SindhUniv. Res. Jour. (Sci. Ser.) Vol.49(1) 183-188 (2017)

SINDH UNIVERSITYRESEARCH JOURNAL (SCIENCESERIES)

On Non-Optimality of Direct Exponential Approach Method for Solution of Transportation Problems

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Received 16th April 2016 and Revised 9th November 2016

Abstract: We investigate global optimality of a recent method – the Direct Exponential Approach (DEA) – for minimizing transportation cost. The DEA method is implemented on a balanced and an unbalanced transportation model and results are compared with the MODI method –the widely accepted optimal solution method. The presented comparison outlines the cases where DEA method fails to be optimal. It is finally recommended to use the DEA method as only a basic feasible solution method, instead of an optimal solution method, for minimizing the transportation cost.

Keywords: Transportation; MODI method; Direct exponential approach; Non-optimal solution.

1. INTRODUCTION

The transportation problem, a sub-class of the linear programming problem, attempts to determine minimum cost on the shipment of certain products/goods from available sources to desired destinations. This is done in way that the supply and demand requirements are satisfied. Literature comprises of two categories of methods: initial basic feasible solution (BFS) and optimal solution methods, to minimize the cost in transportation problem. Most widely used initial BFS methods include the column minimum method (CMM), the row minimum method (RMM), the north-west corner method (NWCM), the least cost method (LCM); and the Vogel's approximation method (VAM) etc. A classical optimal solution method is an iterative method that usually starts with the goods' allocation guess obtained through any initial BFS method. The modified distribution (MODI) and the steeping stone (SS) techniques are considered classical methods to find optimal solution of transportation problems (Tauha, 2007).

In past many researchers have attempted to propose initial allocation methods and optimal formulations other than MODI and SS techniques. The main focus has also been to investigate claimed properties of the new methods and compare with existing from view-points of computational cost and accuracy. Glover *et al.* (1974) compared available methods to minimize transportation cost from view-points of computational cost and other parameters with reference to MODI method. Goyal (1984) modified the VAM method for obtaining initial allocation guess which was later on improved further by Ramakrishnan (1988). (Goyal *et al.*1991) studied Kirka and Satir's algorithm, argued on its infeasibility and also proposed a refinement to the Kirka and Satir's heuristic algorithm. (Adlakha *et al.*1998) encouraged the use of Gauss-Jordon type formulation to solve transportation problem and introduced a heuristic approach (Adlakha and Kowalski, 2001). A systematic analysis to get alternate optimal solution and related suggestions also appear in the work of (Adlakah and Kowalski 2011).

In recent years, researchers have claimed to propose some direct approaches – non-iterative techniques – for minimizing transportation cost which can be initiated without using any allocation guess by initial BFS methods. The main objective of such direct techniques is to minimize the computational time and amount of work needed to find minimum cost as compared to the classical methods. Qudoos *et al.* (2012) proposed a direct optimal solution method,the ASM method, for this purpose and the method provided minimum cost directly that was also confirmed by MODI method. Deshmukh (2012) proposed the NMD method – another direct method – for finding optimal or nearly optimal solution of transportation problems. The NMD method directly resulted in optimal or nearly optimal solution, benefitting at large in reducing the number of iterations to reach optimal solution as compared to those in MODI and SS methods. (Vannan and Rekha2013) also claimed to propose a new direct method, the direct exponential approach (DEA) method, for obtaining optimal solution of transportation problems.

A demanding task of the time in theory of new direct optimal solution methods (Vannan and Rekha, 2013);

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Quddoos *et. al.* 2012, Deshmukh, 2012, Pandian and Natarajan, 2010) for transportation problem is to apply the methods on different models in order to investigate their global optimality by using the classical optimal solutions as comparison criteria. In this work we intend to test the claimed optimality of the DEA method (Vannan and Rekha, 2013) and outline two transportation problems in next sections for which DEA method fails to be optimal. In rest of the paper we quote algorithm of the DEA method and provide discuss a balanced and an un-balanced transportation models where DEA method fails to find optimal solution directly. Results of MODI method are used in same problems for comparison.

