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Investigation of Incompressible Porous Flow in Maxwell Fluid

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Abstract: This article investigates the effects of porous media over the oscillating flows of Maxwell fluid under the absence and presence of porosity. Exact solutions are obtained by employing the techniques of integral transforms for velocity field and adequate shear stress. The generalized solutions are expressed in the compact form with elementary functions. Both the solutions for velocity field and adequate shear stress satisfy the initial as well as boundary conditions. The corresponding solutions are discussed and reduced under the effects of porous media in the absence and presence conditions. Finally, the comparisons including graphical illustrations for pertinent rheological parameters are made into four types of flow namely (i) Maxwell fluid in the absence of porosity, (ii) Maxwell fluid in the presence of porosity, (iii) Newtonian fluid in the absence of porosity and (iv) Newtonian fluid in the presence of porosity.

Keywords: Maxwell fluid, Porous Media, Oscillating plane, Integral Transforms, Comparisons and Graphical illustrations

1. INTRODUCTION

The partial differential equations of the non-linear type mostly arise in the flows of non-Newtonian fluids. Due to wide-ranging importance of technological advancements and applications in engineering and industries, such non-Newtonian fluids have become the focused study of fluids extensively. The rheological behaviors lie in polymers, biological materials and plastics which are controlled by several chemical industries such as paper coating, lubricants, oil explorations, chemical catalytic reactors, ice rinks and engine cooling, pharmaceutical. Seeing these complexities of rheology among the fluids, lot of constitutive equations and models of non-Newtonian fluids are proposed. These fluids are mainly categorized into integral, rate and differential types. The response of relaxation and retardation time is described by rate and differential type models. The recent research of non-Newtonian fluids can be found in (Rajagopal,1982) (Bandelli, *et al.,*1995).(Fetecau, *et al.,*2007) (Fetecau, 2005 (Erdogan, and Imrak, 2005), "Among the various industries, it has been accepted that non-Newtonian fluid is more applicable and suitable than Newtonian fluid and the flow of Newtonian fluid is characterized by Navier-Stokes equation. It is commonly known fact that exact analytical solution of Navier-Stokes equation is not an easy task. This research paper is related to porous medium in which impacts of porosity are discussed under the conditions of absence and presence of porosity. The study of fluid flows of porous media has gained much interest of engineers and scientists due to several engineering and industrial applications, particularly porosity of flow in presence and absence of fluid, for instance chromatography, ceramics processes, filtration process, biomechanics, packed bed reactors, insulation system and several others (Nield, 1999) (Mahomed, *et al.,* 2007) "Even most study for porous media is presented for non-Newtonian fluid which is come across in industrial applications and technological advancements, such as the porous pipe design, polymer solutions, petroleum industry, micro emulsions, reservoirs in different enhanced oil recovery processes and many more. The main objective of this research article is to investigate the effects of porous media over the oscillating flows of Maxwell fluid under the impacts of absence and presence of porosity. General solutions are obtained by mathematical techniques of integral transforms. The similar solutions are found out and reduced from general solutions among four directions namely (i) Maxwell fluid in the absence of porosity when weset $\emptyset = 0$, (ii) Maxwell fluid in the presence of porosity when we set$\emptyset \neq 0$, (iii) Newtonian fluid in the absence of porosity when we set$\emptyset = 0$ and $\lambda = 0$and (iv) Newtonian fluid in the presence of porosity when we set$\lambda = 0$ and $\emptyset \neq 0$. Finally graphical illustrations for pertinent rheological parameters are also described.

2. MATHEMATICAL MODEL OF THE PROBLEM

The equations which govern the incompressible Maxwell fluid are [16-21]

$$\boldsymbol{T} = -p\boldsymbol{I} + \boldsymbol{S}, \boldsymbol{trD} = \boldsymbol{0}, \ \boldsymbol{S} + \lambda \frac{\delta \boldsymbol{S}}{\delta t} = \mu \boldsymbol{A_1}, \qquad (1)$$

Where $\boldsymbol{D}$ is the rate of strain tensor, Sis the extra stress tensor, $\boldsymbol{T}$ is cauchy stress tensor,$-p\boldsymbol{I}$denotes the

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indeterminate spherical stress, μ the dynamic viscosity, λ is the relaxation time and the first Rivlin Ericksen tensor $\boldsymbol{A_1}$ is termed as

$$\boldsymbol{A}_1 = \boldsymbol{L} + \boldsymbol{L}^T, \quad (2)$$

$$\frac{\delta \boldsymbol{S}}{\delta t} = \frac{d\boldsymbol{S}}{dt} - \boldsymbol{LS} - \boldsymbol{SL}^T = \boldsymbol{S} - \boldsymbol{LS} - \boldsymbol{SL}^T, \quad (3)$$

