



Evaluating the Performance of MIMO System

R. H. SHAH, J. H. AWAN, K.U.R. KHOUMBATI, S. ABBASI, R. B. SYED, M. HAMMAD, U. SALAM*

Faculty of Engineering and Technology, University of Sindh, Jamshoro

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Abstract: The modeling of the system is an important and basic function, which compares, analyses the performance of the system. In this modern age, the modeling and stochastic processes are effective to test, validate the throughput, expected outcome, reliability, stability of the system, service, or application. In addition, the maintenance and redesigning of the above-mentioned features is not an easy task; it becomes costly and has a negative impact on business organizations as well for speech recognition, and anomaly detection. So, the organizations utilize more amounts on testing to validate their products and processes before moving them to market or providing alpha and beta versions to users/customers of their products or processes. Moreover, the authors have contributed in this paper by evaluating the performance of a dynamic system (MIMO System). Step Response, Settling Time, and Rise Time are set to be primary parameters to measure the performance of the system. The results of the experimental work have also been illustrated and discussed in the Experimental work section.

Keywords: Modelling, Performance, Evaluate, MIMO System, Markov

1. INTRODUCTION

Generally, the Markov chain helps the mathematician and statistical professionals to formulate algebraic expressions and measures the performance of the system by developing such conditions as the nature of steady-state (Starnini *et al.*, 2017). The generated transition matrix validates or matches the conditions as well as computes the required results. In practice, it is easy to measure steady-state from transitions. For example, video sharing is become easy and possible due to the dynamic recommendation of websites. These websites track follow on videos played and liked videos (Zhang *et al.*, 2016). In this way, the large number of videos is displayed during viewing new video defines the esteem of every video, which is comparative to the steady-state distribution and defined by followed videos. Mostly, we inversely determined that from the matrix of steady-state. This is the most highlighted problem in the field of Performance modeling to overcome such properties underlying transitional or steady-state transition probabilities as required. The researchers and professionals are trying to solve these issues by using a set of constraints. At the initial level, the graph of Markov is to be known, and known transitions are to be zero or equal to the known set. Youtube, Yelp, Amazon, and various websites are offering similar services (He *et al.*, 2017). The aim of this research is to examine and compute the step response of dynamic System, MIMO system as well as

from response data and settling time or rise time of chosen system's data. In addition, the paper is structured as an Introduction, Related work, background study, and experimental work of a chosen system. Hence, Section I introduces the paper's aim, objective and structure as followed above. Section II background study gives an overview about Markov Process from its birth to categorization into two types. Such as DMTCs and TMDCs. Section III discusses what has been done in this field means the literature about the system and ongoing research. At last paper illustrates and discusses the experimental work of MIMO, a dynamic system.

2. BACKGROUND STUDY

Andrei Andreyevich Markov is the pioneer of Markov Process; he used this process to predict the particles available in gas containers (Kirstein, 2018). Markov Process is a stochastic process used to evaluate the probability of the random variables over the state space. The state-space probability distribution is a system that reconciles into a normal blueprint of behavior. It is also widely used because of its usage having simple and effective way, and the user knows how to use it. Markov process has memoryless property, which means that once the particular state is achieved and the future result will neither affect the previous one nor depends on that, only depends on the current state. In this manner, transition represents the

dynamic behavior of a system. Moreover, Markov Process is further associated with the following approaches.

A. State transition diagrams

State transition diagram is used for small Markov processes; it is also the easiest way to characterize the process. In-state transition diagram, each state is represented as a node of a graph, and arcs are represented as transitions between states. So, an exponential distribution governs every transition, and the rate of the transition is also a parameter of that distribution. Hence, each state transition diagram must be powerfully associated (Cox, 2017).

B. Generator matrices

Generator Matrices of the Markov process have n states and $n \times n$ infinitesimal generator matrix (denoted as Q). We can say it is the addition of the parameters. Such as node and arcs. In addition, the diagonal elements are chosen to guarantee that each row is the addition of the elements, which equals zero, i.e

$$q_{ii} = -\sum_{j \in S, j \neq i} q_{ij} \quad (\text{Dau et al., 2014})$$

C. Steady-state distribution

Steady-state distribution is an approach used to analyze the performance of system. Besides this, Performance analysis is also concerned with the dynamic behavior of systems, networks or applications. It is also important to select a primary state for model to measure system performance. The bias problem is avoided in Markovian models; it is because of individual sample paths and the probability distribution over the states (Dui et al., 2015).

