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Mathematical Modelling of Newtonian Flow Through a Sinusoidal Tube

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Abstract: In this research work, time–marching finite element method for numerical simulations has been used to investigate the flow of Newtonian fluid through a sinusoidal tube. A semi–implicit Taylor–Galerken/pressure–correction scheme is utilised to obtain the steady–state results. A main objective of present work is to investigate the effects of inertia and impact of undulation on the flow field.

Keywords: Finite Element Method, Newtonian Fluids, Sinusoidal Tube.

1. INTRODUCTION

Numerical simulation of flow of fluid through sinusoidal tube has been given remarkable interest by Numerical researchers in variety of engineering and bio medical applications (Baloch, *et al.* 1995, 2007 and Shah, *et al.* 2013, 2015). Mostly it is encountered in food processing, polymer, blood flow through arteries and capillaries, paper making etc. Various studies regarding the Flow through both tubes and channels having corrugated nature are available in the field of CFD (Computational Fluid Dynamics) using the numerical methods such as Finite volume, Finite Differences and Finite element methods (Phan–Thein, *et al.* 1985, Lahbabi, and Chang 1986, Pilitsis and Beris 1989) and (Mai–Duy 2004) . Few of others used some analytical methods which based on easy hypothesises and usually transaction with extraordinarily unique cases (Mukhopadhyay and Layek 2009), (Taha 2010), (Patricio *et al.* 2011).

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In present investigation Newtonian fluid flows through sinusoidal tube are investigated. The main object for choosing this particular problem is due to containing both converging and diverging sections. The geometry is relatively simple and may reduce the shear and elongational motion. Inertial influence and undulation effects on flow structure are analysed here by selecting three different levels of undulations, i.e. forty percent, fifty percent and sixty percent. Critical points (where recirculation starts) are also marked in selected levels of undulation. Finite element time–stepping technique is adopted for numerical predictions which based on the semi implicit Taylor–Galerken/pressure–correction scheme.

2. PROBLEM FORMULATION

Flow geometry is considered for Newtonian fluids through sinusoidal tube as a domain of interest and is shown in **(Fig.–1).** It is assumed that in circular tubes, flows are axi–symmetric so flow is examined for upper half of sinusoidal tube only. The local radius along sinusoidal tube along the axis of flow is given by

$$r_w = R\left[1+\lambda\, Cos\left(\frac{2\pi z}{L}\right)\right]$$

Where R is the characteristic radius of the corresponding straight tube, z is the axial direction in the Newtonian flow. "*r"* is along the radial direction, while α and L represents dimensionless amplitude of the undulation and wave length respectively. Due to symmetry conditions only one half of the geometry of single cell is considered and illustrated in Figure–1 with boundary conditions. Completing the problem formulation, necessary to prescribe initial condition in present study at low Reynolds number quiescent flow initial condition, i.e., $\boldsymbol{v} = 0$ at $t = 0$ is adopted. Whilst, for boundary conditions a mixed Dirichlet and Neumann boundary conditions are employed and described here in short. Since undulation wall is solid, therefore no slip boundary conditions are taken as zero. At the exit both radial velocity and pressure are taken to be zero, whilst axial velocity is unknown. Traction free condition and radial velocity are also taken to be zero at axis of symmetry and in the end for Newtonian fluid flow at inlet axial velocity is considered as:

$$v_z = v_{\max}\left\{1-\left(\frac{r}{R}\right)^2\right\}$$

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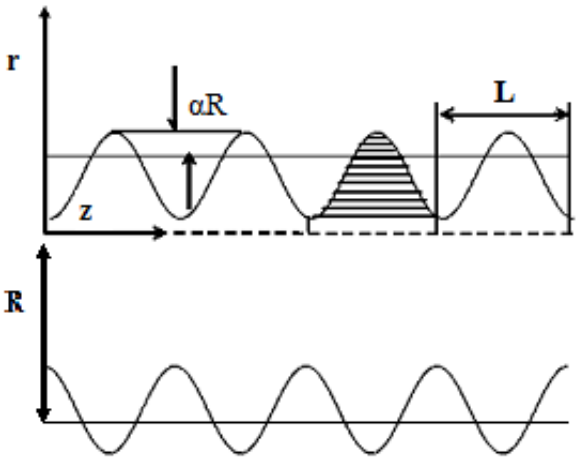


