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Modified Free Derivative Open Method for Solving Non-Linear Equations

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Abstract: In this paper a free derivative open method has been developed to find the root of nonlinear equations. Throughout the study it has been observed that developed algorithm is superior to all existing root location methods in term of iteration as well as accuracy viewpoint. Couple of numerical examples related with algebraic and transcendental functions are existing in this paper to analysis the order of efficiency of develop method. C++ and EXCELL are used to justify the results and graphical representation of proposed method. It has been observed from the results and comparison of proposed method is that the Modified Free Derivative Open Method is better than existing root location methods.

Keywords: non-linear equation, root location methods, forward difference operator, order of convergence, Absolute percentile error**.**

1. INTRODUCTION

From many years estimating a root of non-linear equation are attracted researcher attentions. Lots of variants of accelerated methods had been introduced and given good result for solving non-linear equation.

$$\text{i.e. } F(x)=0 \qquad (1)$$

In fact, solving non-linear equation (1) is utmost important problem in Applied Mathematics, which arises in many scientific and engineering fields, for example: Distance, rate, time problem, population change, Trajectory of a ball, etc. Due to their importance, several methods have suggested and analyzed under certain condition. Such as most commonly used root location techniques of non-linear equation include bisection method, regula-falsi method and secant method. These methods are sure to converge but slow bracketing methods which require two initial guesses (Sangah *et al,* 2016) (Siyal *et al,* 2016) developed a new bracketing algorithm to reduce no: of iterations in contrast with the well-known bracketing methods i.e. bisection method and regula-falsi method and (Allame *et al,* 2012) using mid-point to construct a new iterated method, which is more quickly convergence than Newton raphson method, hybrid or new hybrid iterated method and midpoint Newton raphson method. Whereas some root location techniques required only one initial guess includes Steffensen method and Newton raphson method. These methods are fast converging Open methods and these methods are effective more for solving nonlinear equation. (Ahmed *et al,* 2014) designed a computed method which had been compared with well-known Steffensen method for composed method had been observed to better approximation and (Akram *et al,* 2015) describe all about Newton raphson method which is all-Inclusive to solve the non-square and non-linear problem. This paper has been discussed a Newton raphson formula, Algorithm, Use & Drawback of Newton raphson method.

Moreover, in numerical analysis order of convergence and computational efficiency become an efficient way to obtain approximate solution and it has been vigorous field of research. Whereas Order of convergence is a speed at which a given iterated sequence converges to the root and computational efficiency show economy of iterated method. In similar manners, various numerical methods have been developed for order of convergence and computational efficiency using different techniques including finite differences (Srivastava *et al,* 2010) and (Kumar *et al,* 2015). On the other hand, in recently years piles of free derivative techniques had been developed that are simpler, easier to use and that are free from any pitfall by (Soomro *et al.,* 2016) and (Liu *et al,* 2014).

In the light of above given research, this paper has been developed also free derivative open method by using forward difference operator for solving non-linear equation. The proposed method is simpler and easier to use. The purpose of new iterated method is proposing a mathematical tool for solving all possible root of polynomial of higher degree function and transcendental function. The proposed iterated method is fast converging to approaching the root and free from pitfall. A new iterated method greatly improves order of convergence and computational efficiency. EXCEL and C++ are used to more defend the proposed method. A New iterated method seems to be very easy to employ with reliable result.

2. PROPOSED METHOD

Let us consider derivative of function can be computed by using Taylor series expansion.

$$f(x_n + h) = f(x_n) + hf`(x_n) + \ldots\ldots\ldots \quad (1)$$

When (1) is truncated after two terms, then (1) become as:

$$\frac{f(x_n + h) - f(x_n)}{h} = f`(x_n) + o(h)$$

Where $o(h)$ is a truncation error, then we get

$$f`(x_n) = \frac{f(x_n + h) - f(x_n)}{h} \quad .(2)$$

Newton Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f`(x_n)}$$

Now (2) substitute in Newton Raphson Method instead of derivative of function, we get

$$x_{n+1} = x_n - \frac{hf(x_n)}{f(x_n + h) - f(x_n)} \quad (3)$$

Thus 'h' can be taken from Arithmetic Sequence such as:

$$h = \frac{b-a}{m+1} \text{ For } m \geq 10$$

Where a, b is end points of interval and m is any integer under given condition.