D. Derivation of Performance Measures

Performance analysis is the primary function of Markov Process. In this connection, three ways are defined as under to measure the performance, which is derived from the steady-state distribution (Dui et al., 2015).

1. State-based measures
2. Rate-based measures
3. Other measures which fall outside the above categories

State-based measures generally correspond to the existing state and probability of the working model or its subsets, which are satisfied by some conditions. Like Utilization.

$$U_{mem} = \pi_2 + \pi_3 + \pi_4 + \pi_5 \quad (i)$$

Rate-based measures also predict and correspond to the event occurrence rate. Then, the rate-based measure becomes the product of the rate of the event and the event probability. Like below:

$$X_{mem} = (\mu_A + (\pi_2 + \pi_4)) + (\mu_B + (\pi_3 + \pi_5)) \quad (ii)$$

At last, we may use either operational law or Little's law to the common memory to derive the information, which is required during solution or execution of model given as below.

$$W_{mem} = N_{mem} / X_{mem} \quad (iii)$$

E. Types of Markov Process

1) Discrete-Time Markov Chains

Discrete-Time Markov Chains (DTMCs) are state-based transitions, which are improved via probabilities. In addition, state, transition, and probability are important elements of DTMC. The state is a set of states, which represents the potential configurations of the system during modeling (Puterman, 2014). Transition is also the shifting of the system's states, which occurs in discrete time-steps. The probability is also measured via discrete probability distributions among the transitions of the system.

2) Continuous-Time Markov Chain

The continuous Time Markov Chain (CTMC) approach is based on time, and time is used as discrete steps in a DTMC proceeding. In addition, two potential interpretations are also applied. The first one is an accurate model of discrete units of time, like clock ticks in the model of an embedded device. The second one is the Time abstract, in which no information is assumed for the time transitions to be taken (Kulkarni, 2016). CTMCs are also dense time models, in which transitions occur at any real-valued, modeled via exponential distributions. DTMCs are also suitable for reliability models, control systems, queuing networks, biological pathways, and chemical reactions.

3. MATERIALS AND METHODS

From the literature collection and studies, it is to be noticed that the issue of inversion of a steady-state distribution is not discussed or considered for literature (Kumar et al., 2015).

In (Gurupur et al., 2015), each state is considered as (i) along with score (si) of the Markov process as well as mapping of transition matrix's scores. This mapping is constant and monotonic. In addition, the mapping is directly proportional to transition probability. If an increase in a score i takes place, then the probability may also be increased. Thus, the score becomes adequately high, and transition probability

reaches randomly close to 1, which is unboundedness. Moreover, natural mapping persuaded the condition and the probabilities results due to the transition of neighbor node and score.

Tomlin (Bogolubsky *et al.*, 2016) defined a model. In which, the author illustrated the link configuration of the web pages and identified the transition probabilities on the edges. In addition, optimization problems, regularization of maximum entropy, and iterative matrix scaling are also under constrained.

Notably, the transition probabilities only depend on the endpoints of an edge rather than nodes of the neighborhood. At last, the applied method generates a score of nodes existing in the graph.

The discussed model is primarily dependent on an authentic transition probability that takes only associated nodes rather than regularization. Hence, the number of nodes and parameters are always the same. Temperature Rank is used as baselines in this model and comprises individuality and a polynomial rate of convergence. In the literature (Ge *et al.*, 2016), stationary distribution and integration times are distinctive problems in Markov chains.