2. ALGORITHMOFTHEDIRECT EXPONENTIAL APPROACH (DEA) METHOD

The detailed algorithm of the DEA method can be found in (Vannan and Rekha, 2013). For purpose of analysis in this paper we briefly describe the algorithm comprising of mainly following steps:

1. Formulation: Arrange the transportation problem in tabular form.

2. Row and column reduction: To have at least one zero in every row and column, subtract from every row corresponding minimum entries and then from every column corresponding minimum entries.

3.Assigning exponential penalties: To every zero (starting with first in every row) in the reduced cost matrix, assign exponential penalties which are the number of zeros in corresponding rows and columns without counting the zero being assigned penalty.

4.Optimality test: Allocate maximum possible products/goods to a cell having zero with minimum exponential penalty. In case of tie, allocate to that cell for which average of corresponding supply and demand values is smallest. In case of tie again, check the corresponding value in the rows and column and select the minimum.

5. Discard the row or column from onward calculation if corresponding demand or supply is exhausted.

6. If reduced matrix contains at least one zero in each column and row, then go to next step, otherwise repeat step2.

7. Repeat steps 3 to 6 as long as all supplies and demands requirements are met.

8. Calculate the minimum optimal cost using already made allocations.

3. NUMERICAL PROBLEMS

The DEA method was implemented on many transportation models. The worth mentioning are the following two models which also highlight main findings of this work.

Problem 1. (Balanced Model): Demand = 22 = Supply

	1	2	3	4	5	6	Supply
A	9	12	9	6	9	10	5
B	7	3	7	7	5	5	6
C	6	5	9	11	3	11	2
D	6	8	11	2	2	10	9
Demand	4	4	6	2	4	2	22

Problem 2. (Unbalanced Model): Demand = 65, Supply = 50. "A" being a dummy source.

	1	2	3	4	Supply
A	0	0	0	0	15
B	3	48	14	2	24
C	4	2	30	10	24
D	36	8	12	12	2
Demand	6	12	3	44	65

4. RESULTS AND DISCUSSION

The step by step implementation of the DEA method on both problems is presented in Appendix. Both problems were also solved using MODI method – a classical optimal solution method – with initial basic feasible solution obtained by VAM. The solutions by MODI and DEA methods are listed in **(Table-1).**

Table1.1. Comparison of minimum cost (in $)

Problem	MODI Method	DEA Method
1.	112	114 (Not optimal)
2.	180	188(Not optimal)

For balanced model (Problem 1), the minimum cost obtained by DEA method is $114 where as the MODI method finds the minimum cost as $112, which is infact optimal. In the case of unbalanced model (Problem2), the optimal value of minimum cost by MODI method is $180 and by DEA method is$ 188; which is more than the MODI method's minimum cost.

It can be noticed in context of the discussed problems that the minimum costs determined by DEA method can further be minimized under similar sets of supply and demand conditions; as also evident from results by MODI method. Therefore, the DEA method is not an optimal solution method for minimizing cost in transportation problems. However, it can be recommended as a good addition to initial BFS methods like NWCM, LCM and VAM methods etc.

APPENDIX

A. Solution to Problem 1 (Balanced Model)

1. Formulation.

	1	2	3	4	5	6	Supply
A	9	12	9	6	9	10	5
B	7	3	7	7	5	5	6
C	6	5	9	11	3	11	2
D	6	8	11	2	2	10	9
Demand	4	4	6	2	4	2	22

2.(i). Row reduction.

	1	2	3	4	5	6	Supply
A	3	6	3	0	3	4	5
B	4	0	4	4	2	2	6
C	3	2	6	8	0	8	2
D	4	6	9	0	0	8	9
Demand	4	4	6	2	4	2	22

2. (ii). Column reduction.

