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Finite Element Modeling of Shear –Thinning Flow of Inelastic Non–Newtonian Fluid Past Expansion Pipe

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Abstract: Here presented a study of finite element Taylor–Galerkin pressure correction scheme to predict the transport fluid phenomena of shear thinning Non–Newtonian fluid through the ratio 1:4 expansion pipe. The fluid flow problem is non–linear when shear thinning non–Newtonian fluid occurs due to change in viscosity in terms of shear rate. The main important model power law employed to investigate the shear thinning non-Newtonian fluid behave with different index rates 0.95, 0.90 and 0.80.The numerical finite element scheme study will affect the flow structure, streamlines patterns of Shear thinning Non-Newtonian fluids with enhancing the value of Reynolds number and employing power law Computations on various index rates of the recirculation flow rate (Ψ_v), vortex length(X are plotted. The numerical results are compared and excellent results are achieved.

Keywords: backward step channel flows, Finite Element Method, non –Newtonian Fluids.

1. INTRODUCTION

The importance of incompressible flow of Non–Newtonian fluid past abrupt expansion pipe lies in the numerous engineering and science applications such as, polymer processing in industrial engineering, injection moulding in Mechanical Engineering, cardiovascular biomechanics in science and others are de–watering devices, pumping of slurries and foodstuffs, extrusion, thermoforming etc. The flow through sudden expansion occurs often in many applications due to the complex geometric but not examined so simple flow behaviour. For Newtonian fluids, diverging flows analytically, experimentally and numerically are greatly investigated (Back and Roschke, (1972), Habib and Whitelaw (1982) and Tenstrom, *et al*. (2006).

Since (1950), the incompressible flow of inelastic non–Newtonian fluids in the presence of porous material has obtained a large amount of attention, due to the importance in industrial applications such as polymer solutions, micro emulsions and foam in petroleum industries and the heavy oil flow of polymer solutions in the presence of porous material mostly performs alike a power–law non–Newtonian fluid. Also drilling and hydraulic fracturing fluids expended in the oil industry that are known as Non- Newtonian liquids. Thestudy of Non–Newtonian fluid dynamics is a moderately novel division of applied sciences. The expanding attraction of non- Newtonian fluids has been renowned in those meadows with material whose flow conduct of stress and rate of shear is not obeys the Newton's law of viscosity. Basically the fluids divided into two types one is Newtonian and second is Non-Newtonian. Newtonian fluids just obeys the Newtonian‘s law of constant viscosity or satisfied the power law with unit index rate and Non-Newtonian fluids obeys the variable viscosity or satisfied the power law with index rate is lower and higher than unity.

The examples of the fluids which obeys the Non-Newtonian characteristics especially shear-thinning fluids which are given as molten chocolate, blood, wastewater sludge’s, xanthan gum solutions, polymer solutions and muds (Pinho, *et al*. 2003), Zinani and Frey (2006), Solangi (2012)and Ray, *et al*. (2012). The literature is much available such as (Halmos and Boger (1975) examined experimentally limited mean laminar flow features in a 1:2 sudden expansion geometry and their quantities revealed that the vortex size and length was increased due to the shear-thinning intensity

Similarly (Pinho, *et al*. (2003) examined the laminar flow structure of Non–Newtonian fluids through 1:2.6 ratio expansion geometry. The finite volume method was exercised through the commercial Computational Fluid Dynamics code and transient form of pressure–velocity coupling was allocated with Semi Implicit Pressure Linked Equations for Correction (SIMPLEC) algorithm. The power law employed for the inelastic and shear–thinning fluids and presented the recirculation flow rate in terms of Reynolds Number and local loss coefficient in terms of Reynolds Number and shear–thinning intensity. Pinho concluded that the vortex intensity was decreased at high Reynolds number

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with shear–thinning fluids and vortex length and size was increased at low Reynolds Number. (Bhargava, *et al.* 2007) studied the pulsatile flow of Non–Newtonian fluids through Porous Medium Conduit and rheological model with a Darcy–Forchhemeir through porous channel was used and solve by finite element method. The velocity pulsatile profile was connived by described the effects of fluid inertia and other Rheological effects such as Darcy and Forchhemeir numbers. The velocity depressed continuously in the porous channel and increased velocities due to increase permeability and decreased velocities due to changing of Forchhemeir number and concluded that due to depress in Non–Newtonian behaviour the velocities continuously increased. The numerical results were compared with the (Bhatnagar, 1979).