$\frac{\delta}{\delta t}$is the upper convected derivative and $\frac{d}{dt}$ is material time derivative. For the problem under consideration we assume a velocity field **V** and an extra-stress tensor **S** of the form

$$\boldsymbol{V} = \boldsymbol{V}(y,t) = u(y,t)\mathbf{i}, \qquad \boldsymbol{S} = \boldsymbol{S}(y,t), \quad (4)$$

Where, the unit vector denoted by **i**along the x-coordinate direction. For these flows the constraint of incompressibility is automatically satisfied. If the fluid is at rest up to the moment$t = 0^+$, then

$$\boldsymbol{V} = (y,0) = 0, \qquad \boldsymbol{S} = (y,0) = 0, \quad (5)$$

and Equations (1)and (4) imply $S_{yy} = S_{yz} = S_{zz} = S_{xz} = 0$, and gives

$$\mu \frac{\partial u(y,t)}{\partial y} = \left(1 + \lambda \frac{\partial}{\partial t}\right)\tau(y,t), \quad (6)$$

In the absence of body forces, the balance of linear momentum reduces to

$$\frac{\partial p}{\partial x} = \frac{\partial \tau(y,t)}{\partial t} - \rho \frac{\partial u(y,t)}{\partial t}, \quad (7)$$

where $\tau(y,t) = S_{xy}(y,t)$ are the non-zero shear stresses. Eliminating τ between Equations (6) and (7), we have the governing partial differential equations under the expression

$$\frac{\partial^2 u(y,t)}{\partial^2 t} = \frac{1}{v}\left\{\left(\frac{\partial p}{\partial x} + \lambda \frac{\partial}{\partial t}\frac{\partial p}{\partial x}\right)\frac{1}{\rho} + \frac{\partial u(y,t)}{\partial y}\left(1 + \lambda \frac{\partial}{\partial t}\right)\right\}, \quad (8)$$

Where, the kinematic viscosity of the fluid is$\nu = {}^{\mu}/_{\rho}$.

The expression stated below satisfy Darcy's resistance in the Oldroyd-B fluid [22]

$$-\left(1 + \lambda_r \frac{\partial}{\partial t}\right)\frac{\emptyset\mu}{k}u(y,t) = R\left(1 + \lambda \frac{\partial}{\partial t}\right), \quad (9)$$

Where, k is the permeability of porous medium, $\emptyset$ is the porosity and $\lambda_r$is the retardation time. When $\lambda_r = 0$, we get an expression for Maxwell fluid as

$$-u(y,t)\frac{\emptyset\mu}{k} = R\left(1 + \lambda \frac{\partial}{\partial t}\right), \quad (10)$$

The governing differential equations for an incompressible Maxwell fluid in porous media and in the absence of pressure gradient performing the same motion are

$$\frac{\partial u}{\partial t} + \lambda \frac{\partial^2 u}{\partial t^2} = \nu \frac{\partial^2 u}{\partial^2 t} - \frac{\emptyset\nu}{k}\left\{u + \lambda \frac{\partial u}{\partial t}\right\}, \quad (11)$$

$$\tau + \lambda \frac{\partial \tau}{\partial t} = \mu \frac{\partial u}{\partial y}. \quad (12)$$

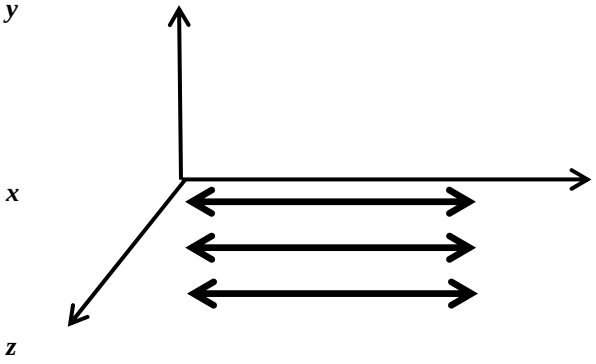


Fig. 1. Geometry of $UH(t)\sin(\omega t)$ $and UH(t)\cos(\omega t)$.

Let us Maxwell fluid haying the porous space onthe plate that is orthogonal to the *y*-axis. In the staring, the fluid is at rest and at the moment $t = 0^+$ the plate is impulsively brought to the oscillating velocity in its plane. The fluid slowly and gradually moved due to the tangential shear stress. The related problem under the following conditions is

$$y > 0, \quad \frac{\partial u(y,0)}{\partial t} = 0, \ \tau(y,0) = u(y,0) = 0, \quad (13)$$

$$u(0,t) = UH(t)\sin(\omega t) \ or \ \cos(\omega t), t \geq 0, \quad (14)$$

Naturally,

$$u(y,t), \frac{\partial u(y,t)}{\partial t} \to 0 \ as y \to \infty and t > 0, \quad (15)$$

have to be also satisfied.

