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Use of the Daftardar-Jafari polynomials in Optimal Homotopy Asymptotic Method for the solution of one Dimensional Heat and Advection-Diffusion Equations

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Abstract: In this paper Daftardar-Jafari (DJ) polynomials have been introduced in the Homotopy of Optimal Homotopy Asymptotic Method (OHAM) for finding the approximate solution of one dimensional heat and advection-diffusion equations. The results of proposed method are compared with exact solution to find absolute errors. The obtained results reveal that the proposed scheme is more reliable, explicit and effective.

Keywords: Optimal Homotopy Asymptotic Method with DJ-Polynomials (OHAM-DJ), Heat Equation, Advection-Diffusion Equation.

1. INTRODUCTION

Partial Differential Equations play a vital role in different fields of Engineering Sciences and Physics such as Thermodynamics, Heat Transform, fluid flow and Wave Propagation. Mostly Partial Differential Equations are non-linear and solution of such type of equations is difficult, so the researchers show their interest in finding the approximate solutions as the exact solutions are rare. Various techniques have been proposed to find the approximate solutions of nonlinear differential equations.The well-known methods for finding the approximate solution of differential equations are RBFM (Hon, and Mao1999). ADM (Wazwaz, 2007), (Wazwaz, 2000), VIM (Noor and Mohyuddin, 2007 and 2008), HPM (Ganji, and Sadighi, 2006), (Liang, and Jeffrey, 2009), and HAM (Liao, 2004), OHAM was introduced by Marinca, *et al.*, 2008, for finding approximate solution of Differential Equations (Marinca, *et al.*, 2008), (Marinca, *et al.*, 2009), (Herisanu, *et al.*, 2008), (Haq *et al.*, 2010) (Ali. *et al.*, 2010) (Nawaz, *et al.*, 2010). This method combines the properties of both asymptotic methods and homotopy based methods. Recently, a new scheme of using Daftardar-Jafari polynomials in the homotopy of OHAM for the analytic solution of non-linear differential equation has been introduced by Ali, *et al.*,2013, which gives us more accurate results than that of OHAM (Ali, *et al.*, 2013). The author has shown the efficiency of proposed scheme for Nonlinear Klein-Gordon and Advection equations (Shah, and Nawaz, 2016), (Shah, *et al.*, 2016), Here we use the Optimal Homotopy Asymptotic Method with Daftardar-Jafari (DJ) polynomials (OHAM-DJ) for finding approximate solution of following one dimensional advection-diffusion equation (Mohebbi and Dehghan, 2010).

$$\frac{\partial u}{\partial t}+\beta\frac{\partial u}{\partial s}=\alpha\frac{\partial^2 u}{\partial s^2},\ a\ \le\ s\ \le\ b;\ t\ \ge 0, \quad (1.1)$$

subject to the initial condition:

$$u(x,0)=\phi(x);\ s\in[a,b], \quad (1.2)$$

and the boundary conditions are:

$$u(a,\ t)\ =y_0(t),$$
$$u(b,\ t)\ =y_1(t),\ \ t\in[0,T]. \quad (1.3)$$

For $\beta=\ 0$, the advection-diffusion equation reduces to one-dimensional heat equation. The Advection-Diffusion Equationis of primary significance in many physical systems, especially those involving fluid flow (Dehghan, 2004). It arises regularly in transferring heat energy, vorticityand mass in engineering and chemistry (Noye, 1990). and water transport in soils (Caglar, *et al.*, 2008)

2. FUNDAMENTAL MATHEMATICAL THEORY OF OHAM-DJ.

The basic mathematical theory of OHAM-DJ [18-20] is as follow. Let us consider the nonlinear differential equation

$$L(u(r))=F(u(r))+f(r),\ r\in\Omega\ \ B(u,\frac{\partial u}{\partial n})=0, \quad (2.1)$$

with boundary conditions

$$B(v(r,q),\frac{\partial v(r,q)}{\partial n})=0. \quad (2.2)$$

Her L is linear operator, F is Non-linear operator, B is Boundary operator is and f come with common meanings. In view of OHAM-DJ, we erect a homotopy as:

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$$(1-q)[L(v(r,q))-f(r)]=h(q))[L(v(r,q))-f(r)+F(v(r,q)]. \quad (2.3)$$

In Eq. (2.3) q is an embedding parameter such that $q\in[0,1]$, $h(q)$ is non-zero auxiliary function for $q\neq 0$, and it is 0 for $q=0$. The unknown function $v(r,q)$ start from $v(r,0)=u_0(r)$ to $v(r,1)=u(r)$ as q starts from 0 to 1.