In this chapter discussed the laminar flow of shear-thinning and shear thickening fluids through 4: 1 expansion geometry with a viscosity by applying the power law model filled with and without porous material and to investigate the effects of fluid inertia in recirculation flow rate, length and size, vortex enhancement and excess pressure drop. Here describe the fluid motions through conservation laws as mass and brinkman momentum to predicate numerically for Non–Newtonian especially shear–thinning flows by using the Semi implicit finite element scheme with Crank–Nicolson choice (ϴ = 0.5) with a viscosity obeying the power law model. The basic governing equations consisting the continuity and momentum Darcy Brinkman equation for two–Dimensional to model through ratio 4: 1 expansion pipe are presented in cylindrical polar coordinates with well posed boundary condition are fixed at solid walls and parabolic velocity profile are fixed and non–Dimensional form of governing equations are discussed.

2. GOVERNING SYSTEM OF EQUATIONS

The basic governing equations such as continuity and momentum equation dominated through nonlinear partial differential equations in cylindrical coordinate's form that is described as:

$$\frac{1}{r}\frac{\partial(r\boldsymbol{u}_r)}{\partial r} + \frac{\partial \boldsymbol{u}_z}{\partial z} = 0$$

r component

$$\frac{\partial \boldsymbol{u}_r}{\partial t} + \boldsymbol{u}_z\frac{\partial \boldsymbol{u}_r}{\partial z} + \boldsymbol{u}_r\frac{\partial \boldsymbol{u}_r}{\partial r} = -\frac{\partial p}{\partial r} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \boldsymbol{u}_r}{\partial r^2} + \frac{1}{r}\frac{\partial \boldsymbol{u}_r}{\partial r} + \frac{\partial^2 \boldsymbol{u}_r}{\partial z^2} - \frac{\boldsymbol{u}_r}{r^2}\right)$$

z component

$$\frac{\partial \boldsymbol{u}_z}{\partial t} + \boldsymbol{u}_z\frac{\partial \boldsymbol{u}_z}{\partial z} + \boldsymbol{u}_r\frac{\partial \boldsymbol{u}_z}{\partial r} = -\frac{\partial p}{\partial z} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \boldsymbol{u}_z}{\partial r^2} + \frac{1}{r}\frac{\partial \boldsymbol{u}_z}{\partial r} + \frac{\partial^2 \boldsymbol{u}_z}{\partial z^2}\right)$$

Where Re and Da show the Reynolds Number and Darcy Number that are given above.

$$\mathrm{Re} = \frac{\rho UL}{\mu_0}$$

3. NUMERICAL SCHEME

The finite element method is adopted and using the Taylor–Galerkin scheme and this scheme is divided into three steps to calculate the velocity and pressure and implemented the two–step Lax– Wendroff process to achieve the excellent 2nd order accuracy in time. Various researchers are applied this scheme in the field of fluid dynamics to examine the fluid flow phenomena through various mathematical geometries in the presence and absence of materials such as porous and fibre materials. (Townsend and Webster. (1987), Baloch, *et al.* (1994). The detail numerical process is already given in personnel published research papers Shaikh, *et al.* (2012, 2013)

4. PROBLEM DEFINITION AND MATHEMATICAL FORMULATION

The power law model in two–dimensional for shear thinning and shear thickening fluids have been examined to confine fluid flow behaviour, therefore, the influence of non-Newtonian viscosity that is a function of shear-dependent through expansion pipe are investigated given in (Fig.1). The various Reynolds numbers with increasing inertia are tested. The problem is related to industrial engineering such as moulding material (Satish, *et al.* 2013). Two different pipes with large width are joined to each other to visualize the expansion pipe.