	1	2	3	4	5	6	Supply
A	0	6	0	0	3	2	5
B	1	0	1	4	2	0	6
C	0	2	3	8	0	6	2
D	1	6	6	0	0	6	9
Demand	4	4	6	2	4	2	22

3. Assign exponential penalties.

	1	2	3	4	5	6	Supply
A	0^3	6	0^2	0^3	3	2	5
B	1	0^1	1	4	2	0^1	6
C	0^2	2	3	8	0^2	6	2
D	1	6	6	0^2	0^2	6	9
Demand	4	4	6	2	4	2	22

4. Choosing a zero having minimum exponential penalty.

Here tie exists for cell value (B, 2) and (B, 6).

a. (6, 4)= (6+4)/2=5

b. (6, 2)= (6+2)/2=4

	1	2	3	4	5	6	Supply
A	0^3	6	0^2	0^3	3	2	5
B	1	0^1	1	4	2	0^1 (2)	6-2=4
C	0^2	2	3	8	0^2	6	2
D	1	6	6	0^2	0^2	6	9
Demand	4	4	6	2	4	2-2=0	20

5. After performing Step4, demand of 6this zero.Hence delete column 6.

6. Check whether resultant matrix possesses at least one zero in each row and column. If not, repeat step2. Otherwise, go to step 7.

	1	2	3	4	5	Supply
A	0^3	6	0^2	0^3	3	5
B	1	0^0 (4)	1	4	2	4-4=0
C	0^2	2	3	8	0^2	2
D	1	6	6	0^2	0^2	9
Demand	4	4-4=0	6	2	4	16

The demand of 2nd,and supply of "B" is zero. So, deleting column 2 and row B, we have:

	1	3	4	5	Supply
A	0^3	0^2	0^3	3	5
C	0^2	3	8	0^2	2
D	1	6	0^2	0^2	9
Demand	4	6	2	4	16

Again there is tie for the cells(A, 3), (C, 1), (C, 5), (D, 4) and (D, 5).

- **a.** (6, 5)=(6+5)/2=5.5
- **b.** (4, 2)=(4+2)/2=3
- **c.** (4, 2)=(4+2)/2=3
- **d.** (2, 9)=(2+9)/2=5.5
- **e.** (4, 9)=(4+9)/2=6.6

	1	3	4	5	Supply
A	0^3	0^2	0^3	3	5
C	0^2 2	3	8	0^2	2-2=0
D	1	6	0^2	0^2	9
Demand	4-2=2	6	2	4	14

The supply of "C" is zero. So, deleting2ndrow.

	1	3	4	5	Supply
A	0^2	0^2	0^3	3	5
D	1	6	0^2	0^1 4	9-4=5
Demand	2	6	2	4-4=0	10

Thedemand of 5this zero. Hence, deleting5thcolumn.

	1	3	4	Supply
A	0^2	0^2	0^3	5
D	1	6	0^2	5
Demand	2	6	2	10

There is tie for cells (A, 1), (D, 3) and (D, 5).

- **a.** (2, 5)=(2+5)/2=3.5
- **b.** (6, 5)=(6+5)/2=5.5
- **c.** (2, 5)=(2+5)/2=3.5

	1	3	4	Supply
A	0^2	0^2	0^3	5
D	1	6	0^1 2	5-2=3
Demand	2	6	2-2=0	12

The demand of 4th is zero. Hence, deleting it gives:

	1	3	Supply
A	0	0	5
D	1	6	3
Demand	2	6	12

Subtracting"1" from row "D", gives:

	1	3	Supply
A	0^2	0^1	5
D	0^1	5	3
Demand	2	6	12

Again, there is tie for (A, 3) and (D, 1).

- **a.** (6, 5)=(6+5)/2=5.5
- **b.** (2, 3)=(2+3)/2=2.5

	1	3	Supply
A	0^2	0^1 5	5-5=0
D	0^1 2	5 1	3-2=1 1-1=0
Demand	2-2=0	6-1=5 5-5=0	12

7. Repeat steps 3 to 6 till all the demands are satisfied and all supplies are exhausted. Finally:

	1	2	3	4	5	6	Supply
A	9	12	9 5	6	9	10	5
B	7	3 4	7	7	5	5 2	6
C	6 2	5	9	11	3	11	2
D	6 2	8	11 1	2 2	2 4	10	9
Demand	4	4	6	2	4	2	22

8.The total cost associated with these allocations is:
$(9\times5+3\times4+5\times2+6\times2+6\times2+11\times1+2\times2+2\times4)=(45+12+10+12+12+11+4+)=\$114.$

B. Solution of Problem 2 (Unbalanced Model)

1.Formulation.