The auxiliary function $h(q)$ is chosen in the form,

$$h(q)=\sum_{i=1}^{m} q^i C_i. \quad (2.4)$$

The convergence control parameters $C_1, C_2, ...,$ are to be evaluated latter.

Next, we use Taylor's series to expand the function $v(r,q)$ about q,

$$V(r,q)=u_0(r)+\sum_{m=1}^{\infty} u_m(r,C_1,C_2,...,C_m)q^m. \quad (2.5)$$

The nonlinear function $F\left(v\left(r,p\right)\right)$ is decomposed as

$$F(v(r,p))=F(u_0)+q[F(u_0+u_1)-F(u_0)]+q^2[F(u_0+u_1+u_2)-F(u_0+u_1)]+...$$

The expression on the right hand side

$$F\left(u_0\right),\ \left[F\left(u_0+u_1\right)-F\left(u_0\right)\right],\ [F(u_0+u_1+u_2)-F\left(u_0+u_1\right)],\ \ldots$$

are DJ-Polynomials. Precisely, these polynomials are the terms of the Taylor's series of the nonlinear term. The convergence of these polynomials has been proved (Bhalekar*et al.,*2011).

For simplicity and convenience these polynomials are expressed as:

$$F_0=F(u_0),$$

$$F_m=F(\sum_{i=0}^{m} u_i)-F(\sum_{i=0}^{m-1} u_i),$$

we can now express

$$F(v(r,q))=F_0+\sum_{i=0}^{m-1} q^k F_k). \quad (2.6)$$

Making use of Eq. (2.4), (2.5) and (2.6) into Eq. (2.3) and comparing the coefficient of same powers of θ, the following linear equations can be directly integrated:

0th order problem:

$$Lu_0=f(r), \qquad B(u_0,\frac{\partial u_0}{\partial x})=0. \quad (2.7)$$

1st order problem:

$$Lu_1=\left(1-C_1\right)Lu_0-C_1\left(F_0-f\right),$$

$$B\ (u_1,\frac{\partial u_1}{\partial x})=0. \quad (2.8)$$

2nd order problem:

$$Lu_2=\left(1-C_1\right)Lu_1-C_1F_1+C_2(L-F_0-f), \qquad B\left(u_2,\frac{\partial u_2}{\partial x}\right)=0, \quad (2.9)$$

although we can build higher order problems simply but solutions till second order problems are sufficient to generate tremendous results.

If the series converges in (2.5), for $q=1$ then

$$\tilde{v}\left(r\right)=\tilde{u}\left(r\right)=u_0(r)+u_1(r,C_1)+u_2(r,C_1,C_2). \quad (2.10)$$

By substituting Eq. (2.10) into Eq.(2.1) the resulting residual is

$$R(r,C_1,C_2)=L(\tilde{u}\left(r\right))-F(\tilde{u}\left(r\right))-f(r), \quad (2.11)$$

If $R=0$, then the exact solution is u. In other case, we reduce R over domain of the problem.

The Galerekin's method and the method of Least squares are followed here.