3. SOLUTION OF THE PROBLEM

3.1Velocity Field for case-I:$u(0,t) = UH(t)sin(\omega t)$

Calculation of governing equation also owning the equations (Mahomed, *et al.,* 2007) and (Nield, 1999) we shall apply Fourier sine transforms (Debnath, and. Bhatta, 2007)

$$\left(1 + \lambda \frac{\partial}{\partial t}\right)\frac{\partial u_s(\xi,t)}{\partial t} = \sqrt{2/\pi}\,\nu\xi UH(t)sin\omega t - \nu\xi^2 u_s(\xi,t) - \frac{\emptyset\nu}{k}\left(\lambda \frac{\partial u_s(\xi,t)}{\partial t} + u_s(\xi,t)\right), \quad (16)$$

And equation (16) is under the satisfaction of equation (13)

$\xi > 0$, $\frac{\partial u_s(\xi,0)}{\partial t} = u_s(\xi,0) = 0,$ (17)

Applying the Laplace transform to equations(16) and (17), we get

$$\overline{u_s}(\xi,\vartheta) = \frac{U\sqrt{2/\pi}\,\nu\xi\omega}{(\omega^2+\vartheta^2)\left\{\lambda\vartheta^2+\left(1+\frac{\emptyset\nu\lambda}{k}\right)\vartheta+\nu\xi^2+\frac{\emptyset\nu}{k}\right\}}, \quad (18)$$

equation(18)can also be expressed in suitable equivalent result as,

$$\overline{u_s}(\xi,\vartheta) = \frac{U\sqrt{2/\pi}\,\nu\xi\omega}{\left(\frac{\emptyset\nu}{k}+\nu\xi^2\right)}\left[\frac{1}{(\omega^2+\vartheta^2)} - \frac{\left(1+\frac{\emptyset\nu\lambda}{k}+\lambda\vartheta\right)\vartheta}{(\omega^2+\vartheta^2)\left\{\lambda\vartheta^2+\vartheta\left(\frac{\emptyset\nu\lambda}{k}+1\right)+\frac{\emptyset\nu}{k}+\nu\xi^2\right\}}\right], \quad (19)$$

usinginverse Fourier sine formula on (19),

$$\bar{u}(y,\vartheta) = \frac{2U\nu\xi\omega}{\pi\left(\frac{\emptyset\nu}{k}+\nu\xi^2\right)}\int_0^\infty \frac{\sin(y\xi)}{\xi}\left[\frac{1}{(\omega^2+\vartheta^2)} - \frac{\left(1+\frac{\emptyset\nu\lambda}{k}+\lambda\vartheta\right)\vartheta}{(\omega^2+\vartheta^2)}\frac{1}{\left\{\lambda\vartheta^2+\left(\frac{\emptyset\nu\lambda}{k}+1\right)\vartheta+\frac{\emptyset\nu}{k}+\nu\xi^2\right\}}\right]d\xi, \quad (20)$$

Using the facts of integral,

$$\int_0^\infty \frac{bsin(y\xi)}{a^2+b^2}d\xi = \frac{\pi e^{-ay}}{2}, \;\; a>0. \quad (21)$$

Simplifying equation (21) we get,

$$\bar{u}(y,\vartheta) = \frac{e^{-y\sqrt{\emptyset/k}}U\omega}{(\omega^2+\vartheta^2)} + \frac{2\,U\omega}{\pi(\vartheta_2-\vartheta_1)\lambda}\int_0^\infty \frac{\xi\sin(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}\frac{\vartheta}{(\omega^2+\vartheta^2)}\left\{\frac{\lambda\vartheta_1+1}{(\vartheta-\vartheta_1)} - \frac{\lambda\vartheta_2+1}{(\vartheta-\vartheta_2)}\right\}d\xi + \frac{2\,U\nu\omega\emptyset}{k\pi(\vartheta_2-\vartheta_1)\lambda}$$
$$\times\int_0^\infty \frac{\xi\sin(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}\frac{\vartheta}{(\omega^2+\vartheta^2)}\left\{\frac{1}{\vartheta-\vartheta_1} - \frac{1}{\vartheta-\vartheta_2}\right\}d\xi, \quad (22)$$

The algebraic equation$\lambda\vartheta^2+\left(\frac{\emptyset\nu\lambda}{k}+1\right)\vartheta+\frac{\emptyset\nu}{k}+\nu\xi^2 = (\vartheta-\vartheta_1)(\vartheta-\vartheta_2)$ has roots.where,

$$\vartheta_1,\vartheta_2 = -\frac{(1)\pm\sqrt{(1)^2-4(\nu\xi^2)\lambda}}{2\lambda} \quad (23)$$

