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Curvature, Ricci and Matter collineations in Reboucas spacetime

M. J. KHAN*, M. RAMZAN++, M. R. MUFTI**

Department of Mathematics, The Islamia University of Bahawalpur, 63100, Pakistan.

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Abstract: The Reboucas spacetime is considered to study curvature, Ricci and Matter collineations. The curvature collineations are investigated by using the rank of the 6×6 Riemann matrix where as Ricci and Matter collineations are discussed by vanishing Lie derivative of Ricci and energy momentum tensors respectively. Studying all these collineations in the above space time, it has been shown that Ricci collineations coincides with matter collineations. Further it has also been observed that curvature and Ricci collineations of the above spacetime form an infinite dimensional vector space.

Keywords: Rank of the 6×6 Rieman matrix; Curvature collineations;Ricci collineations; Direct integration techinque, infinite dimensional vector space

1. INTRODUCTION

Collineations are the geometric assumptions which concern the "symmetries" of the metric defined by the Lie symmetries of either the metric itself or the tensors defined from the metric (Sharif and Sehar, 2003). This paper is an attempt to investigate the existence of proper curvature collineations (CCs), Ricci collineations (RCs) and Matter collineations (MCs) in the Reboucas spacetime. The curvature collineations which preserve the curvature structure of a spacetimes carry significant information and play an important role in Einstein's theory of general relativity and gravitation. The theory of general relativity, which is actually a field theory of gravitation and is described in terms of geometry and physics, is highly nonlinear (Katzin, *et al.,* 1969). Due to this non-linearity it becomes very difficult to solve the gravitational field equations unless certain symmetry restrictions are imposed on the spacetimes. These symmetry restrictions may be expressed in terms of Killing vector fields (KVF), Homothetic vector fields (HVF), Ricci collineations (RCs) Curvature collineations (CCs) and matter collineations (MCs).

Riemann curvature tensor may also provide some extra understandings which are not provided by (KVF) and (HVF) (Katzin, *et al.,* 1972). It is, therefore, important to study CCs.For the classification of spacetime Ricci collineations give many invariant bases. Ricci and matter collineations have a significant role to obtain the exact solution of EFE,s.The Matter collineations provide the information that in the certain region of spacetime, how the physical fields give the reflection of the symmetries of metric (Tsamparlis,2007).

Here an approach, which is given in (Hall and da Costa, 1991), is adopted to study curvature collineations in the said spacetime by using the rank of the 6×6 Riemann matrix. Here, the non linear coupled partial differential equations are solved by direct integration method. By vanishing Lie derivative of Ricci and energy momentum tensors, Ricci and matter collineations are discussed respectively.

Throughout M represents a four dimensional, connected, Hausdorff spacetime manifold with Lorentz metric g of signature (−, +, +, +). The curvature, Ricci and energy tensors associated with g_{ab}, through the Levi-Civita connection, is denoted in component form by $R_{abcd}, R_{ab} and\ T_{ab}$ respectively. usual covariant, partial and Lie derivatives are denoted by a semicolon, a comma and the symbol L, respectively.

2. MAIN RESULTS FOR CURVATURE COLLINEATIONS

The curvature collineation equation is defined as

$$L_X R^a_{bcd} = 0 \qquad (2.1)$$

or equivalently,

$$R^a_{bcd;e} X^c + R^a_{ecd} X^e_{;b} + R^a_{bed} X^e_{;c} + R^a_{bce} X^e_{;d} - R^e_{bcd} X^a_{;e} = 0 .$$

The vector field X is said to be proper CC if it is not affine (Hall and da Costa, 1991) on M. One can expand the above equation in a set of 22 coupled CC equations which can be seen in (Shabbir, *et al.,* 2003).

Consider a Reboucas spacetime with coordinate (t, r, θ, z) labeled by $(x^0, x^1, x^2 x^3)$ respectively with line element (Krori, *et al.,* 1988)

$$ds^2 = -dt^2 + dr^2 - (1 + 3\cosh 2r)d\theta^2 + dz^2 + 4\cosh 2r dt d\theta. \qquad (2.2)$$

++Corresponding author: Email: muhammad.ramzan@iub.edu.pk

* Department of Mathematics, Govt. post graduate college Khanpur, Rahim Yar Khan, Pakistan

** Department of Computer Science, COMSATS, Institute of Information Technology, Vehari, Pakistan

The above space-time admits three linearly independent Killing vector fields which are (Stephni, *et al.,* 2003).

$$\frac{\partial}{\partial t}, \frac{\partial}{\partial \theta}, \frac{\partial}{\partial z} \tag{2.3}$$

The non-zero independent components of the Riemann tensor are

$\alpha_1 = R_{0101} = 4$, $\alpha_2 = R_{0202} = 4\sinh^2 2r$,

$\alpha_3 = R_{0112} = 8\cosh 2r$,

$\alpha_4 = R_{1212} = -(8 - 24\cosh^2 2r)$.