In method of Least Squares, first we build the function

$$J(C_1,C_2)=\int_{a_1}^{b_1}\int_{a_2}^{b_2}\ldots\int_{a_n}^{b_n} R^2 dr, \quad (2.12)$$

and then, minimizing it, we have

$$\frac{\partial J}{\partial C_1}=\frac{\partial J}{\partial C_2}=0, \quad (2.13)$$

and following the Galerekin's method , we explain the following system:

$$\int_{a_1}^{b_1}\int_{a_2}^{b_2}\ldots\int_{a_n}^{b_n} R\frac{\partial u}{\partial C_1}dr=0, \int_{a_1}^{b_1}\int_{a_2}^{b_2}\ldots\int_{a_n}^{b_n} R\frac{\partial u}{\partial C_2}dr=0. \quad (2.14)$$

3. APPLICATION OF OHAM-DJ

Example. 3.1:Solution of Heat Equation via OHAM-DJ:

Now we solve the heat equation is as follow

$$\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial s^2}=0, \quad (3.1.1)$$

with the initial condition is

$$u_0(s,0)=Sin(\pi s). \quad (3.1.2)$$

Exact solution is

$$w(s,t) = e^{-\pi^2 t} Sin(\pi s), \quad (3.1.3)$$

Applying the basic idea of OHAM-DJ mentioned in section 2, on Eq. (3.1.1) we have the following:

0th order problem:

$$\frac{\partial u_0(s,t)}{\partial t} = 0, \quad u_0(s,0) = Sin(\pi s). \quad (3.1.4)$$

Its solution is

$$u_0(s,t) = Sin(\pi s). \quad (3.1.5)$$

1st order problem:

$$\frac{\partial u_1(s,t)}{\partial t} = \frac{\partial u_0(s,t)}{\partial t} + C_1 \frac{\partial u_0(s,t)}{\partial t} - C_1 \frac{\partial^2 u_0(s,t)}{\partial s^2}, \quad u_1(s,0) = 0. \quad 3.1.6)$$

Its solution is

$$u_1(s,t) = C_1 \pi^2 t\, Sin(\pi s). \quad (3.1.7)$$

2nd order problem:

$$\frac{\partial u_2(s,t)}{\partial t} = C_2 \frac{\partial u_0(s,t)}{\partial t} + \frac{\partial u_1(s,t)}{\partial t} + C_1 \frac{\partial u_1(s,t)}{\partial t} - C_2 \frac{\partial^2 u_0(s,t)}{\partial s^2} - C_1 \frac{\partial^2 u_1(s,t)}{\partial s^2}, \; u_2(s,0) = 0. \quad (3.1.8)$$

Its solution is

$$u_2(s,t) = \frac{1}{2}\pi^2 (2C_1 t + 2C_1^{\,2} t + 2C_2 t + C_1^{\,2}\pi^2 t^2)\mathrm{Sin}(\pi s). \quad (3.1.9)$$

3rd order problem:

$$\frac{\partial u_3(s,t)}{\partial t} = C_2 \frac{\partial u_0(s,t)}{\partial t} + \frac{\partial u_1(s,t)}{\partial t} + C_1 \frac{\partial u_1(s,t)}{\partial t} - \frac{\partial u_2(s,t)}{\partial t} - C_2 \frac{\partial^2 u_0(s,t)}{\partial s^2} - C_1 \frac{\partial^2 u_1(s,t)}{\partial s^2},$$
$$u_3(s,0) = 0. \quad (3.1.10)$$

Its solution is

$$u_3(s,t) = \frac{1}{6}\pi^2 (6C_1 t + 12C_1^{\,2} t + 6C_1^{\,3} t + 6C_2 t + 12C_1 C_2 t + 6C_3 t + 6C_1^{\,2}\pi^2 t^2 + 6C_1^{\,3}\pi^2 t^2$$
$$+6C_1 C_2 \pi^2 t^2 + C_1^{\,3}\pi^4 t^3) Sin(\pi s). \quad (3.1.11)$$

Adding equations (3.3.5), (3.3.7), (3.3.9) and (3.3.11),

$$u(s,t) = u_0(s,t) + s_1(s,t) + u_2(s,t) + u_3(s,t). \quad (3.1.12)$$

With the help of least square method, we obtaind the values of unknown constants,

