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Hybrid Closed Algorithm for Solving Nonlinear Equations in one Variable

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Abstract: In this paper, algorithm has been developed for solving non-linear equation of Van Der Waals equations emerging from different phenomena of science and engineering. Solution of non-linear equation of Van Der Waal equation is the formidable challenge, because the solution of non-linear equation is extremely difficult and complex. Our developed algorithm is an essentially a hybrid one. It works wonder to all existing closed methods in terms of iteration and convergence and better efficiency index. For implementation and performance of newly developed method, standard form of Van Der Waal equation has been solved and superbly demonstrated that it works better than others. In addition to this, it is equally applicable to all class of problems. Mat lab is executed for results and graphical representation.

Keywords: Hybrid, Closed Method, Van Der Waal Equation, Convergence

1. INTRODUCTION

There are innumerable problems emerging in science and engineering, which are beyond the realms of possibility in terms of analytic solutions. So, there is no way out, rather than resort to numerical solutions. Numerical stability is certainly central to numerical method. Accuracy, efficiency, stability is deemed as important motivational factor behind iterative solution. As for as fast rate of convergence, relatively more close to exact solution are concerned, it leaves much to be desired. More realistically speaking, numeric-cum-analytic solutions are taxing the brains of nation's experts. Tolerably robust solution of non-linear equation is essentially a daunting task in scientific-applications problems emerging from physical phenomena from time to time. Relatively convergence is a matter of prime concern while creating any numerical method. The recent research is geared to develop such development of algorithm that must be instrumental in expediting the process of bracketing method ie (BM+RFPM) for solution of van derwaal equation. AS a lately, numerous alterations in iterative techniques such as Bisection, Regulafalsi and newton methods have been proposed in writing, which has either equivalent or preferable executions over these strategies. A similar investigation of different strategies for finding root has been poked and probed by (Ehiwario, *et al.,* 2014). (Biswa, 2012) has analysed that extensive body of literature exist and are in compatibility to this fact that bisection is always inclined to convergence but slowly. (Iwetan,2012) explored that the common root-finding algorithm contains various methods i.e false position, newton Raphson .(Srivastava, 2011) has made critical ,but comparative analysis of convergence of bisection, newton and secant methods in following order secant method> newton method>bisection. (Ahmed *et al* 2012) new subordinate free iterative technique for illuminating non-direct mathematical statements has been investigated (tanakan,2013) the most straightforward root-discovering calculation is the binary search method. it performs well when f is real and continuous. it requires past information of two beginning estimates ,a and b such that f(a).f(b)<0 then discover midpoint of [a b] ,and then choose whether the root lies in [a $(\frac{a+b}{2})$] or [$(\frac{a+b}{2})$ b] .Repeat until the interim is adequately little.

(Gyurhan nedzhibov, 2015) introduced an approach that can be applicable to arbitrary iterative process which is linearly or quadratically convergent. (Babajee *et al*, 2016) compared dynamical behavior on quadratic polynomial with one of the newton'sscheme. This comparison is defined in all Cayley Quadratic Test (CQT) which can be used as a first test to check efficiency of such methods. Moreover, they have given a brief insight in cubic polynomials. (Braverman *et al*, 2016) has given some solution of the density-dependent Nagumo equation. (Sangah *et al*, 2016) improved the convergence of Bracketing Mehods by using "improving Brack*et al*gorithm".

2. METHODOLOGY

Step : 01 For non-linear equation of the form f(x)=0 contained in [a b], whereas f(a).f(b)< 0.first initial approximation is derived from

$$X_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

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(Table.1) Example 1: (Wan der waal equation): $10V^3 - 24.3V^2 + 13.6V - 0.0039 = 0$

Iteration	Bisection Method		False position Method		Hybrid CIM	
	x_n	A.E%	x_n	A.E%	x_n	A.E%
1st	2.5		2.23366		2.379302	
2nd	2.25	25	2.341387	10.77275	2.404441	2.5139
3rd	2.375	12.5	2.384157	4.276951	2.408385	0.394433
4th	2.4375	6.25	2.400079	1.592234	2.408994	0.060917
5th	2.40625	3.125	2.405861	0.578247	2.409088	0.009385
6th	2.421875	1.5625	2.407942	0.208093	2.409103	0.001445
7th	2.414063	0.78125	2.408689	0.07464	2.409105	0.000223
8th	2.410156	0.390625	2.408956	0.02674	2.409105	3.43e-05
9th	2.408203	0.195313	2.409052	0.009576	2.409105	5.28e-06

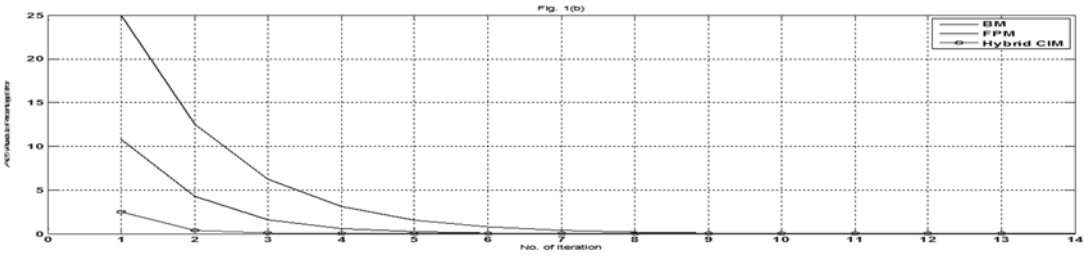


Fig.1(a)