The pipe length of upstream and downstream is given in figure and total number of elements is 2987, total number of nodes is 6220 and degree of freedom is 14057 respectively.

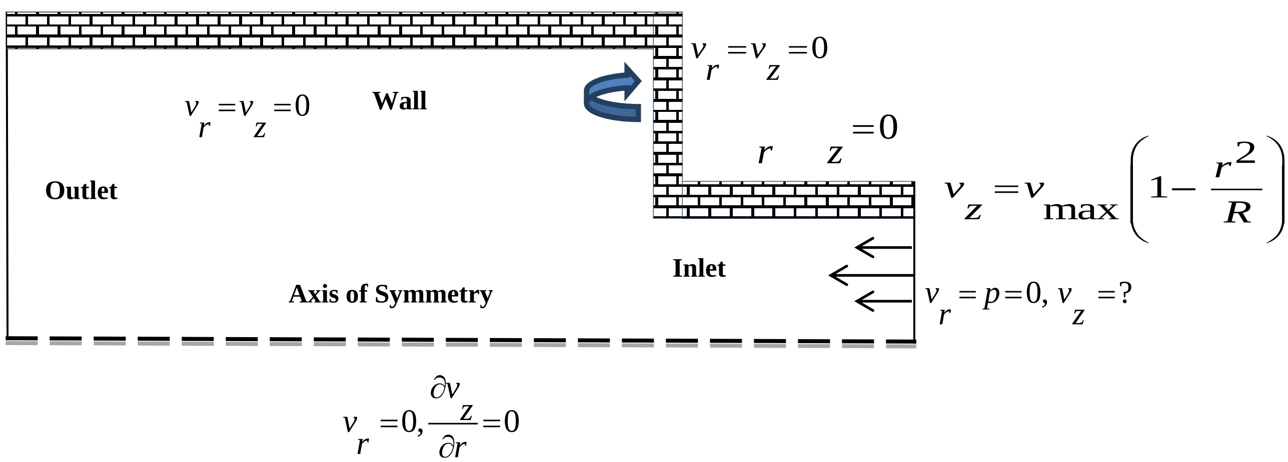
The length and height of the channel (upstream and downstream) is shown in the figure–1. Also no–slip boundary conditions on walls, well posed parabolic velocity profile on inlet and outlet are fixed, also axis of symmetry with mixed conditions Dirchlet and Neumann conditions with zero velocity shown in **(Fig. 1)**. The conditions are verified for high convergence. The approximations initiate from unit Reynolds number and various high Reynolds number are examined for continuum methodology. All numerical results are verified through analytical results and compared with the numerical results and with experimental results (Baloch, *et al.* (1994), Pedrizzetti (1996), Boger and Walters (1993) and Zinani *et al.* (2006). The triangular elements are selected for the geometry.

In industries, engineering and sciences, the pipes are imperative medium to move the various fluids (liquid and gases) from first location to second under the various forces. The good organization is subject to decrease the losses in pipe fluid flows are substantial. The various types of pipes consisting T–junctions, contraction, bends, expansions and various other components are used in applied sciences. These problems related to the pipes especially expansions are complicated to solve analytically or experimentally. Therefore, preferred a numerical solution to examine the fluid behave in the expansion pipes.