$C_1 = -0.9983569043416364$, $C_2 = 0.00000129443026982266 6$,

$C_3 = 4.065725267344232 \times 10^{(-9)}$

and substituting the values of C_1, C_2, C_3 in Eq. (3.1.12), the approximate solution of heat equation is as follow:

$$u(s,t) = \mathrm{Sin}(\pi s) - 9.853387696948161 t\mathrm{Sin}(\pi s) + \frac{1}{2}\pi^2(-0.0032782029295024273 t$$
$$+9.837197638403133 t^2)\mathrm{Sin}(\pi s) + \frac{1}{6}\pi^2(-0.00002388862903046629 t +$$
$$0.09690421324098253 t^2 - 96.92972218268893 t^3)\mathrm{Sin}(\pi s). \quad (3.1.13)$$

The following table shows the Absolute errors of 3rd OHAM-DJ solution at different time levels. The absolute errors are very small which reveals the good agreement of OHAM-DJ solution with exact solution.

Table 3.1.1: Absoluteerrors of 3rdorder OHAM-DJ solution for example 3.1 at different time level.

s∴*t*	*t* = 0.0001	*t* = 0.0005	*t* = 0.001
0.1	2.51504×10^{-12}	2.11553×10^{-12}	2.38698×10^{-14}
0.2	4.78395×10^{-12}	4.024×10^{-12}	4.54081×10^{-14}
0.3	6.58462×10^{-12}	5.53857×10^{-12}	6.23945×10^{-14}
0.4	7.74059×10^{-12}	6.5109×10^{-12}	7.34968×10^{-14}
0.5	8.13893×10^{-12}	6.84597×10^{-12}	7.72715×10^{-14}
0.6	7.74059×10^{-12}	6.5109×10^{-12}	7.34968×10^{-14}
0.7	6.58462×10^{-12}	5.53857×10^{-12}	6.23945×10^{-14}
0.8	4.78395×10^{-12}	4.024×10^{-12}	4.54081×10^{-14}
0.9	2.51504×10^{-12}	2.11553×10^{-12}	2.38698×10^{-14}
1.0	9.9675×10^{-28}	8.38411×10^{-28}	9.44168×10^{-30}

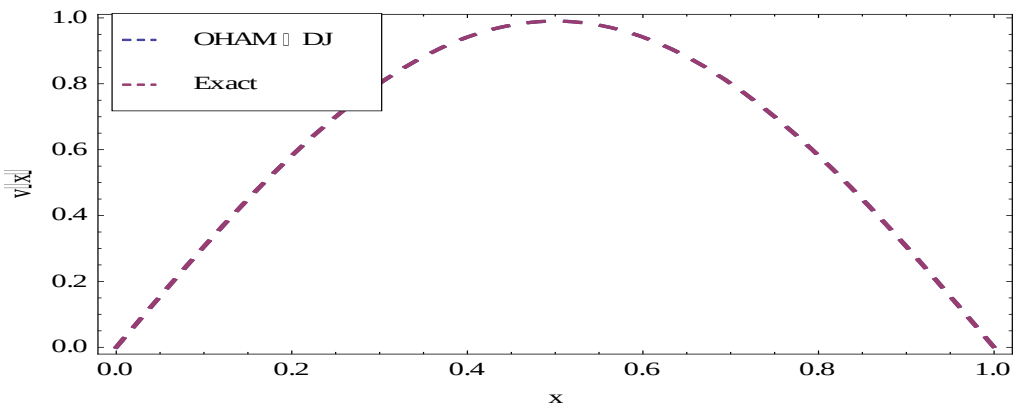

Fig. 1: Comparison of OHAM-DJ solution (solid line) with exact solution (dashed line) for t = 0.001

Example. 3.2.

