\quad (!PC\otimes!Mo^{\bot})\&(!PC^{\bot}\otimes!Mo)\&(PC\otimes!Mo), (!B^{\bot}\&!B)\otimes(!B^{\bot}\otimes!Mo^{\bot}) \vdash !Mo^{\bot}\otimes!B\otimes!PC$

(L→) (id)

$!G, (!G\rightarrow((!PC\otimes!Mo^{\bot})\&(!PC^{\bot}\otimes!Mo)\&(PC\otimes!Mo)), !B^{\bot}\&!B)\otimes(!B^{\bot}\otimes!Mo^{\bot}) \vdash !Mo^{\bot}\otimes!B\otimes!PC$ $\quad !G\vdash!G$

(R⊗,L⊗,L⊗,L⊗) (id)

$!G\otimes(!G\otimes(!G\rightarrow((!PC\otimes!Mo^{\bot})\&(!PC^{\bot}\otimes!Mo)\&(PC\otimes!Mo)))\otimes(!B^{\bot}\&!B)\otimes(!B^{\bot}\otimes!Mo^{\bot})\vdash!Mo^{\bot}\otimes!B\otimes!PC$ $\quad !H\vdash!H$

(R⊗, W!, L⊗)

$!P\otimes!H,(!G\otimes(!G\rightarrow(!PC\otimes!Mo^{\bot})\&(PC^{\bot}\otimes!Mo)\&(!PC\otimes!Mo)))\otimes(!B^{\bot}\&!B)\otimes(!B^{\bot}\otimes!Mo^{\bot})\vdash!Mo^{\bot}\otimes!B\otimes!PC\otimes!G\otimes!H$

(L⊗) (id)

$(!P\otimes!H)\otimes(!G\otimes(!G\rightarrow(!PC\otimes!Mo^{\bot})\&(PC^{\bot}\otimes!Mo)\&(!PC\otimes!Mo)))\otimes(!B^{\bot}\&!B)\otimes(!B^{\bot}\otimes!Mo^{\bot})\vdash !Mo^{\bot}\otimes!B\otimes!PC\otimes!G\otimes!H$ $\quad !M^{\bot}\vdash!M^{\bot}$

(R⊗,W!,L⊗)

$(!H\otimes!M^{\bot}),(!P\otimes!H)\otimes(!G\otimes(!G\rightarrow(!PC\otimes!Mo^{\bot})\&(PC^{\bot}\otimes!Mo)\&(!PC\otimes!Mo)))\otimes(!B^{\bot}\&!B)\otimes(!B^{\bot}\otimes!Mo^{\bot})\vdash!Mo^{\bot}\otimes!B\otimes!PC\otimes!G\otimes!H$

(id) (L&)

$!S\vdash!S$ $\quad (!H\otimes!M^{\bot})\&(!H\otimes!M^{\bot}),(!P\otimes!H)\otimes(!G\otimes(!G\rightarrow(!PC\otimes!Mo^{\bot})\&(PC^{\bot}\otimes!Mo)\&(!PC\otimes!Mo)))\otimes(!B^{\bot}\&!B)\otimes(!B^{\bot}\otimes!Mo^{\bot})\vdash!Mo^{\bot}\otimes!B\otimes!PC\otimes!G\otimes!H$

(L→) (id)

$!S,!S\rightarrow((!H\otimes!M^{\bot})\&(!H\otimes!M^{\bot})),(!P\otimes!H)\otimes(!G\otimes(!G\rightarrow(!PC\otimes!Mo^{\bot})\&(PC^{\bot}\otimes!Mo)\&(!PC\otimes!Mo)))\otimes(!B^{\bot}\&!B)\otimes(!B^{\bot}\otimes!Mo^{\bot})\vdash!Mo^{\bot}\otimes!B\otimes!PC\otimes!G\otimes!H$ $\quad !S\vdash!S$

(R⊗,C!,L⊗, L⊗) (id)