Writing the curvature tensor with components R_{abcd} at p as a 6×6 symmetric matrix (Hall and da costa,1991)

$$\begin{pmatrix} \alpha_1 & 0 & 0 & \alpha_3 & 0 & 0 \\ 0 & \alpha_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_3 & 0 & 0 & \alpha_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

If the rank of 6x6 Riemann matrix is greater than three there exists no proper CCS (Hall and da.Costa, 1991). Here only one case of rank three is survived for which

$$\alpha_1 \neq 0, \quad \alpha_2 \neq 0, \quad \alpha_3 \neq 0, \quad \alpha_4 \neq 0.$$

Substituting the above information in equation (2.2),

the line element is

$$ds^2 = -dt^2 + dr^2 - (1 + 3\cosh 2r)d\theta^2 + dz^2 + 4\cosh 2r dt d\theta.$$

The above spacetime is 1+3 decomposable and belongs to curvature class C. The CCS in this case are (Hall and, da.Costa., 1991)

So, $$X = f(z)\frac{\partial}{\partial z} + X', \tag{2.4}$$

where f(z) is arbitrary function and X' is a

Homothetic vector field in the induced geometry on each of the three dimensional sub manifold of constant z, The induced metric g_{ab} (where a,b=0,1,2) with non-zero components is given by

$$g_{00} = -1, g_{11} = 1, g_{22} = -(1 + 3\cosh 2r),$$
$$g_{02} = g_{20} = 2\cosh 2r. \tag{2.5}$$

The homothetic vector field X' is given by the relation

$$L_{X'} g_{ab} = 2\alpha g_{ab}; \quad \forall \alpha \in R. \tag{2.6}$$

One can write (2.6) explicitly using (2.5) as (Shabbir and Ramzan, 2007), we have following system of nonlinear coupled partial differential equations .

$$X'^0_{,0} - 2\cosh 2r X'^2_{,0} = \alpha. \tag{2.7}$$

$$X'^0_{,1} - X'^1_{,0} - 2\cosh 2r X'^2_{,1} = 0. \tag{2.8}$$

$$4\sinh 2r X'^1 - X'^0_{,2} + 2\cosh 2r X'^0_{,0} - (1 + 3\cosh^2 2r)X'^2_{,0} + 2\cosh 2r X'^2_{,2} = 4\alpha\cosh 2r. \tag{2.9}$$

$$X'^1_{,1} = \alpha. \tag{2.10}$$

$$2\cosh 2r X'^0_{,1} + X'^1_{,2} - (1 + 3\cosh^2 2r)X'^2_{,1} = 0. \tag{2.11}$$

$$6\cosh 2r \sinh 2r X'^1 - 2\cosh 2r X'^0_{,2} + (1 + 3\cosh^2 2r)X'^2_{,2} = \alpha(1 + 3\cosh^2 2r). \tag{2.12}$$

From the equations (2.7), (2.8) and (2.10) we get

$$\left.\begin{aligned} X'^0 &= \alpha t + \frac{1}{2}\coth 2r A^1_t(t,\theta) + A^3(r,\theta) \\ X'^1 &= \alpha r + A^1(t,\theta) \\ X'^2 &= \frac{1}{4\sinh 2r} A^1_t(t,\theta) + A^2(r,\theta) \end{aligned}\right\}. \tag{2.13}$$

Here, A^1,A^2 and A^3 are functions of integration to be determined. If one proceeds further, after a straight forward calculation in remaining equations , one can find that the proper homothetic vector fields do not exist because equation (2.9) gives that α = 0. So, the only homothetic vector fields in the induced geometry are the Killing vector fields which are

$$\left.\begin{aligned} X'^0 &= 2\cos ech 2r(-c_6\sin 2\theta + c_7\cos 2\theta) + c_5 \\ X'^1 &= c_6\cos 2\theta + c_7\sin 2\theta \\ X'^2 &= \coth 2r(-c_6\sin 2\theta + c_7\cos 2\theta) + c_3 \end{aligned}\right\},$$

where $c_3, c_5, c_6, c_7 \in R$. (2.14)

CCS in this case are given by $X = (X'^0, X'^1, X'^2, f(z))$,

where f(z) is an arbitrary function of z. One can write the above equation after subtracting Killing vector fields as:

$$X = (0, 0, 0, f(z)) \tag{2.15}$$

CCs clearly form an infinite dimensional vector space.