The need of such algorithm development to process the low Reynolds number and as well as high Reynolds

number for the fluid flow regimes may be verified and justified. Therefore, preferred here a finite element scheme due to the robustness and accuracy and it is easy to handle due to the various stages (Boughamoura, *et al*. (2000) and Satish, *et al*. (2013). Initially this scheme proposed by Donea (1984) for the problems of fluid flows, after that (Van. 1986). Contributes and prolonged the accuracy and stability of finite element scheme also adds the pressure correction term for the problems of especially constant viscosity incompressible flow materials in 2nd order. Consequently, several scholars and researcher such as (Townsend and Webster (1987), Baloch and Webster (2003), Baloch *et al*. (2008), Solangi *et al*., 2012) and Memon (2013) contributed and exonerates the strength of a 2nd order Semi implicit pressure–correction finite element scheme to determine the solution of Navier–stokes equations.

(Fig.1) and table shows the detail initial and boundary conditions fully developed and well posed boundary conditions on stationary walls, Dirchlet and Neumann boundary conditions on axis of symmetry and parabolic velocity profile is imposed at inlet and outlet in the two dimensional expansion pipe.



Fig–1: Schematic diagram 1:4 planner expansion pipe with boundary conditions.

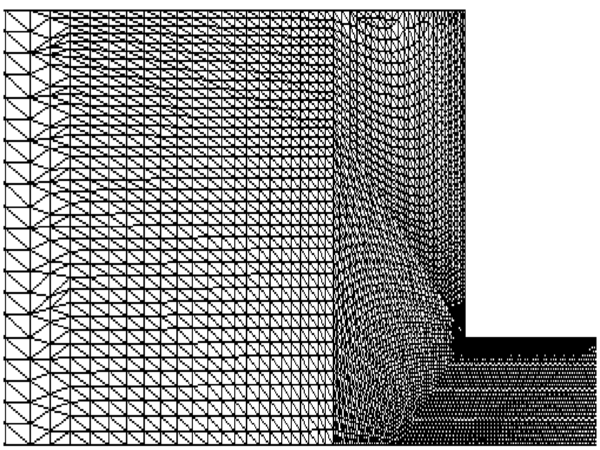
	MOMENTUM EQUATION
Initial Conditions:	***$v_z(r, 0) = 0$***
Boundary conditions:	**1. $v_r = v_z = 0$, no-slip at walls:** **2. $v_r = p = 0$, $v_z = ?$, at exit:** **3. $v_r = 0$,** $\frac{\partial v_z}{\partial r}$ **, at axis of symmetry** **4.** $v_z = v_{max}\left\{1 - \left(\frac{r}{R}\right)^2\right\}$ **at Inlet**

An interesting and alternative approach commonly used for flow features through expansion pipes is to report a Couette correction to examine the total pressure across the whole domain. Here empirical rapport has been established which predicting the extra pressure drop through analytically acquired related pressure drop in both small and large pipe in the 1:4 ratio. The following equations are known as Couette–Correction

(***CC***) and are applicable for excess pressure drop that shares with the fluid flow appearance in the geometry.

$$C = \frac{\mathrm{Re}\ \delta \mathrm{p} - \left(\mathrm{L_u}\ \nabla P_u\ + L_d \nabla\ P_d\ \right)}{2\tau_w}$$

Where Re and δP indicates a Reynolds number and approximated total pressure in the domain. Downstream and upstream lengths of the pipe shows the L_d and L_u and ∇P_d and ∇P_u shows the downstream and upstream pressure gradient correspondingly calculated analytically. τ_w shows the wall shear stress $\tau_w = \left.\frac{\partial \mathbf{u}}{\partial y}\right|_w$ in the fully developed downstream flow. Here two meshes Curse (M1) and other is refined Mesh (M2) are occupied for simulations and visualised in **(Fig.2)**.



Fig–2: Refined Finite Element Mesh (M) for two dimensional 1:4 geometry

5. NUMERICAL RESULTS AND DISCUSSION

The numerical results of streamlines patterns of expansion pipe are presented the flow structure at silent corner of the pipe and examined the vortex intensity, effect of inertia, effect of power law index and excess pressure drop. The Non-Newtonian fluids shear thinning or pseudo plastic fluids analysed via power law model. The results are compared with the numerical results Boughamoura, *et al*. (2000), Pinho, *et al*. (2003), Satish, *et al*. (2013).

