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Power Optimization of Wind Turbines by the Adjoint Method

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Abstract: This study aims to simulate the position of wind farms such as to find those positions of the wind turbines that maximize the power production and minimize the cost of operation and maintenance, keeping in view the constraints of space (available land) and the minimum distance between adjoining turbines.

Keywords: Wind Turbine Micrositing, Adjoint Method, Finite Element Method (FEM), N.O. Jensen model, Firefly Algorithm (FFA).

1. INTRODUCTION

This paper follows the approach of (Funke and Team, 2014) (Ryanking, 2015) It also denotes the specific mathematical models for optimization which is referenced as per the turbine locations. The constraints of this optimization problem are discussed at length by the use of the Finite Element Method (FEM).

The optimization of wind farms is critical for getting the maximum power output from a limited space with the least cost. This present study presents several options for the wind farm layout optimization for reducing the losses due to the wind wakes. The wind farms in question can be either on-shore or offshore.

It is well documented that the wake of a wind turbine affects the power production of downstream turbines. It is also known that the wake effects have detrimental effects on the rotor operation and also decrease the life expectancy of moving parts of the turbines (Wang. *et al.,* 2014)

It is also common knowledge that there is a finite limit on the number of turbines that can be placed in a wind farm. If a certain number is reached; adding more turbines is no longer feasible nor cost effective. Hence, the problem of designing a wind farm with the optimal number of turbines is essential (Wang. *et al.,* 2014)

The present research findings suggest that placing wind turbines in open areas is a complex problem of optimization which cannot be solved by deterministic methods. No matter how simple the terrain, the solution set will always be non-linear because of the losses due to the wakes and the non-linear nature of the typical energy captured. Thus the wind farm developer is faced with finite and real constraints of spacing and size (Lückehe *et al.,* 2015)

The Finite Element Method is the best suited method for elegant solutions to the Wind Turbine Optimization problem. The layouts presented show the uniqueness and the global acceptability of the candidate solution as the best solution for the problems presented. The proposed output consists of not more than 64 cascaded turbines to show the effectiveness of the method. The computational costs of the simulation are higher with higher number of turbines used and by the use of greater area under question.

The problem of wind turbine micrositing is fundamental to the success of an Energy project. The main factors for wind farm layout being,

1) Landscape and terrain of the site
2) Patterns of Wind flow:
3) Cost of investment
4) Cost of maintenance
5) Optimal control system design
6) Location of each individual turbine
7) Specifications of the selected turbines
8) Total number of turbines to be installed
9) Total number of turbines to be installed

This study undertakes a rational view at achieving the design optimization by providing a comprehensive answer to the above questions.

This study is particularly based at achieving the design optimization for the location as well as maximizing the total number of turbines that may be installed in each area. The Open Tidal Farm software was for this study (Funke and Team, 2014) (Ryanking, 2015) (Funke, *et al.,* 2014) (Culley *et al.,* 2016). The first and foremost questions to be asked before designing the wind farm is the budget for the wind farm. This dictates the size of the farm completely. Once the budget and the relevant financing is decided then comes the decision of the site location and the type of turbines to be installed. Our intervention plans to work effectively at this stage of time in the decision process.

Once given inputs on the basic parameters of site design the FEM optimization algorithm can be run to

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give the best location of the individual turbines to maximize the AEP (Annual Energy Production) and Minimize the OEM (Operation and Equipment Maintenance) costs.

The latest publications in this area have demonstrated the use of novel techniques that are being developed and tested in this field of research. One such technique is the Finite Element Method as referenced in (Culley *et al.,* 2016), Funke *et al.,* 2014), (Funke and Team, 2014). The use of optimization algorithms is of prime importance and they offer unique advantages. They can decrease the search space by a factor of at least ten times in some cases and they can also employ design constraints at the very beginning to efficiently carry out the search strategy (Culley *et al.,* 2016)

2. MATERIALS AND METHODS

2.1 Methodology

The method used in this work is of quasi-experimental type in which an optimization is run over very high number of iterations to reach at the best global solution. The tools as well as the software used in this research are both well-known and tested. It is also worthy to mention that the work of other researchers is well documented and known in this field of science.

The work examines a particular method for the solution of a well-known and documented problem of wind turbine micro-siting. The application of the FEM is relatively new to in this field of research.