3.MAIN RESULTS FOR RICCI COLLINEATIONS

The Ricci tensor is usually defined as

$$R_{ab} = R^{c}_{acb}. \tag{3.1}$$

The non zero components of Ricci tensor are

$$R_{00} = 8, \; R_{11} = 4 \; R_{02} = R_{20} = -16\cosh 2r$$

$$R_{22} = 36\cosh^2 2r - 4.$$

The vector field X is said to be Ricci collineations if it satisfies the relation

$$L_X R_{ab} = 0. \tag{3.2}$$

Equivalently,

$$R_{ab,c}\, X^{c} + R_{bc}\, X^{c}_{,a} + R_{ac}\, X^{c}_{,b} = 0,$$

where *a, b, c = 0, 1, 2,3.*

The explicit form of the above equation (3.2) can be written as set of non linear coupled partial differential equations:

$$X^0_{,0} - 2\cosh 2r X^2_{,0} = 0. \tag{3.3}$$

$$2X^0_{,1} + X^1_{,0} - 4\cosh 2r X^2_{,1} = 0. \tag{3.4}$$

$$8\sinh 2r X^1 - 2X^0_{,2} + 4\cosh 2r X^0_{,0} - (9\cosh^2 2r - 1)X^2_{,0} + 4\cosh 2r X^2_{,2} = 0. \tag{3.5}$$

$$X^0_{,3} - 2\cosh 2r X^2_{,3} = 0. \tag{3.6}$$

$$X^1_{,1} = 0. \tag{3.7}$$

$$4\cosh 2r X^0_{,1} - X^1_{,2} - (9\cosh^2 2r - 1)X^2_{,1} = 0. \tag{3.8}$$

$$X^1_{,3} = 0. \tag{3.9}$$

$$18\cosh 2r \sinh 2r X^1 - 4\cosh 2r X^0_{,2} + (9\cosh^2 2r - 1)X^2_{,2} = 0. \tag{3.10}$$

$$4\cosh 2r X^0_{,3} - (9\cosh^2 2r - 1)X^2_{,3} = 0. \tag{3.11}$$

From equations (3.3), (3.4),(3.7) and (3.9), we get

$$\left.\begin{aligned} X^0 &= -\frac{1}{2}\coth 2r A^1_t(t,\theta) + \int 2\cosh 2r A^3_t(r,\theta,z)\,dr + A^5(\theta,z) \\ X^1 &= A^2(t,\theta) \\ X^2 &= -\frac{1}{8}\cos ech 2r A^2_t(t,\theta) + A^3(r,\theta,z) \end{aligned}\right\}, \tag{3.12}$$

where $A^1(t,\theta)$, $A^2(t,\theta)$, $A^3(r,\theta,z)$ and $A^5(\theta,z)$ are the functions of integration to be determined. If one proceeds further, after straight forward calculations of remaining equations then we get

$$\left.\begin{aligned} X^0 &= -\cos ech 2r(c_5\cos 2\theta + c_6\sin 2\theta) + c_8 \\ X^1 &= -\frac{1}{2}(c_5\sin 2\theta - c_6\cos 2\theta) \\ X^2 &= -\frac{1}{2}\coth 2r(c_5\cos 2\theta + c_6\sin 2\theta) + c_4 \end{aligned}\right\} \tag{3.13}$$

The Ricci collineations in this case turn out to be

$$\left.\begin{aligned} X^0 &= -\cos ech 2r(c_5\cos 2\theta + c_6\sin 2\theta) + c_8 \\ X^1 &= -\frac{1}{2}(c_5\sin 2\theta - c_6\cos 2\theta) \\ X^2 &= -\frac{1}{2}\coth 2r(c_5\cos 2\theta + c_6\sin 2\theta) + c_4 \\ X^3 &= \xi(x^\alpha), \alpha = 0,1,2,3 \end{aligned}\right\} \tag{3.14}$$

In the above equation (3.14), $\xi(x^\alpha)$ is an arbitrary function of all four variables involved. Now, Ricci collineations after subtracting the Killing vector fields are

$$X = (0, 0, 0, \xi(x^\alpha)). \tag{3.15}$$

Clearly, it forms an infinite dimensional vector space.

4. MATTER COLLINEATIONS

If we set the cosmological constant and the coupling constant equal to zero and unity respectively. Then from the Einstein field equations, the expression for energy momentum tensor reduced to be

$$T_{ab} = R_{ab} - \frac{1}{2}Rg_{ab}, \tag{3.16}$$

where R_{ab} is Ricci tensor and R is Ricci scalar defined by $R=g^{ad}R_{ad}$ .Its value for the above spacetime is zero. Therefore, equation (3.16) yields

$$T_{ab} = R_{ab},$$

which implies that

$$L_X R_{ab} = L_X T_{ab}. \tag{3.17}$$

From the information given in (3.17), one can conclude that matter collineations coincide with Ricci collineations.

5. CONCLUSION

From the above discussion we come to the following conclusions:

(i) The spacetime under consideration admits proper curvature collineations which form an infinite dimensional vector space as shown in equation (2.15).
(ii) When the Ricci collineations of the above space time has been discussed, it is found that Ricci collineations also form an infinite dimensional vector space as determined by the equation (3.15)
(iii) For Matter collineations, we proved algebraically that matter collineations coincide with the Ricci collineations as discussed in section 4.

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