To substitute 'h' in (3), we get

$$x_{n+1} = x_n - \frac{(\frac{b-a}{m+1}) f(x_n)}{f(x_n + \frac{b-a}{m+1}) - f(x_n)}$$

Hence this is the Modified Free Derivative Open Method.Where a, b and m are fixed values under given condition. The Proposed method follows as same process as Newton Raphson Method.

3. ANALYSIS OF CONVERGENCE

Proof:

By using Taylor series, we are expanding $f(x_n)$and $f(x_n + h)$ only second order term about `a`, we obtain

$$\mathrm{f}(x_n) = f`(a)(e_n + ce^2{}_n) \qquad ---(i)$$

$$f(x_n + h) = f`(a)[(e_n + h) \ + c(e_n + h)^2 \ --(ii)$$

$$[\text{Note c} = \frac{f``(a)}{2f`(a)}]$$

By using$(i) and\ (ii)$, we get

$$f(x_n + h) - f(x_n) = f`(a)[h \ + c(2he_n + h^2)$$

Thus,

$$\frac{f(x_n + h) - f(x_n)}{h} = f`(a)(1 + hc + 2ce_n) \ \ (iii)$$

By using$(i)\ and\ (iii) in$ MFDOM, we get

$$e_{n+1} = e_n - \frac{e_n\, f`(a)(1 + e_n c)}{f`(a)(1 + hc + 2ce_n)}$$

$$e_{n+1} = e_n(h + e_n]c \qquad ---(iv)$$

Thus,

$$h = \frac{e_{n+1} - e_n}{m+1}$$

By using $h\ in\ (iv)$, we get

$$e_{n+1} = e_n(\frac{e_{n+1} - e_n}{m+1} + e_n]c$$

$$e_{n+1} = e_n(\frac{e_{n+1} - e_n + me_n + e_n}{m+1}]c$$

$$\frac{e_{n+1}}{e_n(e_{n+1} + me_n)} = \frac{c}{m+1} \qquad ---(iv)$$

For finding the order of convergence we use lemma, such as

$$\frac{|e_{n+1}|}{|e_n|^p} = c$$

Here p is a convergence rate and there exists some constant C, that is

$$e_{n+1} = Ce^p{}_n \text{Similarly, } e_n = Ce^p{}_{n-1} \qquad --(v)$$

equation (iv) become

$$\frac{Ce^p{}_n}{e_n(Ce^p{}_n + \ me_n)} = \frac{c}{m+1}$$

$$\frac{Ce^p{}_n}{e^2{}_n(Ce^{p-1}{}_n + \ \mathrm{m})} = \frac{c}{m+1}$$

$$\frac{Ce^{p-2}{}_n}{(Ce^{p-1}{}_n + \mathrm{m})} = \frac{c}{m+1} --- \ (vi)$$

The Relation *(vi)* can be only satisfied if,

$$p - 2 = 0$$

Finally, we get

$$p = 2$$

Hence this shows that the MFDOM has quadratic convergence.