The research work to be undertaken as part of this work is of a quantitative and empirical nature. It is proposed to view the layout of a theoretical wind farm consisting of a maximum of 64 turbines and the results of the power output and the cost are documented. The interaction of the wake on the wind speed and its overall effect on the wind turbines is also discussed (Samorani, *et al.,* 2013).

The findings presented in this work are systematic as well as based on solid theory of mathematics. The results are factored around the language of nature; mathematics. The measurements to be undertaken are based on this solid groundwork laid down by a well-documented mathematical model.

3. PROBLEM FORMULATION

3.1 Optimization

The main goal of optimization is to find out the overall return of investment of the array aka R_{fin} for a number, say n, of wind turbines in a specified area. The domain of the problem is a two dimensional Cartesian space and the encoding of the turbines are done in an area of n^2 digits. Where the vector m represents (Culley *et al.,* 2016),

$$m = (x_1, y_1, x_2, y_2, ..., x_n, y_n)^T \quad (1)$$

Thus the function R_{fin} can be maximized as follows,

$$\max_m R_{fin}(m) \quad (2)$$

Hence, incorporating all the cost functions together to formulate the complete cost function as follows,

$$\max_m I_p(P(m)) - C_C(L(m)) \quad (3)$$

Where

$P(m)$ is the power extracted from the turbines,

$I_p : \Re \longrightarrow \Re$ is the relationship between the power produced and the income from the farm

$L(m)$ is the cost of cabling and infrastructure

$C_c : \Re \longrightarrow \Re$ is the relationship between the cost of infrastructure and the financial cost

It is given that the functions P as well as L are defined as functions of m as they are both interlinked

3.2 Application to OpenWindFarm –

The maximization of the power production has been described and adapted for use by (Ryanking, *et al.,* 2015).

Hence, we may write the optimization problem as,

$$\max_m P(m) \quad subject \quad to \; b_1 \leq \underset{g(m)\leq 0}{m} \leq b_u \quad (4)$$

The bounds of the area are defined from b_l to bu and the minimum distance between the turbines is defined by the value of $g_m \leq 0$ (Culley *et al.,* 2016).

Hence, the total power produced by the turbines at every iteration is given by the equation (Culley, et al., 2016)

$$P(m) = \int_\Omega \rho C_t(m) \|u\|^3 dx \quad (5)$$

Where, the value of ρ gives the fluid density, the value of $C_t(m)$ gives the enhanced friction of the turbines and $\|u\|^3$ is the cube of the wind speed and η is the free surface displacement (Culley, et al., 2016).

Hence, we may write a general solution of the form (Culley, et al., 2016),

$$u\nabla u - v\nabla^2 u + g\nabla n + \frac{c_b + c_t(m)}{H}\|u\|u = 0 \quad (6)$$

Where,

$$\nabla.(Hu) = 0$$

Where,

v = viscosity coefficient

g = acceleration due to gravity

H = Height of the turbine

c_b = Constant background bottom friction

In the case of air flow coming unidirectionally the solution of the above problem is quite trivial and the optimal positioning can easily be generated. The air flow patterns are resolved using the Finite Element Method by using triangular meshes by the Taylor-Hood element pair (Taylor, *et al.,* 1973)

Furthermore the value of dP/dm can be calculated and hence the value of the gradient dP/dm the next iteration improves the value of m such that an optimized solution is reached.

The optimization algorithm used is packaged in the Ubuntu-Python software of SciPy and is a widely known as Sequential Quadratic Programming SLSQP

The adjoint equations enable the resolution of the Power produced with respect to their position by the evaluation of the gradient of dP/dm. This also enables the use of the flow model to determine the best positions for the turbines.

This model is much more realistic, complex as well as computationally feasible and suited to be run on a cloud environment for evaluation of real world scenarios

4. EXPERIMENT

4.1 Mesh Size -

basin_x (*float*)– The width of the Site = 640 m
basin_y (*float*)– The width of the Site = 640 m

4.2 Parameters -

turbine type – Bump Turbine (2.9 MW)
diameter (float) – the diameter of the turbine = 20 meters
min_dist (*float*) – the minimum distance between turbines = 20 meters
num_x (*int*) – the number of turbines in the x direction = 8 turbines
num_y (*int*) – the number of turbines in the y direction = 8 turbines
x_start (*float*)–The minimum x-coordinate of the site=160 m
x_end (*float*)–The minimum x-coordinate of the site = 480 m
y_start (*float*)–The minimum y-coordinate of the site=160 m
y_end (*float*) –The minimum y-coordinate of the site= 480 m