Finally,we apply the inverse Laplace transform to (23),we get

$$u_S(y,t) = UH(t)\sin(\omega t)\,e^{-y\sqrt{\emptyset/k}} + \frac{2\,UH(t)\,\omega}{\pi(\vartheta_2-\vartheta_1)\lambda}\int_0^\infty \frac{\xi\sin(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}\cos(\omega t) * \left\{(\lambda\vartheta_1+1)e^{\vartheta_1 t} - (\lambda\vartheta_2+1)e^{\vartheta_2 t}\right\}d\xi$$
$$+\frac{2\,UH(t)\emptyset\omega\nu}{k\pi(\vartheta_2-\vartheta_1)\lambda}\int_0^\infty \frac{\xi\sin(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}\cos(\omega t) * \left(e^{\vartheta_1 t}-e^{\vartheta_2 t}\right)d\xi. \quad (24)$$

3.2Shear stress for case-I: $u(0,t) = UH(t)sin(\omega t)$

Apply Laplace transform to equation (12), the expression for shear stressis

$$\bar{\tau}(y,\vartheta) = \frac{\mu\,\partial\bar{u}(y,\vartheta)/\partial y}{(1+\lambda\vartheta)}, \quad (25)$$

Calculating $\partial\bar{u}(y,\vartheta)/\partial y$from equation (22),

$$\frac{\partial\bar{u}(y,\vartheta)}{\partial y} = -\frac{U\sqrt{\emptyset/k}\,\omega e^{-y\sqrt{\emptyset/k}}}{(\omega^2+\vartheta^2)} - \frac{2\,U\omega}{\pi\lambda}\int_0^\infty \frac{\xi^2 cos(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}\left\{\frac{(1+\vartheta\lambda)\vartheta}{(\omega^2+\vartheta^2)(\vartheta-\vartheta_1)(\vartheta-\vartheta_2)}\right\}d\xi + \frac{2\,U\emptyset\nu\omega}{k\pi(\vartheta_2-\vartheta_1)\lambda}$$
$$\times\int_0^\infty \frac{\xi^2 cos(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}\left\{\frac{(1+\vartheta\lambda)\vartheta}{(\omega^2+\vartheta^2)(\vartheta-\vartheta_1)(\vartheta-\vartheta_2)}\right\}d\xi, \quad (26)$$

Setting equation (26) in (25),

$$\bar{\tau}(y,\vartheta)=\frac{\mu}{(1+\lambda\vartheta)}\left[-\frac{U\sqrt{\emptyset/k}\omega e^{-y\sqrt{\emptyset/k}}}{(\omega^2+\vartheta^2)}-\frac{2U\omega}{\pi\lambda}\int_0^\infty\frac{\xi^2cos(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}\frac{(1+\vartheta\lambda)\vartheta}{(\omega^2+\vartheta^2)(\vartheta-\vartheta_1)(\vartheta-\vartheta_2)}d\xi+\frac{\emptyset\omega\nu\ 2\ U}{\pi k(\vartheta_2-\vartheta_1)\lambda}\right.$$
$$\left.\times\int_0^\infty\frac{\xi^2cos(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}\frac{\vartheta}{(\omega^2+\vartheta^2)(\vartheta-\vartheta_1)(\vartheta-\vartheta_2)}d\xi\right], \tag{27}$$

On simplification equation (27)

$$\bar{\tau}(y,\vartheta)=-\sqrt{\frac{\emptyset}{k}}\frac{U\mu\omega e^{-y\sqrt{\emptyset/k}}}{(\omega^2+\vartheta^2)(1+\lambda\vartheta)}-\frac{2\ U\mu\omega}{\pi\lambda}\int_0^\infty\frac{\xi^2cos(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}\frac{\vartheta}{(\omega^2+\vartheta^2)(\vartheta-\vartheta_1)(\vartheta-\vartheta_2)}d\xi\ +\frac{\emptyset\nu\omega\mu\ 2\ U}{\pi k(\vartheta_2-\vartheta_1)\lambda}$$
$$\times\int_0^\infty\frac{\xi^2cos(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}\frac{\vartheta}{(\omega^2+\vartheta^2)(1+\lambda\vartheta)(\vartheta-\vartheta_1)(\vartheta-\vartheta_2)}d\xi\Bigg], \tag{28}$$

bydecomposing equation (28),

$$\bar{\tau}(y,\vartheta)=-\sqrt{\frac{\emptyset}{k}}\frac{U\mu\omega e^{-y\sqrt{\emptyset/k}}}{(\omega^2+\vartheta^2)(1+\lambda\vartheta)}+\frac{\omega\mu\ 2\ U}{\pi(\vartheta_2-\vartheta_1)\lambda}\int_0^\infty\frac{\xi^2cos(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}\frac{1}{(\omega^2+\vartheta^2)}\left\{\frac{\vartheta}{(\vartheta-\vartheta_2)(\vartheta-\vartheta_1)}\right\}d\xi$$
$$+\frac{\emptyset\omega\mu\nu\ 2\ U}{\pi k(\vartheta_2-\vartheta_1)\lambda}\int_0^\infty\frac{\xi^2cos(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}\frac{\vartheta}{(\omega^2+\vartheta^2)}\left\{\frac{\lambda^2(\vartheta_1-\vartheta_2)}{(1+\lambda\vartheta_1)(1+\lambda\vartheta_2)(1+\lambda\vartheta)}+\frac{1}{(1+\lambda\vartheta_1)(\vartheta-\vartheta_1)}\right.$$
$$\left.-\frac{1}{(1+\lambda\vartheta_2)(\vartheta-\vartheta_2)}\right\}d\xi, \tag{29}$$