The advection-diffusion equation with $\beta = 1, \alpha = 0.1$ is as follow

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial s} = 0.1\frac{\partial^2 u}{\partial s^2}, \quad 0 < s < 1, \qquad t > 0, \tag{3.2.1}$$

Withinitial condition

$$u(s,0) = e^{5s}(\mathrm{Cos}(\frac{\pi s}{2}) + 0.25\mathrm{Sin}(\frac{\pi s}{2})). \tag{3.2.2}$$

Exact solution of Eq. (3.2.1) is

$$w(s,t) = e^{5(s-\frac{t}{2})} e^{(\frac{-\pi^2 t}{40})}(\mathrm{Cos}(\frac{\pi s}{2}) + 0.25\mathrm{Sin}(\frac{\pi s}{2})). \tag{3.2.3}$$

Applying the proposed method OHAM-DJ as discussed in section 2, we obtain
0th order problem:

$$\frac{\partial u_0(s,t)}{\partial t} = 0, \quad u_0(s,0) = e^{5s}(\mathrm{Cos}(\frac{\pi s}{2}) + 0.25\mathrm{Sin}(\frac{\pi s}{2})). \tag{3.2.4}$$

Its solution is

$$u_0(s,t) = 1.\mathrm{e}^{5.s}\mathrm{Cos}(1.5707963267948966s) + 0.25\mathrm{e}^{5.s}\mathrm{Sin}(1.5707963267948966s). \tag{3.2.5}$$

1st order problem:

$$\frac{\partial u_1(s,t)}{\partial t} = \frac{\partial u_0(s,t)}{\partial t} + C_1\frac{\partial u_0(s,t)}{\partial t} + C_1\frac{\partial u_0(s,t)}{\partial s} - 0.1C_1\frac{\partial^2 u_0(s,t)}{\partial s^2},\ u_1(s,0) = 0. \tag{3.2.6}$$

Its solution is

$$u_1(s,t) = t(2.7467401100272344\mathrm{c1}\mathrm{e}^{5.s}\mathrm{Cos}(1.5707963267948966s) + 0.6866850275068086\mathrm{c1}\mathrm{e}^{5.s}\mathrm{Sin}(1.5707963267948966s)). \tag{3.2.7}$$

2nd order problem:

$$\frac{\partial u_2(s,t)}{\partial t} = C_2\frac{\partial u_0(s,t)}{\partial t} + \frac{\partial u_1(s,t)}{\partial t} + C_1\frac{\partial u_1(s,t)}{\partial t} + C_2\frac{\partial u_0(s,t)}{\partial s} - 0.1C_2\frac{\partial^2 u_0(s,t)}{\partial s^2}$$
$$-C_1\left(\frac{\partial u_1(s,t)}{\partial s} + 0.1\frac{\partial^2 u_0(s,t)}{\partial s^2} - 0.1\left(\frac{\partial^2 u_0(s,t)}{\partial s^2} + \frac{\partial^2 u_1(s,t)}{\partial s^2}\right)\right),\quad u_2(s,0) = 0. \tag{3.2.8}$$

Its solution is

$$\begin{aligned} u_2(s,t) = e^{5s}(&2.7467401100272344C_1\,t\,Cos(1.5707963267948966s) + \\ &2.7467401100272344C_1^{\,2}tCos(1.5707963267948966s) + \\ &2.7467401100272344\ C_2\ t\ Cos(1.5707963267948966\ \mathrm{s}) + \\ &3.7722906160162113C_1^{\,2}t^2Cos(1.5707963267948966s) + \\ &0.6866850275068086C_1\ t\ Sin(1.5707963267948966\ \mathrm{s}) + \\ &0.6866850275068086C_1^{\,2}tSin(1.5707963267948966s) + \\ &0.6866850275068086C_2\ t\ Sin(1.5707963267948966\ \mathrm{s}) + \\ &0.9430726540040528C_1^{\,2}t^2Sin(1.5707963267948966s)). \end{aligned} \tag{3.2.9}$$