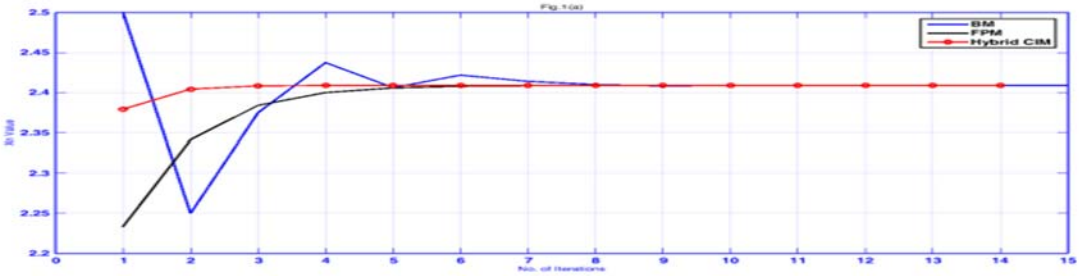


Fig.1(b)

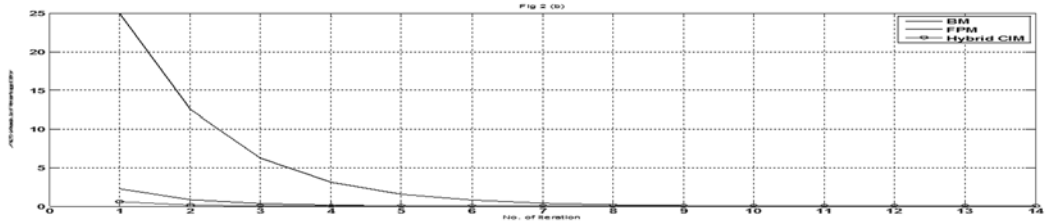


Fig.2(a)

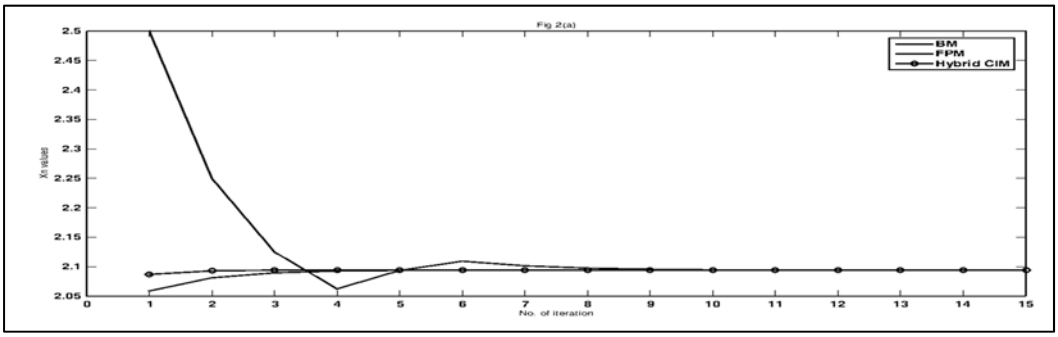


Fig.2(b)

Example 2 : (Algebraic equation) $x^3 - 2x - 5 = 0$(Table.2)

Iterations	Bisection Method		False position Method		Hybrid CIM	
	x_n	A.E%	x_n	A.E%	x_n	A.E%
1st	2.25		2.058824		2.087047	
2nd	2.125	25	2.081264	2.244013	2.092991	0.594387205
3rd	2.0625	12.5	2.089639	0.837555	2.094228	0.123674068
4th	2.09375	6.25	2.09274	0.310036	2.094484	0.025672231
5th	2.109375	3.125	2.093884	0.114413	2.094538	0.005326423
6th	2.101563	1.5625	2.094305	0.042174	2.094549	0.001105003
7th	2.097656	0.78125	2.094461	0.015539	2.094551	0.000229236
8th	2.095703	0.390625	2.094518	0.005725	2.094551	4.75553E-05
9th	2.094727	0.195313	2.094539	0.002109	2.094551	9.86542E-06
10th	2.094238	0.097656	2.094547	0.000777	2.094551	2.04659E-06
11th	2.25	0.048828	2.09455	0.000286	2.094551	4.24569E-07