Applying inverse Laplace transform on(29)

$$\tau_S(y,t)=-UH(t)\frac{\mu}{\lambda}\sqrt{\frac{\emptyset}{k}}e^{-y\sqrt{\emptyset/k}}\sin(\omega t)*e^{-t/\lambda}+\frac{\mu\omega\ 2\ U}{\pi(\vartheta_2-\vartheta_1)\lambda}\int_0^\infty\frac{\xi^2cos(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}cos(\omega t)*\left(e^{\vartheta_1 t}-e^{\vartheta_1 t}\right)d\xi$$
$$-\frac{\emptyset\omega\mu\nu\ 2\ U}{\pi k(\vartheta_2-\vartheta_1)\lambda}\int_0^\infty\frac{\xi^2cos(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}cos(\omega t)*\left\{\frac{e^{\vartheta_2 t}}{(1+\vartheta_2\lambda)}-\frac{e^{\vartheta_1 t}}{(1+\vartheta_1\lambda)}-\frac{\lambda e^{-t/\lambda}(\vartheta_1-\vartheta_2)}{(1+\vartheta_2\lambda)(1+\vartheta_1\lambda)}\right\}d\xi. \tag{30}$$

(30)

is obtained.

3.3 Velocity Field and shear stress for case-II: $u(0,t)=UH(t)cos(\omega t)$

Applying similar manners, we have investigated the velocity field and corresponding shear stress

$$u_C(y,t)=UH(t)\cos(\omega t)\,e^{-y\sqrt{\emptyset/k}}+\frac{2\ UH(t)\ \omega}{\pi(\vartheta_2-\vartheta_1)\lambda}\int_0^\infty\frac{\xi sin(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}sint(\omega t)*\left\{(\lambda\vartheta_1+1)e^{\vartheta_1 t}-(\lambda\vartheta_2+1)e^{\vartheta_2 t}\right\}d\xi$$
$$+\frac{2\ UH(t)\ \nu\lambda\emptyset}{k\pi(\vartheta_2-\vartheta_1)\lambda}\int_0^\infty\frac{\xi sin(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}\cos(\omega t)*\left(e^{\vartheta_1 t}-e^{\vartheta_2 t}\right)d\xi. \tag{31}$$

and

$$\tau_C(y,t)=-UH(t)\frac{\mu}{\lambda}\sqrt{\frac{\emptyset}{k}}e^{-y\sqrt{\emptyset/k}}cos(\omega t)*e^{-t/\lambda}+\frac{\mu\omega\ 2\ U}{\pi(\vartheta_2-\vartheta_1)\lambda}\int_0^\infty\frac{\xi^2cos(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}sin(\omega t)*\left(e^{\vartheta_1 t}-e^{\vartheta_1 t}\right)d\xi$$
$$-\frac{\mu\lambda\nu\emptyset\omega\ 2\ U}{\pi k(\vartheta_2-\vartheta_1)\lambda}\int_0^\infty\frac{\xi^2cos(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}cos(\omega t)*\left\{\frac{e^{\vartheta_2 t}}{(1+\vartheta_2\lambda)}-\frac{\lambda e^{-t/\lambda}(\vartheta_1-\vartheta_2)}{(1+\vartheta_2\lambda)(1+\vartheta_1\lambda)}-\frac{e^{\vartheta_1 t}}{(1+\vartheta_1\lambda)}\right\}d\xi. \tag{32}$$

4. LIMITING CASE

4.1. Absence of porosity in Maxwell Fluid when $\emptyset\to 0$

Letting$\emptyset\to 0$ into equations (24) and (30) the solutionsare in the absence of porosity

$$u_S = UH(t)\sin(\omega t) + \frac{H(t)\omega\ 2\ U}{\pi(\vartheta_2-\vartheta_1)\lambda}\int_0^\infty \frac{\sin(y\xi)}{\xi}\cos(\omega t) * \left\{(\lambda\vartheta_1+1)e^{\vartheta_1 t} - (\lambda\vartheta_2+1)e^{\vartheta_2 t}\right\}d\xi, \quad (33)$$

$$\tau_S = -\frac{H(t)\mu\omega\ 2\ U}{\pi\lambda}\int_0^\infty \cos(y\xi)\cos(\omega t) * \left(e^{\vartheta_1 t} - e^{\vartheta_2 t}\right)d\xi\,. \quad (34)$$

are achieved.