Page Rank (Wesley-Smith *et al.*, 2016) advertised and introduced Markov chains via web search engines and data mining techniques. In this process, the author used a random suffer model to link the structure of the web graph. PageRank is the stationary distribution of the system in which each node remains stationary in the graph and yields score for all nodes by the proposed method but varies as compared to Page Rank. Page Rank model is also consistent on the neighborhood of a foundation node, and the proposed system score is weighted from the destination node. Hence, the normalization factor is also proportional to the size of a primary node and its neighbors. Moreover, the available data in (Ohsaka *et al.*, 2015) shows and illustrates the score-based method. In addition, Berkhin (Berkhin *et al.*, 2016) and Langville and Meyer (Srivastava *et al.*, 2018) have also discussed link analysis methods in their published surveys.

In (Kumar *et al.*, 2015), the authors have discussed that user's navigation was applied between states as per Markov process to examine the steady-state of the applied approach and learned the transition matrix. Moreover, a particular structure of the transition matrix is used, and the linear number of variables is also under constrained. Thus, each node consists of an unidentified score and the transitional probability achieved from the source of the last state, which is reliant on the score of the targeted state. The researcher proposed an iterative

algorithm to solve this inverse issue as discussed above, as well as applied an iterative algorithm along with natural settings. Besides this, the proposed algorithm predicts state transitions in user-friendly behavior rather than strong baselines. In addition, the researcher inquired several research questions. Such as Parallelization, efficiency, and scalability. The studied model in PageRank is based on a random teleportation component. This model seems fruitful for opinion ratings of application domains.

In (Hu *et al.*, 2020), the researchers proposed an analytical model to distinguish the performance in outcome and represent access interruption of Multi-Dimensional Carrier Sense Multiple Access (MDCSMA) algorithm in varied MIMO network. In addition, nodes having additional antennas than contention winner permitted to persist to complete carrier sense but in a space orthogonal to the enduring transmissions. In (Franch *et al.*, 2020), Different scenario technologies are generally seen in a number of industrial applications. The mapping of such systems includes comparative tests of the system output in determining the parameters of the model so that it can be compatible with the original system mathematically. Such models would be used, among other applications, for modeling, predictive analytics, error handling. In (Panda, 2020), under channel-varying scenarios due to the sudden mobility, researchers study uplink and downlink efficiency of the heterogeneous cell Large Multiple Input Multiple Output (MIMO) system under channel varying environment due to swift consumer mobility. In the uplink, symmetric pilot patterns are transmitted to the core network for estimating channel status information (CSI) before data is transmitted. Besides Air-to-Air (A2A) connectivity using a 3D model, a non-stationary spatial mmWave multi-input (MIMO) channel model will be proposed in (Ma *et al.*, 2020). In (Dou & Yang, 2020), The spatial bandwidth keeps increasing immediately in the last years with the fast growth of satellite navigation services. Given the difficulties in the satellites and spectrum resource shortages, MIMO technologies are being used to increase spectrum utilization and network throughput in satellite technology which have become the primary research pattern.

4. RESULTS AND DISCUSSION

In experimental work, the author has chosen a dynamic system named as MIMO (Multiple Input and Multiple Output) dynamic system. Moreover, the model comprises continuous or discrete units such as tf (term frequency algorithm), genss (Generalized state-space model). These models need a toolbox named as Control toolbox, which is used to compute the step response as shown in **Fig 1**.

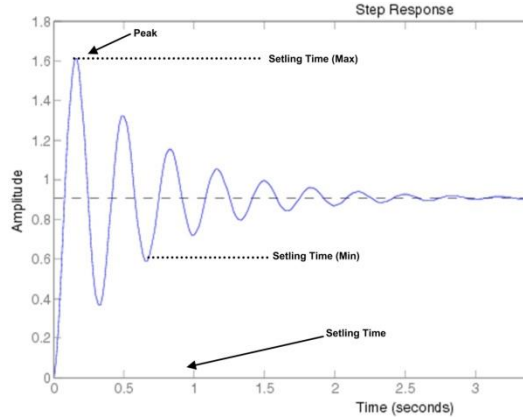


Fig. 1 General illustration of Step Response of Dynamic System

The author has calculated the step response of dynamic System (i.e: MIMO system) from the data retrieved from response and settling time or rise time as illustrated in Fig 1.

