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An Identity Related to Generalized Derivations on Semiprime Γ-Rings

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Abstract. Let M be a 2-torsion free semiprime Γ -ring and let $F : M \to M$ be an additive mapping with associated derivation D satisfying $x\alpha y\beta z = x\beta y\alpha z$. If $F(x\alpha y\beta x) = F(x)\alpha y\beta x + x\alpha D(y\beta x)$, for all $x, y \in M$, $\alpha, \beta \in \Gamma$. Then F is a Jordan generalized derivation

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1. INTRODUCTION

Nobusawa (Sapanci, and Nakajima, (1997), introduced the notion of a Γ-ring, a notion more general than a ring. (Barnes, 2012) slightly weakened the conditions in the definition of a Γ-ring given by Nobusawa. After the study of work on Γ-ring by Nobusawa (Sapanci, and Nakajima, (1997), and Barnes (Barnes, 2012) many re- searchers have done a lot of work on Γ-rings and have obtained some generalizations of the corresponding results in ring theory (Barnes, 2012) (Khan, *et al.,* 1966), (Khan, *et al.,* 2014), (Khan, *et al.,* 2013), references therein).

If M and Γ are additive abelian groups and there exists a mapping (.,.,.,.):

$M \times \Gamma \times M \to M$ which satisfies the following conditions:

(i) (α, β, b) is an element of M,

(ii) $(a+b)\alpha c = a\alpha c$, $a(\alpha+\beta)b = a\alpha b + a\beta b$, $a\alpha(b+c) = a\alpha(b\beta c)$, for $a, b, c \in M$, $\alpha, \beta \in \Gamma$,

(iii) $(a\alpha b)\beta c = a\alpha(b\beta c)$, for $a, b, c \in M$, $\alpha, \beta \in \Gamma$,

then M is called a Γ-ring (Barnes, 2012).

$0\alpha b = a0b = a\alpha 0 = 0$, *for all* $a, b \in M$, $\alpha \in \Gamma$ [1].

Let M be a gamma ring. Then M is called a prime gamma ring if u Γ M Γ v = 0, then u = 0 or v = 0 and M is called semi-prime Γ -ring if u Γ M Γ u = 0, then u = 0, u, v ∈ M. A Γ -ring M is said to be 2-torsion free if $2x = 0$ implies $x = 0$, for all $x \in M$.

The notions of derivation and Jordan derivation on a Γ -rings were defined by Sapanci and Nakajima Later on many researchers have contributed a lot by using these notions (Nobusawa, 1964), and references therein). An additive mapping D: $M \to M$ is called a derivation on M if $D(x\gamma y) = D(x)\gamma y + x\gamma D(y)$ for all $x, y \in M$ and $\gamma \in \Gamma$. An additive mapping F: $M \to M$ is called a Jordan generalized Derivation if there exists a derivation D on M such that $F(x\gamma y) = F(x)\gamma x + x\gamma D(x)$ for all $x \in M$ and $\gamma \in \Gamma$.

2. RESULTS

Theorem 2.1. *Let M be a 2-torsion free semiprime Γ- ring and let $F : M \to M$ be an additive mapping with associated derivation D satisfying $x\alpha y\beta z = x\beta y\alpha z$. If $F(x\alpha y\beta x) = F(x)ay\beta x + xaD(y\beta x)$, for all $x, y \in M$, $a, \beta \in \Gamma$. Then F is a Jordan generalized derivation.*

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(1) $F(x\alpha y\beta x) = F(x)\alpha y\beta x + x\alpha D(y\beta x),\ x, y \in M,\ \alpha, \beta \in \Gamma.$

Linearizing (2), we get $F(x\alpha y\beta x + x\alpha y\beta z + z\alpha y\beta z) = F(x)\alpha y\beta x + F(x)\alpha y\beta z + F(z)\alpha y\beta x + F(z)\alpha y\beta z + x\alpha D(y\beta x) + x\alpha D(y\beta z) + z\alpha D(y\beta x) + z\alpha D(y\beta z)$. That is, $F(x\alpha y\beta x)F(x\alpha y\beta z + x\alpha y\beta z) + F(z\alpha y\beta z) = F(x)\alpha y\beta x + x\alpha D(y\beta x) + F(z)\alpha y\beta(z) + z\alpha D(y\beta z) + F(x)\alpha y\beta z + x\alpha D(y\beta z) + F(z\alpha y\beta x) + z\alpha D(y\beta x)$. The last relation along-with (1) implies,