4.2. Presence of porosity in Newtonian Fluid when $\lambda \to 0$

Letting$\emptyset \to 0$ into equations (24and (30) the solutions are in the presence of porosity

$$u_S(y,t) = UH(t)\,sin(\omega t)\,e^{-y\sqrt{\emptyset/k}} - \frac{H(t)\omega\ 2\ U}{\pi}\int_0^\infty \frac{\xi sin(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}cos(\omega t) * e^{-\left(\frac{\emptyset}{k}+\nu\xi^2\right)t}d\xi - \frac{H(t)\nu\emptyset\omega\ 2\ U}{k\pi}$$
$$\times\int_0^\infty \frac{\xi sin(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}cos(\omega t) * e^{-\left(\frac{\emptyset}{k}+\nu\xi^2\right)t}d\xi. \quad (35)$$

$$\tau_S = -UH(t)\mu\sqrt{\frac{\emptyset}{k}}e^{-y\sqrt{\emptyset/k}} + \frac{H(t)\mu\omega\ 2\ U}{\pi}\int_0^\infty \frac{\xi^2 cos(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}sin(\omega t) * e^{-\left(\frac{\emptyset}{k}+\nu\xi^2\right)t}d\xi - \frac{\mu\nu\omega\emptyset\ 2\ U}{k\pi}\int_0^\infty \frac{\xi^2 cos(y\xi)}{\left(\frac{\emptyset}{k}+\xi^2\right)}$$
$$\times cos(\omega t) * e^{-\left(\frac{\emptyset}{k}+\nu\xi^2\right)t}d\xi \quad (36)$$

are obtained

4.3. Absence of porosity in Newtonian Fluid when $\lambda \to 0$ and $\emptyset \to 0$

Letting$\emptyset \to 0$ into equations (24 and (30) the solutions are

$$u_S = UH(t)\,sin(\omega t) - \frac{H(t)\omega\ 2\ U}{\pi}\int_0^\infty \frac{\xi sin(y\xi)}{\xi}cos(\omega t) * e^{-\nu\xi^2 t}d\xi \quad (37)$$

$$\tau_S = -\frac{H(t)\mu\omega\ 2\ U}{\pi}\int_0^\infty cos(y\xi)\,cos(\omega t) * e^{-\nu\xi^2 t}d\xi \quad (38)$$

5. RESULTS AND CONCLUSION

The main purpose of this investigations is establish the exact solutions related to the presence and absence of porosity for the flow of Maxwell fluid under the sine and cosine oscillations which depend upon the rheological pertinent parameters such as the permeability of fluid$k$,the relaxation timeλ,the viscosity of fluidν, amplitude of fluidω, non-zero constant$U$, dynamics viscosityμ, time$t$ etc. The exact solutions are reduced from general solutions among four directions namely (i) Maxwell fluid in the absence of porosity when we set $\emptyset = 0$, (ii) Maxwell fluid in the presence of porosity when we set $\emptyset \neq 0$, (iii) Newtonian fluid in the absence of porosity when we set $\emptyset = 0$ and $\lambda = 0$and (iv) Newtonian fluid in the presence of porosity when we set $\lambda = 0$ and $\emptyset \neq 0$.Viewing the effects of porosity, the graphical illustrations for velocity field and shear stress are depicted against t and y for distinct valuesof the rheological pertinent parameters $U, \mu, \nu, \lambda, k, \omega$ etc.**(Fig. 2)** is depicted in which velocity field is increasing and shear stress is decreasing function with respect to increasing values of time. The influence of permeability and relaxation time on the fluid is drawn in the **(Fig. 3)** which seem to be similar. **(Fig. 4, 5, 6)**prepared to show the impacts of viscosity and frequency of oscillations in the motion of fluid. The oscillating behavior of motion for fluid can easily be seen from **(Fig. 7)**. From **(Fig. 8)**, profile of velocity field and shear stress is drawn for comparison under the impacts of the presence and absence of porosity. Finally four models are to gather depicted for revealing some relevant physical aspects which are (Maxwell fluid in the presence of porosity, Maxwell fluid in the absence of porosity, Newtonian fluid in the presence of porosity and Newtonian fluid in the absence of porosity).As compared to Newtonian fluid with porosity moves the fastest than Maxwell with porosity, Maxwell without porosity, and Newtonian fluid without porosity. It is pointed out that the pertinent rheological parameter permeability *k* has significant impacts on the motion of fluid and also due to permeability *k* the motion of fluid slow down near the oscillating plane. The units of all rheological pertinent parameters are in SI units.