The effect of fluid inertia at fixed power law index "n "

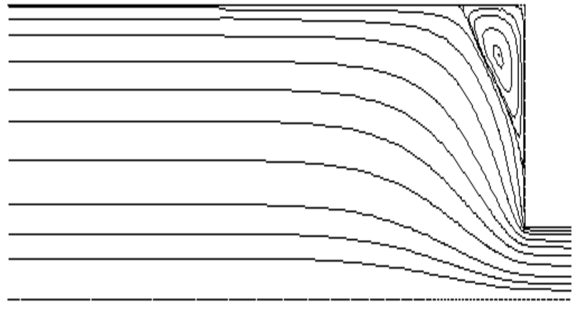
5.1 Streamlines patterns of expansion pipe of Non–Newtonian Fluids (Shear thinning fluids)

In these numerical results, the stream lines patterns of velocity fields are achieved at various Reynolds number and employing power law models of the different flow regimes available in the literature. Comprehensive study of flow structures are presented to a wide range of Reynolds number and power law indices, also generate the critical Reynolds Number (Re_c). **(Fig-3 (a–c))** showed the flow of shear thinning fluids structure of streamlines patterns power law index (= 0.95) at various Reynolds number ($1 \le \mathrm{Re} \le 50$) and recirculation flow rate is increased due to decrease power law index (= 0.95) relatives to the Newtonian fluids at unit Reynolds Number. Consequently recirculation flow rate is enhanced due to increase Reynolds number at fixed power law index (= 0.95) and at Reynolds number (= 50) the vortex cell size and length is fully enhanced and filled the region of the 1 : 4 expansion pipe shown in figure–4. The flow phenomena of shear thinning fluids are displayed in figure.5 at power law index (n = 0.90) also, the same phenomena of flow structure visualized as compared with power law index (n = 0.95). At unit Reynolds number the vortex size and length is small at silent corner but size and length is higher than the unit Reynolds Number at power law index (n= 0.95). Consequently the vortex size and length is enhanced due to increase Reynolds number up to 50. Recirculation flow rate is linearly increased with increasing Re's. Also developed and fitted the computational data of vortex length and size (X) in terms of various power law indices (n = 01, 0.95, 0.90 and 0.80) is expressed through empirical relationship as:

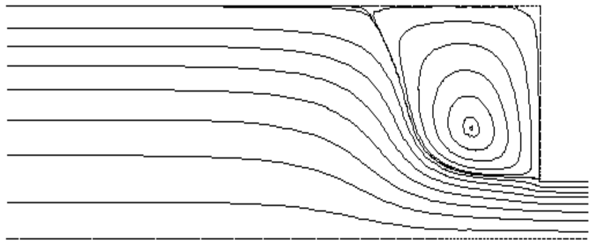
$\mathbf{X} = 0.0698\mathbf{Re} + 1.5516$	$01 \le \mathbf{Re} \le 50$, n = 01
$\mathbf{X} = 0.2283\mathbf{Re} + 1.1551$	$01 \le \mathbf{Re} \le 50$, n = 0.95
$\mathbf{X} = 0.1064\mathbf{Re} + 1.7431$	$01 \le \mathbf{Re} \le 50$, n = 0.90
$\mathbf{X} = 0.1046\mathbf{Re} + 1.3889$	$01 \le \mathbf{Re} \le 50$, n = 0.80