3rd order problem:

$$\begin{aligned} \frac{\partial u_3(s,t)}{\partial t} = {} & C_3\frac{\partial u_0(s,t)}{\partial t} + C_2\frac{\partial u_1(s,t)}{\partial t} + \frac{\partial u_2(s,t)}{\partial t} + C_1\frac{\partial u_2(s,t)}{\partial s} + C_3\frac{\partial u_0(s,t)}{\partial s} \\ & -0.1C_3\frac{\partial {u_0}^2(s,t)}{\partial s^2} + C_2\frac{\partial u_1(s,t)}{\partial s} - 0.1\frac{\partial {u_0}^2(s,t)}{\partial s^2} + 0.1\left(\frac{\partial {u_0}^2(s,t)}{\partial s^2} + \frac{\partial {u_1}^2(s,t)}{\partial s^2}\right) \\ & C_3\left(\frac{\partial u_2(s,t)}{\partial s} - 0.1\left(\frac{\partial u_0(s,t)}{\partial s} + \frac{\partial u_1(s,t)}{\partial s} + \frac{\partial u_2(s,t)}{\partial s} + \frac{\partial {u_1}^2(s,t)}{\partial s^2}\right) + \right. \\ & \left. 0.1\left(\frac{\partial {u_0}^2(s,t)}{\partial s^2} + \frac{\partial {u_1}^2(s,t)}{\partial s^2}\right)\right), \end{aligned} \tag{3.2.10}$$

with boundary condition:

$$u_3(s,0) = 0.$$

Whose solution is

$$u_3(s,t) = -e^{5s}t(-4.853429173528852C_1\ Cos(1.5707963267948966\ \mathrm{s}) -$$

$$
\begin{aligned}
&5.493480220054469C_1^2Cos(1.5707963267948966s)-\\
&2.746740110027235C_1^3Cos(1.5707963267948966s)-\\
&2.7467401100272344C_2\ Cos(1.5707963267948966\ \mathrm{s})-\\
&5.493480220054469C_1C_2\ Cos(1.5707963267948966\ \mathrm{s})-\\
&2.7467401100272344C_3\ Cos(1.5707963267948966\ \mathrm{s})-\\
&13.331108582108026C_1^2tCos(1.5707963267948966s)-\\
&10.437844907070225C_1^3tCos(1.5707963267948966s)-\\
&10.437844907070225C_1C_2\ t\ Cos(1.5707963267948966\ \mathrm{s})-\\
&6.102848442267399C_1^3t^2Cos(1.5707963267948966s)+\\
&0.2887166941154069C_1Sin(1.5707963267948966\ \mathrm{s})-\\
&1.3733700550136172C_1^2Sin(1.5707963267948966s)-\\
&0.6866850275068087C_1^3Sin(1.5707963267948966s)-\\
&0.6866850275068086C_2\ Sin(1.5707963267948966s)-\\
&1.3733700550136172C_1C_2\ Sin(1.5707963267948966\ \mathrm{s})-\\
&0.6866850275068086C_3\ Sin(1.5707963267948966\ \mathrm{s})\ +\\
&0.7930297241612521C_1^2tSin(1.5707963267948966s)-\\
&0.5465577919234269C_1^3tSin(1.5707963267948966s)-\\
&0.5465577919234268C_1C_2\ t\ Sin(1.5707963267948966\ \mathrm{s})+\\
&0.36304109196625733C_1^3t^2Sin(1.5707963267948966s).
\end{aligned}
\tag{3.2.11}
$$

From equations (3.2.5), (3.2.7), (3.2.9), and (3.2.11), we obtain:

$$u(x,t)=u_0(s,t)+u_1(s,t)+u_2(s,t)+u_3(s,t) \tag{3.2.12}$$

The values of unknown constants are obtained by least square method, for $t=\ 0.0001$ the values of $C_1=-0.0007977074697195564$, $C_2=-451.8410780151379$, and $C_3=901.9643760326104$.

Now putting the values of C_1, C_2 and C_3 in Eq. (3.2.12), we get the approximate solution of heat equation:

$$
\begin{aligned}
u(s,t)=\ &e^{5s}Cos(1.5708s)+0.25e^{5s}Sin(1.5708s)+t(-0.0021911e^{5s}Cos)1.5708s)-\\
&0.000547774e^{5s}Sin(1.5708x))-e^{5s}t(-1238.35Cos(1.5708s)-3.76219tCos(1.5708s)\\
&+3.09787\times10^{-9}t^2Cos(1.5708s)-309.588Sin(1.5708s)-0.196999tSin(1.5708s)\\
&-1.84284\times10^{-10}t^2Sin(1.5708s))+e^{5s}(-1241.09tCos(1.5708s)+\\
&2.40045\times10^{-6}t^2Cos(1.5708s)-310.273tSin(1.5708s)+6.00112\times10^{-7}t^2Sin(1.5708s))
\end{aligned}
\tag{3.2.13}
$$

The following table shows the Absolute errors of 3rd OHAM-DJ solution at different time levels. The absolute errors reveal the good agreement of OHAM-DJ solution with exact solution.