4. RESULT AND DISCUSSION

To justify the Modified free derivative open method, the developed algorithm is applied on few examples including algebraic and transcendental functions to compare a developed algorithm with the root location methods such as: bisection method, regula-falsi method, secant method, Steffensen method and Newton Raphson Method (Table-1) and present a figure for more justification. From observing algebraic and logarithm problem (i.e. 1 and 4), the existing root location methods taking almost same iterations in both problem such as 27,4,4,3 or 3 and 22,4,4,3 or 3 with accuracy, AE%=0.0000007451, AE%=0.0000104308, AE%=0.0001, AE%0.0003980850 or AE%=0.0003980-850 and AE%=0.00004768, AE%=0.00023842, AE%=0.00004, AE%=0.001535 or AE%=0.000095 respectively, while developed algorithm has taken accuracy AE%=0 at 3 number of iterations in both problem **(Table.1, Fig.1).** Similarly, for trigonometric function i.e. 2 problem, the existing root location methods taking 18,7,6,5, or 5 number of iterations with accuracy AE%=0.000762939, AE%=0.00003, AE%=0.00001, AE%=0.0394106 or AE%=0.000011-921 respectively, while developed algorithm has taken accuracy AE%=0 at 5 number of iterations **(Table.1, Fig.2).** and finally, for exponential function i.e. 3 problem, the existing root location methods taking 25,6,4,4 or 4 number of iterations with accuracy AE%=0.00000298, AE%=0.00000298, AE%=0.0001, AE%=0.00000298 or AE%=0.00000298 while developed algorithm has taken accuracy AE%=0 at 4 number of iterations (**Table.1, Fig. 3).** it has ensured that developed algorithm significant execution and superlative performance with existent algorithm.

Table-1

Function	Methods	No: of Iteration	X	A E %
x^3 -9x+1 (0,1)	**Bisection method**	**27**	**0.111264**	**0.0000007451**
	R F Method	**4**		**0.0000104308**
	Secant method	**4**		**0.0001**
	Steffensen method	**3**		**0.0003980850**
	N R Method	**3**		**0.0003980850**
	Modified method	**3**		**0**
Sinx-x+1 (0.5,2.5)	**Bisection method**	**18**	**1.93456**	**0.000762939**
	R F Method	**7**		**0.00003**
	Secant method	**6**		**0.00001**
	Steffensen method	**5**		**0.0394106**
	N R Method	**5**		**0.000011921**
	Modified method	**5**		**0**
e^x–5x (0,0.5)	**Bisection method**	**25**	**0.259171**	**0.00000298**
	R F Method	**6**		**0.00000298**
	Secant method	**4**		**0.0001**
	Steffensen method	**4**		**0.00000298**
	N R Method	**4**		**0.00000298**
	Modified method	**4**		**0**
2x-lnx-7 (3,5)	**Bisection method**	**22**	**4.21991**	**0.00004768**
	R F Method	**4**		**0.00023842**
	Secant method	**4**		**0.00004**
	Steffensen method	**3**		**0.001535**
	N R Method	**3**		**0.000095**
	Modified method	**3**		**0**

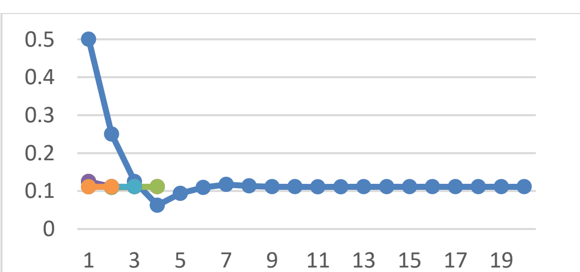

Fig.1. Comparison of accuracy analysis for the case of problem 1

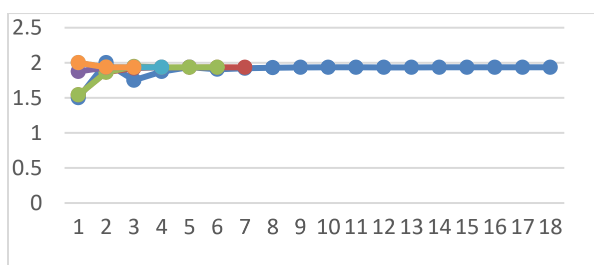

Fig.2.Comparison of accuracy analysis for the case of problem2

5. CONCLUSION

In this paper, we have proposed a Modified free derivative open method to solve non-linear problems. Throughout the study, it has been concluded that the developed algorithm performs magnificent to any existing root location techniques such as: bisection method, regula-falsi method, secant method, Steffensen method and Newton Raphson Method from accuracy and iterative perception. Besides a new developed method is fast converging to approaching the root and free from pitfall. Hence the proposed method is performing well, more efficient and easy to employ with reliable results for solving non-linear equations.

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