Fig.–1:Flow domain for Sinusoidal Tube

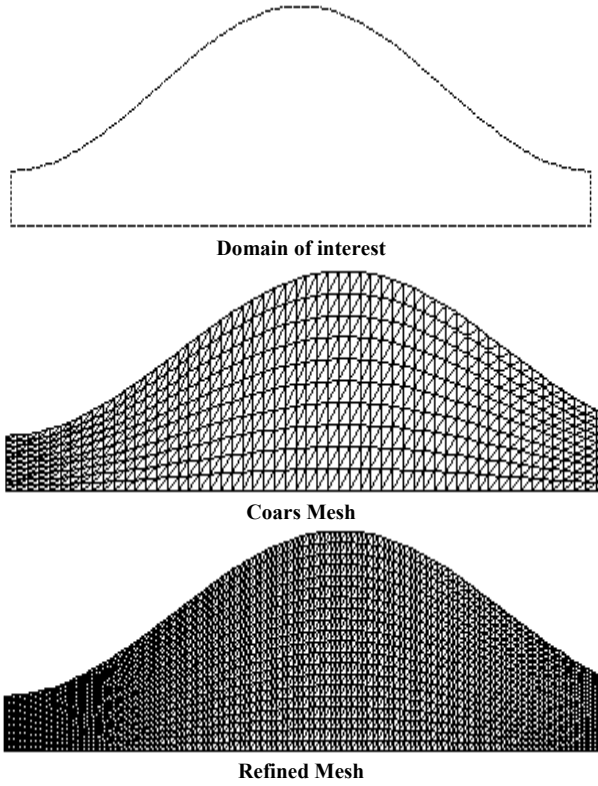


Fig.2: Domain of Interest and Finite Element Meshes for PCT.

3. GOVERNING SYSTEM OF EQUATIONS

For Newtonian fluid through sinusoidal tube, the flow governing system consists of conservation equation of mass and momentum transport equation in cylindrical polar co–ordinates. The non–dimensional components for momentum transport equation in the absence of body force can be written as:

Continuity equation: $\nabla . \boldsymbol{w} = 0$

Momentum Equation:

$$\frac{\partial w}{\partial t} = \frac{1}{\mathrm{Re}} \nabla^2 w - (w.\nabla) w - \nabla p$$

Where" w_r " and "w_z " be the velocity components in "r" and "z" direction, "p" the identical fluid pressure and "t" represents the time. Reynolds number (Re) determined here as: $\mathrm{Re} = \dfrac{\rho W R}{\mu_0}$

4. NUMERICAL SCHEME

Numerical scheme employed here for the Newtonian flows is the time–stepping finite element method (Shah *et al.* 2013, 2015). This considered algorithm was originally designed for incompressible Newtonian flows, and after that by adopting Petrove–Galerken streamlines up-winding technique, this algorithm used to solve the complex flows of Viscoelastic fluids. Describing this technique, short précis is presented here for wholeness. The method holds temporal discretisation in Taylor expansion with the combination of predicator–correction technique. Higher order time derivatives are captured by adopting two steps Lax–Wendroff approach. Regarding finite difference and Euler–Galerken FE systems (Baloch *et al.* 1995, 2007), these schemes illustrate considerable development in both accuracy and stability. The Pressure–correction technique is for assurance in second order accuracy by linearised energy analysis. When this technique is combined to "predicator- corrector Taylor–Galerken method", then basis for numerical proposal employed here in this investigation to get the steady–state solutions.

The numerical solutions process is conducted in three fractional stage forms. In two steps non–divergence free velocity field is calculated at stage one. In first part of stage one, on right hand side, the fluid is given in cylindrical structure of orientation at half time step and at the start; result is obtained at half time step. Halfway non solenoidal velocity field erstwhile computed in the stage–1(b), whilst by solving Poisson equation, Pressure difference has been calculated at stage two. To obtain the second order level of accuracy, the Crank–Nicolson Scheme is used. At final stage solenoidal velocity has been determined. First and final stages are managed with improved mass matrices and have been iteratively evaluated using Jacoby method. As the pressure matrix positive definite and symmetric with a bounded structure so at the stage two we employed direct Choleski method. To solve the problem equations are evaluated through conventional Galerken weighted residual procedure and merged velocity

pressure preparation are employed for solutions. Here triangular element is selected for geometry. Quadratic and linear piecewise functions are employed for each element i.e. pressure as well as velocity respectively.