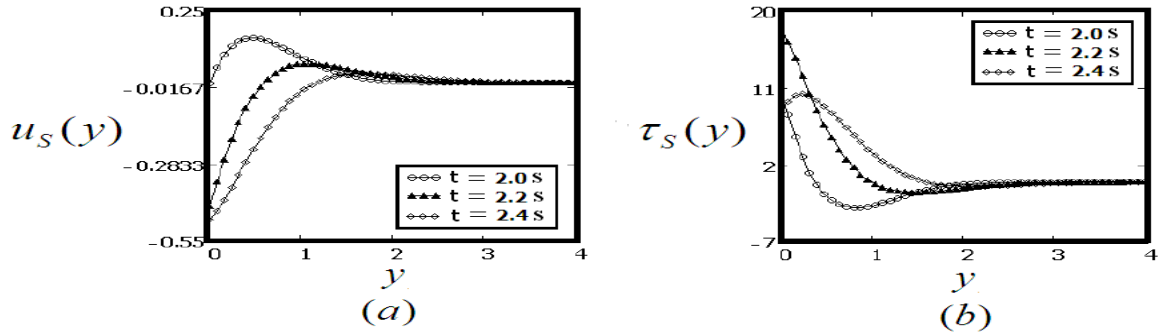


Fig.2:Graphical Illustration of time parameter.

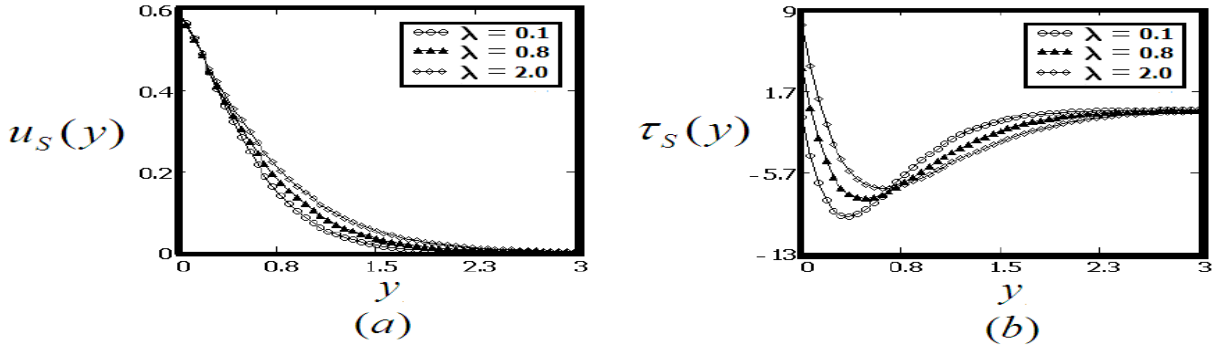


Fig. 3:Graphical Illustration of relaxation time parameter.

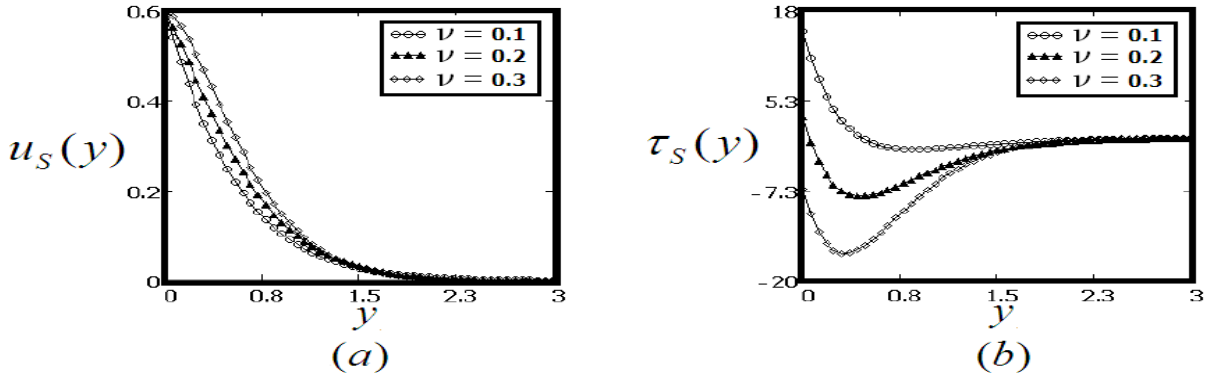


Fig. 4:Graphical Illustration of viscosity.

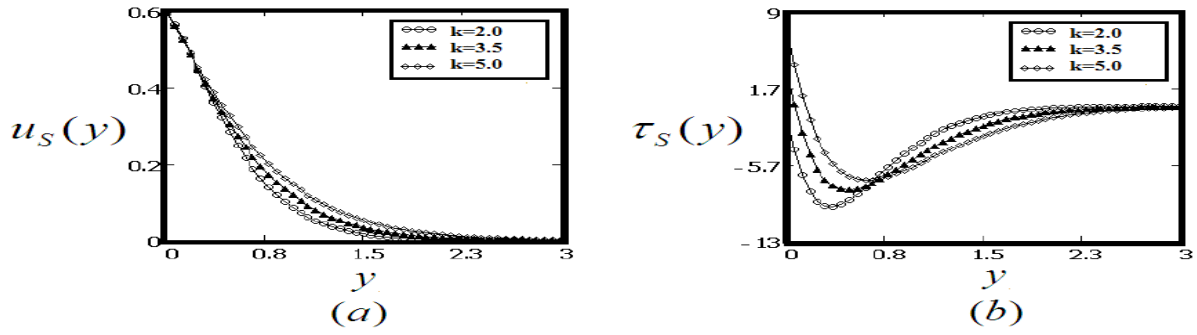


Fig. 5: Graphical Illustration of permeability parameter.