(2) $F(x\alpha y\beta z + z\alpha y\beta x) = F(x)\alpha y\beta z + x\alpha D(y\beta z) + F(z)\alpha y\beta x + z\alpha D(y\beta x).$

Replacing z by $x\gamma x$ in (3), we get

4. $F(x\alpha y\beta x\gamma x + x\gamma x\alpha y\beta x) = F(x)\alpha y\beta x\gamma x + x\alpha D(y\beta x\gamma x) + F(x\gamma x)\alpha y\beta x + x\gamma x\alpha D(y\beta x).$

Substituting $x\gamma y + y\gamma x$ for y in the equation (2), we obtain $F(x\alpha(x\gamma y + y\gamma x)\beta x) = F(x)\alpha(x\gamma y + y\gamma x)\beta x + x\alpha D((x\gamma y + y\gamma x)\beta x),\ x, y \in M,\ \alpha, \beta, \gamma \in \Gamma.$ That is, (5)

$F(x\gamma x\alpha y\beta x + x\alpha y\beta x\gamma x) =$

$F(x)\alpha x\gamma y\beta x + F(x)\alpha y\gamma x\beta x + x\alpha D(x\gamma y\beta x) + x\alpha D(y\gamma x\beta x).$

From (4) and (5), we have

(6) $A_\gamma(x)\alpha y\beta x = 0,\ x, y \in M,\ \alpha, \beta, \gamma \in \Gamma.$

Where $A_\gamma(x) = F(x\gamma x) - F(x)\gamma x - x\gamma D(x)$. We intend to prove that $A_\gamma(x) = 0, x \in M, \gamma \in \Gamma$. In relation (6) if we replace y by $x\beta y\alpha A_\gamma(x)$ then we get, $A_\gamma(x)\alpha x\beta y\gamma A_\gamma(x)\alpha x = 0,$ for all $x, y \in M, \alpha, \beta, \gamma \in \Gamma$ and so, by the semiprimeness of M, we have

(7) $A_\gamma(x)\alpha x = 0, \qquad x \in M, \gamma \in \Gamma.$

The relation (Sapanci, and Nakajima, 1997), alongwith (1), we obtain $x\alpha A_\gamma(x)\beta y\beta\alpha A_\gamma(x) = 0,$ for all $x, y \in M, \alpha, \beta, \gamma \in \Gamma$. By the semiprimeness of M it follows that

(8) $x\alpha A_\gamma(x) = 0,\ x \in M, \gamma \in \Gamma.$

Linearizing equation (7) gives

(9) $A_\gamma(x)\gamma y + \delta_\alpha(x, y)\alpha x + A_\gamma(y)\alpha x + \delta_\alpha(x, y)\alpha y = 0,\ x, y \in M, \alpha, \gamma \in \Gamma,$

where

$\delta_\alpha(x, y) = F(x\alpha y + y\alpha x) - F(x)\alpha y - F(y)\alpha x - x\alpha D(y) - y\alpha D(x).$

Replacing in (9) x by $-x$, we get

(10) $A_\gamma(x)\gamma y + \delta_\alpha(x, y)\beta x - A_\gamma(y)\beta x - \delta_\alpha(x, y)\beta y = 0.$

Adding (9) and (10) along with 2-torsionfreeness of M, we obtain $A_\gamma(x)\alpha y + \delta_\alpha(x, y)\beta x = 0,\ x, y \in M, \alpha, \beta, \gamma \in \Gamma$. The last equation along with (1) and (8), we get $A_\gamma(x)\beta y\gamma A_\gamma(x) = 0,$ for all $x, y \in M, \beta, \gamma \in \Gamma$. Since M is semiprime, we get $A_\gamma(x) = 0,$ for all $x \in M, \gamma \in \Gamma$. We have therefore proved that $F(x\gamma x) = F(x)\gamma x + x\gamma D(x)$ holds for all $x \in M,\ \gamma \in \Gamma$. In other words, F is a Jordan generalized derivation.

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