Table 3.2: Absoluteerrors of 3rd order OHAM solution for example 3.2 at different time levels.

$x \cdot t$	t = 0.0001	t = 0.0005	t = 0.001
0.11	**2.51504×10^{-12}**	**2.11553×10^{-12}**	**2.38698×10^{-14}**
0.12	**4.78395×10^{-12}**	**4.024×10^{-12}**	**4.54081×10^{-14}**
0.13	**6.58462×10^{-12}**	**5.53857×10^{-12}**	**6.23945×10^{-14}**
0.14	**7.74059×10^{-12}**	**6.5109×10^{-12}**	**7.34968×10^{-14}**
0.15	**8.13893×10^{-12}**	**$6.84597\times10^{-}$**	**7.72715×10^{-14}**
0.16	**7.74059×10^{-12}**	**6.5109×10^{-12}**	**7.34968×10^{-14}**
0.17	**6.58462×10^{-12}**	**5.53857×10^{-12}**	**6.23945×10^{-14}**
0.18	**4.78395×10^{-12}**	**4.024×10^{-12}**	**4.54081×10^{-14}**
0.19	**2.51504×10^{-12}**	**2.11553×10^{-12}**	**2.38698×10^{-14}**
0.20	**9.9675×10^{-28}**	**8.38411×10^{-28}**	**9.44168×10^{-30}**

The **(Fig. 3.2 (a))** shows the plot of Approximate solution and the **(Fig. 3.2 (b))** shows the Exact solution, which are in close agreement.

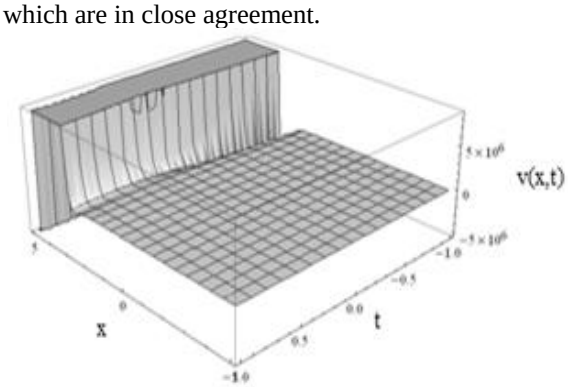

Fig 3.2(a), 3D Approximate solution of $u(s,t)$

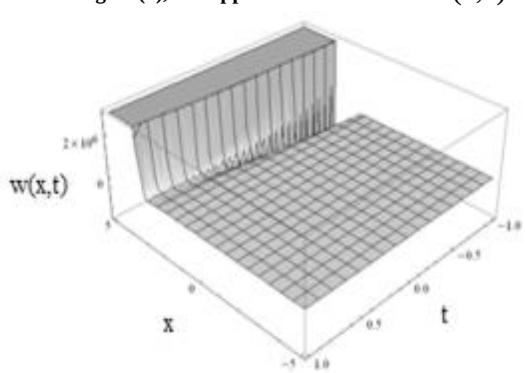

Fig 3.2 (b), 3D Exact solution of $w(s,t)$

4. CONCLUSION

In this paper, we applied the newly developed method namely OHAM-DJ to advection diffusion and heat equation. Table-3.1 gives absolute errors of OHAM-DJ solution for heat equation and Table3.2 gives absolute errors of OHAM-DJ solution for Advection Diffusion equation. All these results show good conformity with the exact solution. We concluded that among the existing numerical techniques, OHAM-DJ may be considered as a nice modification.

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