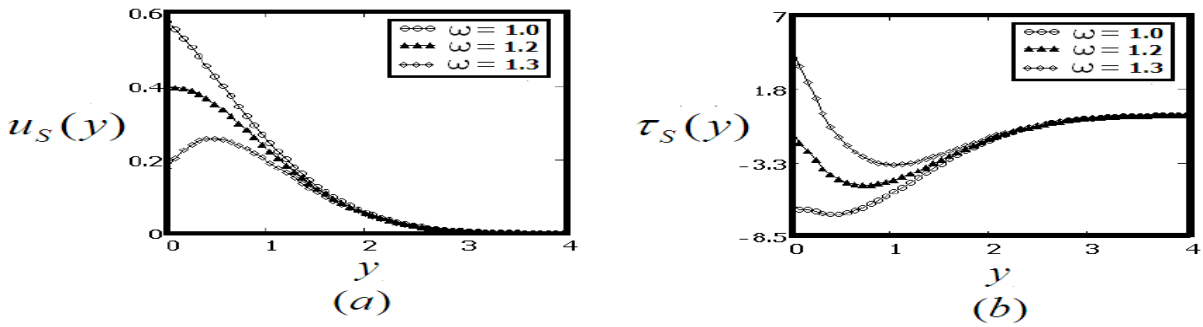


Fig.6:Graphical Illustration of oscillations.

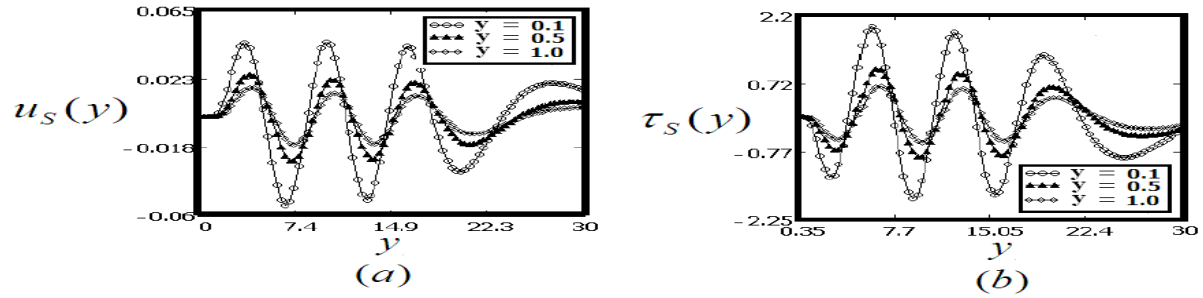


Fig.7:Graphical Illustration of spatial parameter.

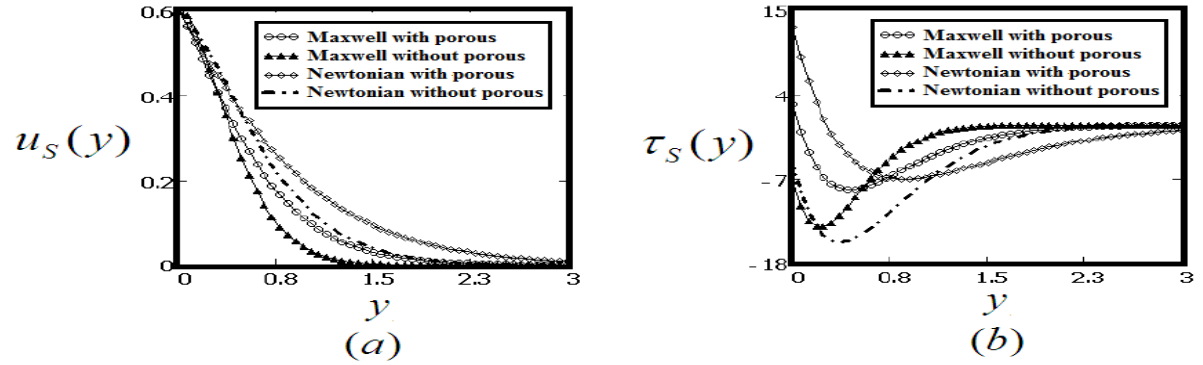


Fig. 8: Graphical Illustration for comparison of four models.

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