Example 3 : (Transcendental equation) $e^x sinx - 1 = 0$**(Table.3)**

Iterations	Bisection Method		False position Method		Hybrid CIM	
	x_n	A.E%	x_n	A.E%	x_n	A.E%
1st	0.5		0.437186		0.575164332	
2nd	0.75	25	0.555986	11.87999	0.587798158	1.26338263
3rd	0.625	12.5	0.581881	2.589524	0.58849262	0.06944615
4th	0.5625	6.25	0.587189	0.530731	0.588530553	0.00379334
5th	0.59375	3.125	0.588262	0.107313	0.588532624	0.00020713
6th	0.578125	1.5625	0.588478	0.021638	0.588532737	1.131E-05
7th	0.585938	0.78125	0.588522	0.004361	0.588532744	6.1755E-07
8th	0.589844	0.390625	0.588531	0.000879	0.588532744	3.372E-08
9th	0.587891	0.195313	0.588532	0.000177	0.588532744	1.8412E-09
10th	0.588867	0.097656	0.588533	3.57E-05	0.588532744	1.0053E-10
11th	0.588379	0.048828	0.588533	7.19E-06	0.588532744	5.4956E-12

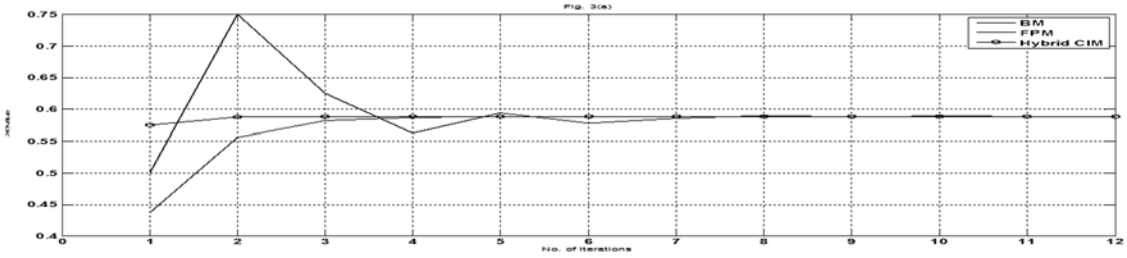


Fig.3(a)

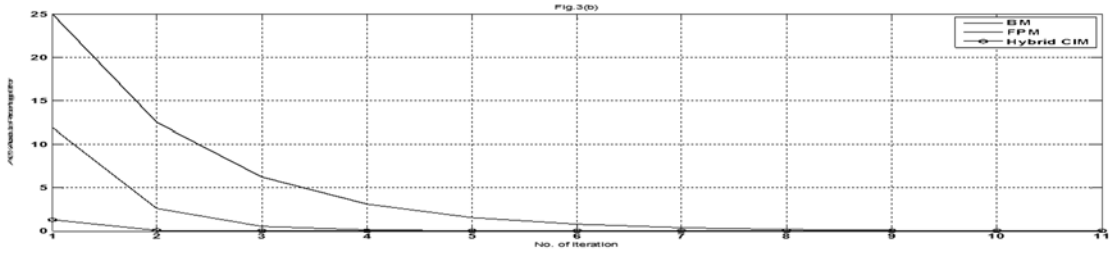


Fig.3(b)

Step:02
Suppose new approximation is X_1,then we find new interval which satisfies the general conditionie f(a).f(X_1) < 0 $orf(X_1).f(b) < 0$.Now if it lies in[$X_1 b$].
Step:03
Decompose $[X_1 b]$ $into$ subintervals by using bisection $[x_1 \ \frac{x_1+b}{2}]$ or $[\frac{x_1+b}{2} b]$ and choose the subinterval where lies root.
Step :04 If it lies in $[\frac{x_1+b}{2} b]$ .second approximation can more likely to be refined by using regula falsi position $x_2 = \frac{cf(d)-df(c)}{f(d)-f(c)}$ where as c= $\frac{x_1+b}{2}$ d=b similarly we find x_3 ,x_4 ,upto x_n by the same process.

3. RESULTS AND DISCUSSION

In the present study, to justify the developed method, couple of illustrations are tested on various problems. The results are also compared with the existing methods. Further, it has been analyzed that the developed algorithm works faster than other prevailing traditional methods. Moreover, time factor and number of iterations on developed algorithm and existing sectioning/bracketing techniques are understandable from the above graphical representation and findings. For the van der waal equation, the existing bracketing methods take 9 and 7 iterations with AE%=0.19, AE% 0.074 respectively, while developed algorithm take 4 number of iterations with same accuracy ie AE%=0.060 **(Table.1, Fig.1(a) and Fig.1(b))**. However, it is critically probed that developed algorithm reaches at 10^{-6} error. .Similarly for algebraic functions, Bisection and False Position techniques take 11 and 6 number of iterations with AE%=0.04.AE%=0.04 respectively, while developed algorithm takes in single iteration with same accuracy. (Table.2, Fig.2(a) and Fig.2(b)) and finally for transcendental function, bisection and regula falsi and developed algorithm take 11 , 6 and 3 number of iteration with same accuracy respectively. Table. 3, Fig. 3(a) and Fig. 3(b)), which ensures for the best execution and remarkable performance of developed algorithm.

4. CONCLUSION

At long last, it has been inferred that developed algorithm performs superior to anything existing bracketing techniques i-e Bisection and False Position Method from precision and iterative perspective, henceforth the created hybrid closed iterative is a decent accomplishment to determine root of nonlinear equations arising out of various phenomena in engineering and sciences

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