Stage–1(a):Given $\left(w^n, p^n\right)$, **find** $w^{n+\frac{1}{2}}$ **such that**

$$\left\{\left(\frac{2}{\Delta t}\left(w^{n+\frac{1}{2}}-w^n\right),w\right)+\frac{1}{2\mathrm{Re}}\left(\nabla\left(\left(w^{n+\frac{1}{2}}-w^n\right),\nabla w\right)\right)\right\}$$

$$=\left(p^n-\frac{1}{\mathrm{Re}}\nabla w^n,\nabla w\right)-\left(\left(\left(w.\nabla\right)w\right)^n,w\right)$$

Stage–1(b):Given $\left(w^n, w^{n+\frac{1}{2}}, p^n\right)$,**find** w^* **such that:**

$$\left\{\left(\frac{1}{\Delta t}\left(w^*-w^n\right),v\right)+\frac{1}{2\,\mathrm{Re}}\left(\nabla\left(w^*-w^n\right),\nabla w\right)\right\}$$

$$=\left(p^n-\frac{1}{\mathrm{Re}}\nabla w^n,\nabla w\right)-\left(\left(\left(w.\nabla\right)w\right)^{n+\frac{1}{2}},w\right)$$

Stage–2: Given w^* **and** p^n **, find** $p^{n+1}-p^n$ **such that:**

$$\left(\theta\nabla\left(p^{n+1}-p^n\right),\nabla q\right)=\frac{-1}{\Delta t}\left(\nabla w^*,q\right)$$

Stage–3:Given v^* **and** $p^{n+1}-p^n$ **, find** v^{n+1} **such that:**

$$\left(\frac{1}{\Delta t}\left(w^{n+1}-w^*\right),w\right)=\left(\theta\left(p^{n+1}-p^n\right),\nabla w\right)$$

5. RESULTS AND DISCUSSIONS

Consider the Newtonian fluid flow through corrugated tube. Three different levels of undulation are selected for relevant domain and two finite element meshes are employed for simulation as shown in Figure–2. Computational outcomes are expressed in streamlines form. Inertial flows of Newtonian fluid are simulated here to understand the influence of inertia and effects of tube wave on flow structure. For each undulation level, critical limit points of Reynolds numbers are point out, where the generation of recirculation starts. Growth in recirculation at various inertial points (vortex intensity) is also realised in present work. The results in this investigation are good agreement with the previously available solutions (Baloch *et al.* 1995, 2007) and (Shah *et al.* 2013, 2015). Flow is considered in each level of undulation in the form of streamline projections and described in core flow and recirculation regions. Seven contours are set for main flow region in each undulation level and in the area where the vortex generation occurs, ψ_{min} has been adopted to determine the slower area. In recirculation area, contours are represented from vortex centre to vortex periphery $\psi_{per:}$ i.e. innermost to outer most contours.

Inertial effects on flow formation

At first, to investigate inertial effects on flow structure, computational results are analysed for pure Newtonian fluid through sinusoidal tube with all selected undulation levels. Streamline patterns are illustrated with escalating values of inertia. It is clear from the simulated data shown in Figure–3 that, stream lines are roughly close to sinuous wall and no generation of recirculation is observed at low values of inertia up to Re = 29, 27 and Re =30 at forty, fifty and sixty percent undulation respectively. In all undulation levels recirculation has been observed at almost nearby undulation upstream wall. In forty percent undulation, generation of recirculation area starts at Reynolds number (Re = 30). In fifty percent undulation, it starts at low Reynolds number as compare to forty percent i.e. Re = 28. Whilst, in sixty percent undulation, recirculation observed surprisingly at higher value of inertia Re = 31, comparing to both lower undulation levels. In Figures: [4–6], it is observed that, as the value of inertia increase to higher value, the centre of recirculation moves to middle of undulation and subsequently to downstream area at high inertia of order Re = 1200. Recirculation region enhance more and strengthens in this process and occupy whole undulation. The streamlines representing main core flow region realized very much straight. In forty percent undulation, at high values of Reynolds number the shape of recirculation region is observed stable but both in fifty and sixty percent undulations it is observed unstable.