$!S\otimes(!S\rightarrow((!H\otimes!M^{\bot})\&(!H\otimes!M^{\bot})))\otimes(!P\otimes!H)\otimes(!G\otimes(!G\rightarrow(!PC\otimes!Mo^{\bot})\&(PC^{\bot}\otimes!Mo)\&(!PC\otimes!Mo)))\otimes(!B^{\bot}\&!B)\otimes(!B^{\bot}\otimes!Mo^{\bot})\vdash!Mo^{\bot}\otimes!B\otimes!PC\otimes!G\otimes!H\otimes!S$ $\quad !P\vdash!P$

(id) (R⊗,L⊗)

$!ES\vdash!ES$ $\quad !P\otimes!S\otimes(!S\rightarrow((!H\otimes!M^{\bot})\&(!H\otimes!M^{\bot})))\otimes(!P\otimes!H)\otimes(!G\otimes(!G\rightarrow(!PC\otimes!Mo^{\bot})\&(PC^{\bot}\otimes!Mo)\&(!PC\otimes!Mo)))\otimes(!B^{\bot}\&!B)\otimes(!B^{\bot}\otimes!Mo^{\bot})\vdash!Mo^{\bot}\otimes!B\otimes!PC\otimes!G\otimes!H\otimes!S$

(L→) (id)

$!ES,(!ES\rightarrow(!P\otimes!S\otimes(!S\rightarrow((!H\otimes!M^{\bot})\&(!H\otimes!M^{\bot})))\otimes(!P\otimes!H)\otimes(!G\otimes(!G\rightarrow(!PC\otimes!Mo^{\bot})\&(PC^{\bot}\otimes!Mo)\&(!PC\otimes!Mo)))\otimes(!B^{\bot}\&!B)\otimes(!B^{\bot}\otimes!Mo^{\bot})))\vdash!Mo^{\bot}\otimes!B\otimes!PC\otimes!G\otimes!H\otimes!S$ $\quad !ES\vdash!ES$

(R⊗,C!,L⊗)

$!ES\otimes(!ES\rightarrow(!P\otimes!S\otimes(!S\rightarrow((!H\otimes!M^{\bot})\&(!H\otimes!M^{\bot})))\otimes(!P\otimes!H)\otimes(!G\otimes(!G\rightarrow(!PC\otimes!Mo^{\bot})\&(PC^{\bot}\otimes!Mo)\&(!PC\otimes!Mo)))\otimes(!B^{\bot}\&!B)\otimes(!B^{\bot}\otimes!Mo^{\bot})))\vdash!Mo^{\bot}\otimes!B\otimes!PC\otimes!G\otimes!H\otimes!S\otimes!ES$

3. CONCLUSIONS

This paper was a step of our ongoing effort of computing the quality of a given feature model. The contributions of the paper is the validation of maturity model of feature diagrams. This validation and computation is backed by the formal sequent calculus of linear logic.

After borrowing the encoding from our previous work we encoded feature diagrams into linear formulas and instances into instance formulas. As, feature diagram generates instances so, for validation we derived instance formula from linear formula and it proved the results of our semantics based quality detection .

We computed the validation of FODA maturity model of feature diagrams. We have proved that semantics of FODA maturity model is also realized at the level of formal semantics of linear logic.

REFERENCES:

Assad, G. M., M. Naeem, and H. A. Wahab, (2014). Towards Cardinality-based Service Feature Diagrams, Computational Ecology and Software: 4(3), 69-73.

Assad, G. M., M. Naeem, and H. A. Wahab, (2015). Semantics of Cardinality-based Service Feature Diagrams, Journal of Selforganizology: 4(2), 102-114.

Batory, D., and D. R. Antonio, (2006). Automated analysis of feature models: challenges ahead. Communications of the ACM, 49(12): 45-47.

Batory, D., (2005). Feature models, grammars, and propositional formulas. Springer Berlin Heidelberg. 1: 07-20.

Benavides, D., S. Segura, and A. R. Cortés, (2010). Automated analysis of feature models 20 years later: A literature review. Information Systems, 35(6): 615-636.

Benavides, D., (2007). On the Automated Analysis of Software Product Lines Using Feature Models. Dissertation, Universidad de Sevilla, Spain.

Böckle, G., and F. V. D. Linden, (2005). Software product line engineering. Ed. Klaus Pohl. Book Vol. 10. Heidelberg: Springer.