A. Step Response Characteristics of Dynamic System

The following function is applied to compute the above-mentioned characteristics of a dynamic system. The syntax, mathematical models, and values are defined below as per function as shown in Fig. 2.

$$\text{Sys} = \frac{S^2 + 3S + 3}{S^4 + 0.65S^3 + 5S^2 + 4.5S + 4} \quad (\text{IV})$$

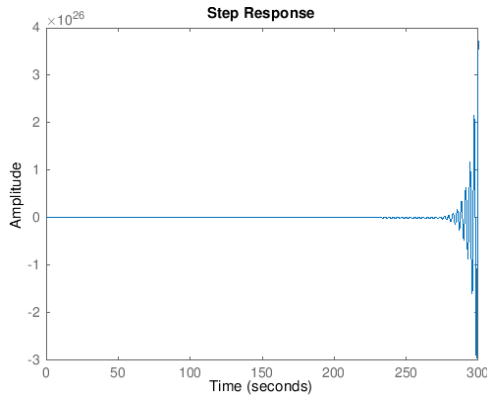


Fig. 2 Step Response of Dynamic System

B. Step Response of MIMO System

The selected dynamic system, like MIMO system, uses two-output and two-input for execution or computation of step-response as shown in fig 3. The input data is mentioned below in W,X and Y array set for computation.

And stepinfo() is script method used for computation. Sys is set of arrays like W, X, and Y. Then, sys becomes equal to ss(W,X,Y, 0.2). At last, stored in a variable like A = stepinfo(sys).

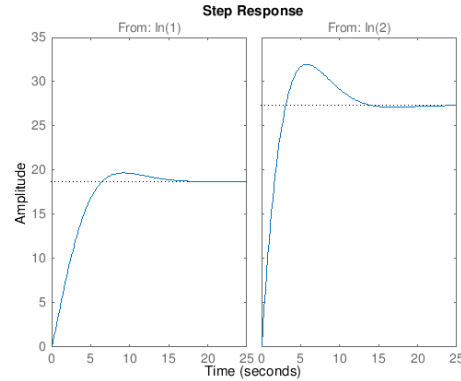


Fig. 3 Step Response Plot of MIMO System
Settling Time or Rise Time

C.

The settling time of a system is the time taken in activity from input to output response, remain within a defined range. In this paper, the following functions are executed and then stored in S1 and S2 objects to compute the rise of the steady state value.

D. -Response Characteristics from Response Data

The following function (load) is applied to compute the above-mentioned characteristics of a dynamic system. The values are defined as per function requirement. In addition, the results have been shown in figure 4. Load Step Info Data t y is also used to compute the Step-Response Characteristics from Response Data. Further, the input response is also stored in y and t variables for execution.

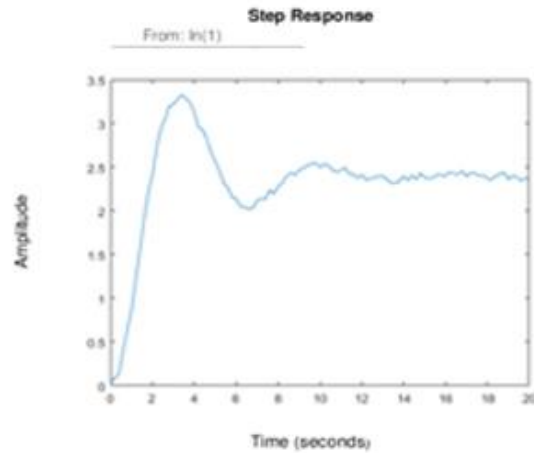


Fig. 4 Step Response gathered from Response Data

5. CONCLUSION

Generally, stochastic modeling is an advanced approach to measure and estimate probability distributions of effective results for dynamic systems. In addition, the use of modeling tool is playing an important and effective role for statistical applications to measure performance, stability, and throughput of application, service, and systems. Hence, organizations, developers, and professionals use such systems in the testing phase to measure the quality of their product, service, and application. From the beginning, the authors have discussed Markov Chains, their functions, and features. In addition, the previous work in the discussed field is also part of this paper. Moreover, the authors have chosen the Control toolbox and MIMO system to measure the step response, response data, and settling time or rise time of dynamic Systems like the MIMO system from collected system's data. The results show variations as illustrated in the Experiment section of the paper.

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