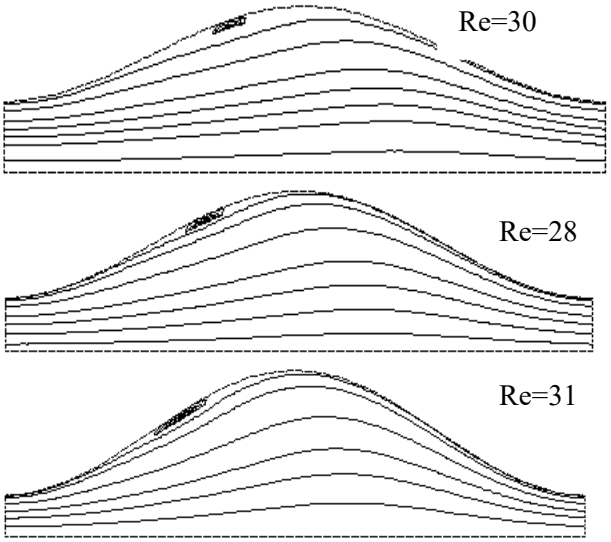


Fig.–3: Streamlines patterns for the flow of Newtonian fluid through sinusoidal tube at critical Reynolds number for selected levels of undulation

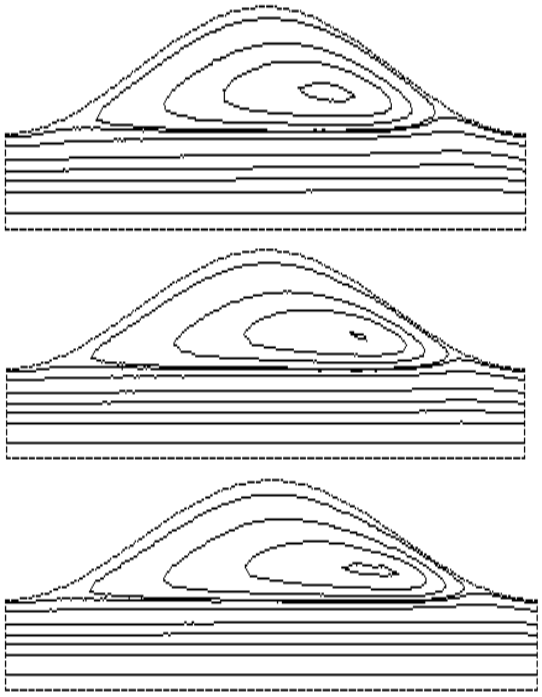


Fig. 4: Streamlines projections for Newtonian flow through sinusoidal tube of forty percent undulation with increasing Reynolds number Re = (400, 800 and 1200)

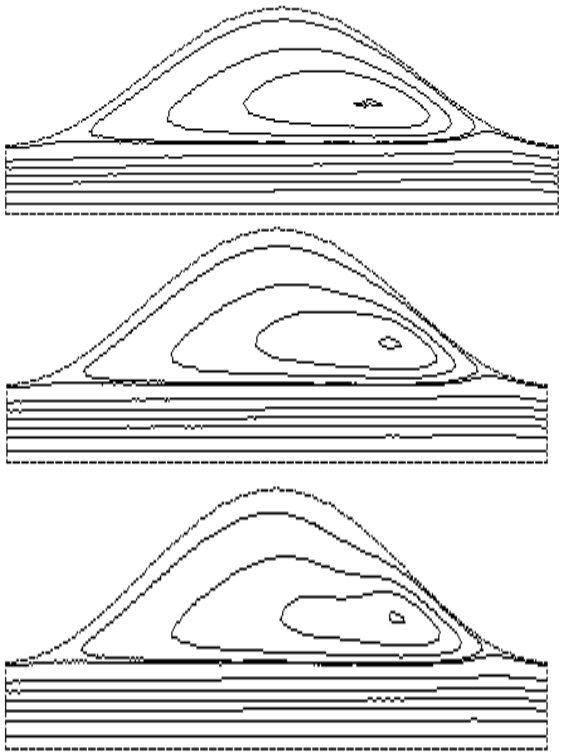


Fig. 5: Streamlines projections for Newtonian flow through sinusoidal tube of fifty percent undulation with increasing Reynolds number Re = (400, 800 and 1200)