Clements, P., (2001). Software Product Lines: Practices and Patterns. Addison-Wesley Longman Publishing Co., Inc., Boston, MA, USA.

Cosmo, D. R., and D. Miller, (2010). Linear Logic. In Edward N. Zalta, editor, The Stanford Encyclopedia of Philosophy. Stanford University, USA.

Felfernig, A., B. David, J. Galindo, and F. Reinfrank, (2013). Towards Anomaly Explanations in Feature Models. In Proceedings of the Proceedings of the 15th International Configuration Workshop (ConfWS-2013): 117~ 124.

Girard, J., (1987). Linear Logic. Theoretical Computer Science, 50:1–102.

Girard, J., (1995). Linear logic: Its syntax and semantics. In: Proceedings of the Workshop on Advances in Linear Logic, 1-41, New York, NY, USA.

Hemakumar, A., (2008). Finding Contradictions in Feature Models. In Proceeding of Workshop on Analyses of Software Product Lines (ASPL 2008), collocated with International Software product Line Conference (SPLC 2008): 183-190, Limerick, Ireland.

Javed, M., M. Naeem, and H. A. Wahab (2014). Towards the Maturity Model for Feature Oriented Domain Analysis, Computational Ecology and Software: 4(3), 170-182.

Javed, M., M. Naeem, and H. A. Wahab, (2015). Semantics of the Maturity Model for Feature Oriented Domain Analysis, Computational Ecology and Software: 4(3), 77-112

Mendonça, M., (2009). Efficient reasoning techniques for large scale feature models. Diss. University of Waterloo.

Naeem, M., and R. Heckel, (2011). “Towards Matching of Service Feature Diagrams based on Linear Logic”, in proceedings of 15th Internatiinal Software Product Line Conference, Vol. 2 (SPLC'11), Ina Schaefer, Isabel John, and Claus Schmidt (Eds.). ACM, New York, NY, USA, Article vol. 13, 8, 21-26., Munich, Germany.

Naeem, M., (2012). Matching of Service Feature Diagrams based on Linear Logic. Dissertation, Department of Computer Science, University of Leicester, UK.

Rosso, C. D., (2006). Experiences of performance tuning software product family architectures using a scenario-driven approach. In Proceedings of the 10th International Conference on Evaluation and Assessment in Software Engineering (EASE 2006): 30~39, Keele, Steffildahire, UK

Rubin J. and and M. Chechik (2012). Combining related products into product lines. In Proceedings of the 15th international conference on Fundamental Approaches to Software Engineering (FASE'12), Juan Lara and Andrea Zisman (Eds.). Springer-Verlag, Berlin, Heidelberg, 285~300

Segura, S., D. Benavides, and A. R. Cortés, (2010). FaMa Test Suite v1.2, Applied Software Engineering Research Group, University of Seville, Spain, Technical Report ISA-10-TR-0:: 1~52

Segura, S., (2011). Extended Support for the Automated Treatment of Feature Models. Dissertation, University of Sevilla, Spain.

Segura, S., D. Benavides, and A. R. Cortés, (2011). Functional testing of feature model analysis tools: a test suite. Software, IET 5, no. 1: 70-82

Thomas, E., (2008). SOA: principles of service design. Vol. 1. Upper Saddle River: Prentice Hall.

Thörn, C., (2007). A Quality Model for Evaluating Feature Models. In Proceedings of second volume (Workshops) of the 11th International Software Product Lines Conference, Kindai Kagaku Sha Co. Ltd., Tokyo, Japan: 184~190.

Thörn, C., (2010). On the Quality of Feature Models. Dissertation, Department of Computer and Information Science, Linköping University, Sweden.

Trinidad, P., D. Benavides, A. Duran, A. Ruiz-Cortes, and M. Toro (2008). Automated error analysis for the agilization of feature modeling. Journal of Systems and Software 81(6): 883-896

Zhang, G., H. Ye, and Y. Lin, (2011). Feature model validation: A constraint propagation-based approach. In 10th International Conference on Software Engineering Reaserch and Practice’, Las Vegas, US.