	1	2	3	4	SUPPLY
A	0	0	0	0	15
B	3	48	14	2	24
C	4	2	30	10	24
D	36	8	12	12	2
DEMAND	6	12	3	44	65

2. Row and column reduction.

	1	2	3	4	SUPPLY
A	0	0	0	0	15
B	1	26	12	0	24
C	2	0	28	8	24
D	28	0	4	4	2
DEMAND	6	12	3	44	41

3. Assign exponential penalties.

	1	2	3	4	SUPPLY
A	0^3	0^5	0^3	0^4	15
B	1	26	12	0^1	24
C	2	0^2	28	8	24
D	28	0^2	4	4	2
DEMAND	6	12	3	44	41

4. Choose a zero with the minimum exponential penalty.

	1	2	3	4	SUPPLY
A	0^3	0^5	0^3	0^4	15
B	1	26	12	0^1 24	24-24=0
C	2	0^2	28	8	24
D	28	0^2	4	4	2
DEMAND	6	12	3	44-24=20	41

5. After performing step 4, supply of "B" is zero. Hence, deleting row "B".

6. Check whether resultant matrix possesses at least one zero in each row and column. If not, repeat step 2. Otherwise, go to step7.

	1	2	3	4	SUPPLY
A	0	0	0	0	15
C	2	0	28	8	24
D	28	0	4	4	2
DEMAND	6	12	3	20	39

To choose a zero with the minimum exponential penalty, there is tie for the cells (D, 2) and (C, 2).

a. (24,12)= (24+12)/2=18

b. (2, 12)=(2+12)/2=7

	1	2	3	4	SUPPLY
A	0^3	0^5	0^3	0^3	15
C	2	0^2	28	8	24
D	28	0^2 2	4	4	2-2=0
DEMAND	6	12-2=10	3	20	39

Here,supplyof "D" is zero. Hence, deleting it.

	1	2	3	4	SUPPLY
A	0^3	0^4	0^3	0^3	15
C	2	0^1 10	28	8	24-10=14
DEMAND	6	10-10=0	3	20	29

The demand of 2nd is zero. So, deleting it gives:

	1	3	4	SUPPLY
A	0	0	0	15
C	2	28	8	14
DEMAND	6	3	20	29

Subtracting"2" from row "C", gives:

	1	3	4	SUPPLY
A	0^3	0^2	0^2	15
C	0^1 6	26	6	14-6=8
DEMAND	6-6=0	3	20	23

Thedemand of 1st is zero, so, deleting column 1.

	3	4	SUPPLY
A	0	0	15
C	26	6	8
DEMAND	3	20	23

Subtracting"6" from row "C", gives:

	3	4	SUPPLY
A	0^1 3	0^2	15-3=12
C	20	0^1	8
DEMAND	3-3=0	12	20

The demand of 3rd is also zero. Deleting the 3rd column.

	4	SUPPLY
A	0^2	12
C	0^2 (8)	8-8=0
DEMAND	20-8=12	12

Finally,

	4	SUPPLY
A	0^2 (12)	12-12=0
DEMAND	12-12=0	0

7.Repeat steps 3 to 6 till all the demands are satisfied and all supplies are exhausted. Therefore:

	1	2	3	4	SUPPLY
A	0	0	0 (3)	0 (12)	15
B	3	48	14	2 (24)	24
C	4 (6)	2 (10)	30	10 (8)	24
D	36	8 (2)	12	12	2
DEMAND	6	12	3	44	65

8. The total cost associated with these allocations is:

$(0 \times 3 + 0 \times 12 + 2 \times 24 + 4 \times 6 + 2 \times 10 + 10 \times 8 + 8 \times 2)$=$(0 + 0 + 48 + 24 + 20 + 80 + 16) = \$\mathbf{188}$.

5. CONCLUSION

A recently proposed direct optimal solution method, the DEA method, was investigated on a balanced and an unbalanced transportation problem and results were compared with the optimal solution obtained by MODI method. The comparison shows that the DEA method fails to find optimal solution of the example transportation models. Consequently, the DEA method should not be considered as an optimal solution method. However, it can be used to find initial basic feasible solution, wherein,it happens to be better than some otherinitial allocation cost methods in literature.

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