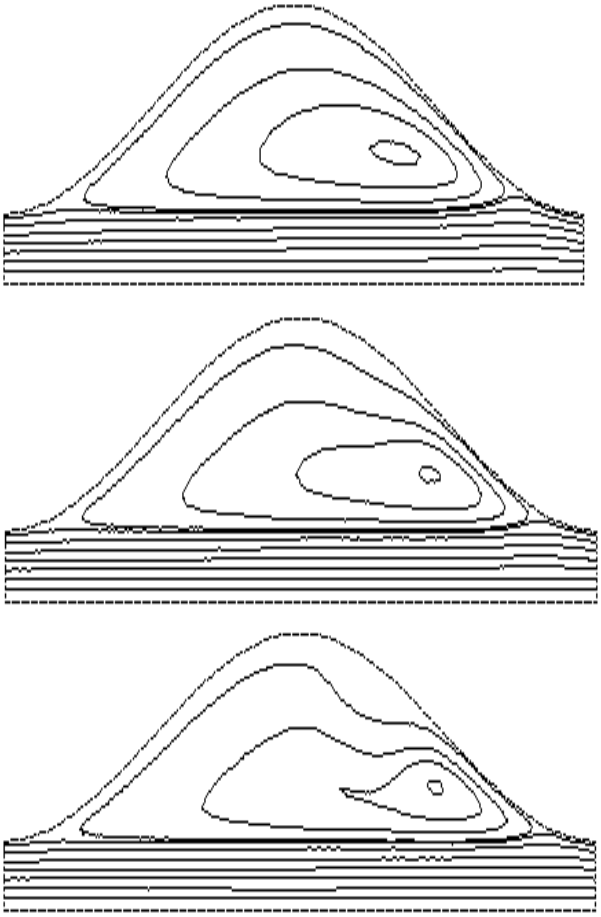


Fig.6: Streamlines projections for Newtonian flow through sinusoidal tube of sixty percent undulation with increasing Reynolds number Re = (400, 800 and 1200)

Pressure drop is presented for the sinusoidal tube with all selected levels of undulation. This is demonstrated in Figure–7 for pure Newtonian flow at Reynolds number Re = 400. In forty percent undulation, pressure contours are plotted from 7.7464e–023 to 0.233923, in Fifty Percent from 8.48763e–023 to 0.298588 and in Sixty percent they are plotted from 8.5345e–023 to 0.405416. It is very much clear from the figure the maximum pressure drop shifts from upstream in the corner of converged wall to downstream in the area of diverging wall. Because of this reason, the non – linear influence of the inertia is observed in the friction factor increase as function of increase in Reynolds number but on the other hand non–linearity is not observed in the recirculation area. Flows through sinusoidal tubes are very much complex. At low Reynolds number where generation of recirculation region is not developed, flow resistance is also low but as inertia increased the flow resistance is observed higher than Stokian value.

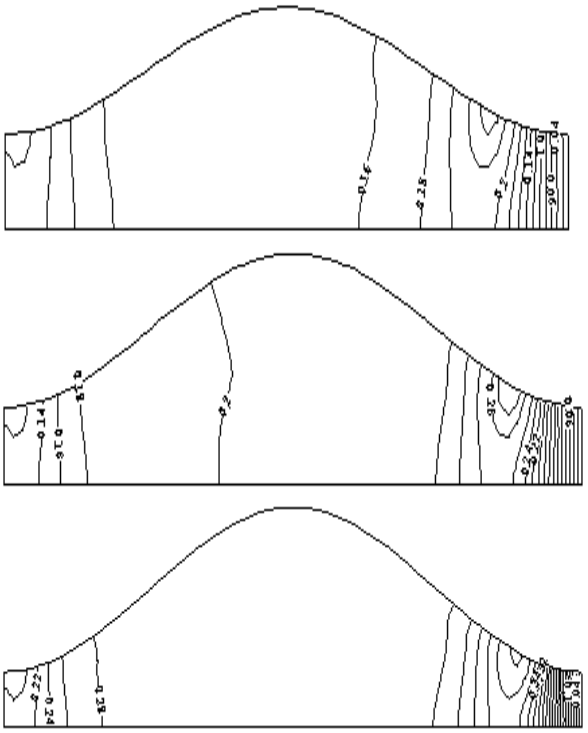


Fig. 7: Graph of Pressure in Forty, Fifty and Sixty percent Undulation At Re = 400

6. CONCLUSIONS

Steady–state numerical predictions are computed by utilizing TGPC Scheme the. The influence of inertia (Re) on flow structure and on various undulation levels, start of recirculation, i.e. "Critical Limit Points of Reynolds number" is realised. it is observed that, as the value of inertia increase to higher value, the centre of recirculation moves to middle of undulation and subsequently to downstream area. Whilst in stability is observed at the higher levels of undulation.

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