5.2 Effect of power law index:

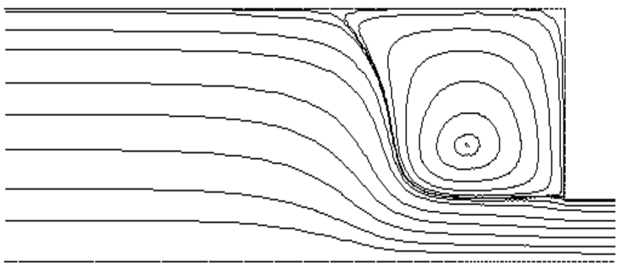
Fig.4 and **Fig.5** plotted that the recirculation flow rate in terms of power law indices at various Reynolds number and demonstrated that the recirculation flow rate observed the highest length initiated from 5.35874e-04 at power law index. **(Fig.4 c, d)** described the recirculation flow rate in terms of all index rate and concluded that recirculation flow rate is a function of power law indices is reduced due to decrease the index rate (n).



(a)



(b)



(c)

Fig.3: (a – c): Stream lines for 1: 4 expansion flow through pipe.

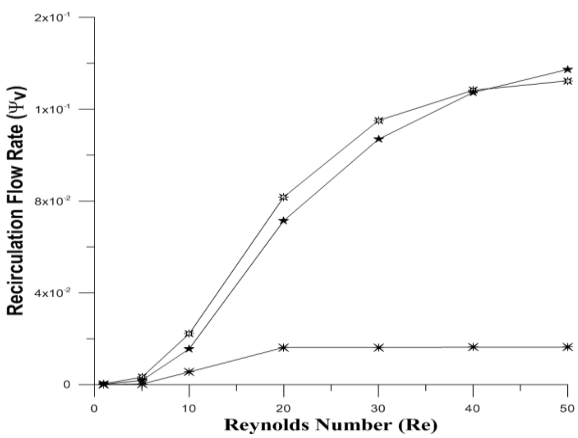
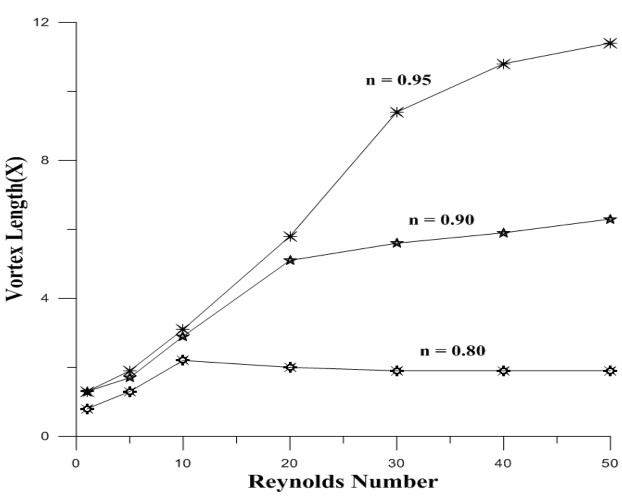


Fig.4: Two dimensional expansion flows, recirculation flow rate (Vortex intensity)(Q_v) is a function of fluid inertia (Re)



Fig–5: Graph of the vortex cell size (X) is a function of Reynolds Number (Re)

6. CONCLUSION

The effect of power law indices is presented to analyse the flow phenomena through vortex intensity, vortex length and size. Due to decrease power law index rate(n = 0.95), the vortex intensity and vortex length and size is increases at unit inertia other than Newtonian fluids and due to increase the Reynolds number, the vortex intensity is enhanced other the Newtonian fluid(n=01) because the power law index rate(n = 0.95) is near to the Newtonian fluid. But when decrease power law index (n= 0.90 and 0.80) the vortex intensity and vortex length and size is decreased due to increase inertia and also on power law index(0.80) the vortex length and size enhanced slowly and when inertia will become dominant after Reynolds number 20, the vortex developments in silent corner move towards the lip vortex in upstream wall and up to Reynolds number 50, the vortex enhancement remains constant in silent corner of the downstream wall of expansion pipe see (Fig.4) and (Fig.5). The good numerical results are achieved and compared with other analytical and numerical results.

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