4.3 Data:

#	Description	Value
1)	Current Function Value	-75572.3079142
2)	Power produced	292.8808 MW
3)	Iterations	199
4)	Function evaluations	351
5)	Gradient evaluations	195

4.4 Graphical Layout and Discussion-

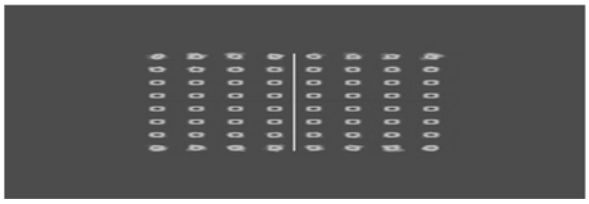


Fig. 1. Frame 0 with all turbines in rectangular array

This is the largest simulation with the size of the turbines as 20 meters and 8 x 8 = 64 turbines are placed in a tight array with minimum distance constraint between the turbines as 20 m.

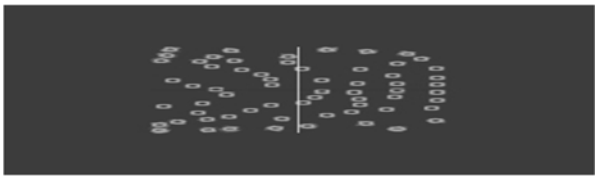


Fig. 2 – Frame 10 with all turbines in transition

The convergence of the turbines begins almost immediately as the turbines start their migration very early in the iterative process. The size of the site is 320 m x 320 m.



Fig. 3 – Frame 20 with all turbines in transition

It is seen very early on that the frontal positions are taken very early and the turbines take formations quickly in tight formations of several closely located turbines. It is reminded that the minimum distance of the turbines is 20 meters.



Fig. 4 – Frame 30 with all turbines in transition

The frontal array is taken up very early then follow the other arrays one after the other. The turbines take up positions in lines one behind the other.



Fig. 5 – Frame 40 with all turbines in transition

The final position is very near the 40th iteration where the final turbine positions can be deduced easily.

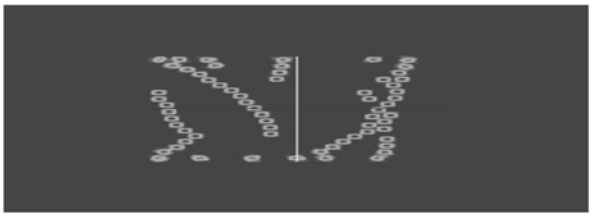


Fig. 6 – Frame 107 with all turbines in transition

Hence, the culmination of the simulation yields three rows of turbines one behind the other maximizing the power output of the turbines.

5. RESULTS AND DISCUSSION

The findings of the Adjoint method overcame the following shortcomings of the earlier publications as follows,

1) Most early studies could not accommodate change of wind speeds. The adjoint method allows for dynamic assignment of wind data to the simulation.
2) Other studies were limited to square grids only. The Adjoint Method allows for a map based data to be used.
3) Most studies could not accommodate uneven terrains and there was no solution for overcoming the real life scenarios. The wind vortices and the complex interactions are better handled by the Adjoint Method. Hence, the real life scenarios are handled in a much better way.
4) In earlier studies, the number of wind turbines was staggered to obtain the maximum number of possible turbines to be installed. Which means that the iterations increased the turbines one by one to reach at the maximum number of wind turbines that can be installed in a wind farm. However, the number of turbines are proposed by the operator before the beginning of the feasibility study by the Adjoint method.
5) Some studies have also presented matrix shaped grids as solutions with even spacing of the turbines which is ergonomically incorrect considering the wind speed changes. The Adjoint method lets nature shape the turbine layout keeping in view the hills, crevices and the local vortices of the terrain.
6) Some earlier studies have left out the option of relocation or migration of the turbines in altered wind conditions. This is an advanced concept, and the Adjoint Method allows relocations of the turbines in altered wind conditions. This can be accomplished in simulation prior to the installation of the turbines.

In conclusion, it said that the Adjoint Method/the Finite Element Method is a big step for the solution of the WTO problem and for improving the wind farm performance. The results are promising and convincing.

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