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On the Development of A New Multi-Step Derivative Free Method to Accelerate the Convergence of Bracketing Methods for Solving $f(x) = 0$

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Abstract: Finding root of non-linear equation $f(x) = 0$ is a classical problem in numerical analysis which arise in many scientific and engineering fields. In this paper the main focus is given to develop an Algorithm which speed up the convergence of bracketing methods for finding root of a non-linear equation. For this purpose, combination of newton's method and truncated Taylor's series is employed. The new method is derivative free, require three preliminary approximations to start and does not have any pitfall. Some numerical examples are also presented in this paper in order to analyze the efficiency of the developed method with other methods. It is observed from the results and comparison of new method with the famous bracketing methods including Bisection method and Regula-Falsi method that the new developed method is performing better than these two methods.

Keyword s: non-linear equation, bracketing methods, No. of iterations, convergence, relative percentile error.

1. INTRODUCTION

This world is full of such physical phenomena that can be modeled mathematically, finding solutions to those physical problems is worth knowing. It is often the case while finding out solutions to the physical world problems by applying mathematics(in the form of an equation of set of equations), that the exact formulas or analytical schemes are not capable of handling complexity of such problems, there numerical techniques are employed to get the solutions. (Grady 2004). Finding root of non-linear equation $f(x) = 0$ is a classical problem in numerical analysis which arise in many scientific and engineering fields (Nusrat *et al*., 2012).

Various numerical methods have been developed using different techniques including finite differences (Nusrat *et al*., 2012), (Kincaid and. Cheney, 1996), (Cordero *et al*., 2010), quadrature rules (Ujevic, 2006), (Anup *et al*., 2012), (Mario *et al*., 2014),Taylor's series, decomposition methods, homotopy techniques, Newton theorem (IDE Nasr-Al-Din, 2013), (Kaleel, *et al*., 2011) etc., in order to carry out the solution of non-linear equations with different convergence rates.

$$f(x) = f(x_n) + \int_{x_n}^{x} f'(t)\,dt$$

Most commonly used numerical methods for root location of non-linear equations includes, Bisection/interval halving method, Regula-falsi/false position method, (Bracketing Methods i.e. the methods which require two initial guesses $a\&b$ such that $x \in [a,b]$ for $f(a)f(b) < 0$) fixed point iteration method, Newton Raphson method (Open Methods i.e. methods that require only single initial guess) etc.There are some numerical techniques that are sure to converge but too slow while approaching to the root of non-linear equations. On the other hand, there are some faster converging numerical techniques but are not reliable i.e. having a kind of pitfall. Other methods involve complex functional evaluations with their derivatives of first or higher order. (Amit *et al*., 2013).

In numerical analysis, to speed up the rate of convergence has always been a vigorous field of research. While solving some practical problems, very often the selected algorithm converges very slowly to the required root. In such situations we have the choice that either we switch to another quicker converging method or we accelerate the first one (Nedzhibov, 2004).

In recent years, research has been carried out in order to increase the convergence rate of Bisection method by introducing some new formulations using different techniques. (Solanki, 2014), (Tanakan, 2013). (Sangah *et al*., 2016) developed a new bracketing algorithm to reduce the no. of iterations in contrast with the well-known bracketing methods, Bisection method and Regula-Falsi method. (Noor, 2006) combined the well-known Regula-Falsi and Newton's method to give rise to two new methods for solving non-linear equations with better accuracy. Many derivative free techniques have also been developed that are simpler and easier to use and that are free from any pitfall. (Nusrat *et al*., 2012), (Soleymani and Hosseinabadi,

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2011), (Ahmad, 2012), (Cordero *et al*., 2010), (Kumar *et. al*., 2015).

Proposed Method

The new method is developed by using,

1.Taylor's series:

$$f(x_n+h)=f(x_n)+hf'(x_n)+\frac{h^2}{2!}f''(x_n)+\frac{h^3}{3!}f'''(x_n)+\ldots$$

Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Truncating the Taylor's series up to two terms,

i.e. $f(x_n + h) = f(x_n) + hf'(x_n)$

By taking h equals to $x_{n-1} - x_n$ and $x_{n-2} - x_n$ in the truncated Taylor's series and simplifying, we get,

$$f'(x_n) = \frac{f(x_{n-1}) - f(x_{n-2})}{x_{n-1} - x_{n-2}}$$

Using this approximation of $f'(x_n)$ in the Newton's method, the new method is formulated as,

$$x_{n+1} = x_n - \frac{x_{n-1} - x_{n-2}}{f(x_{n-1}) - f(x_{n-2})} f(x_n) \quad \text{----1)}$$

Where, x_n, x_{n-1}, and x_{n-2} are first three approximations, taken from the bracketing root location methods for non-linear equation (i.e. bisection method or regula-falsi method). For instance, we approximate x_1, x_2, x_3 , the first three approximations of the root x of the equation $f(x) = 0$ from the bracketing method. Then the first iteration or first approximation of the root

$$x_4 = x_3 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_3)$$

x of the newmethod is evaluated by the following formula,

And the further iterations are carried out by taking the most recent approximated values of root x on the R.H.S of the formula (1).

2. RESULTS AND DISCUSSION

In order to analyze the new developed method (N.D.M), the method is applied on few examples including algebraic and transcendental functions and compared with the Bisection method (B.M) and Regula-Falsi method (R.F.M) (see table 1). We observe from problem 1 and 3 that B.M and R.F.M are respectively taking 18 and 20 iterations in problem 1 and 17 and 11 iterations in problem 3 while, the new developed method taking only 5 and 4 iterations only. So in the case of problem 1 and 3 the new developed method is performing far better than the other two methods w.r.t no. of iterations and accuracy of functional value f(x)=2.×10^{-6}and R.E%=2×10^{-4} in problem 1 **(Fig.1)** and f(x)=-5×10^{-6} in problem 3. In the case of problem 2, 4 and 5 B.M is taking 20, 15 and 17 iterations. In these cases, the R.F.M is taking fewer iterations i.e. 6, 4 and 3 in the problem 2, 4 and 5 respectively but the new developed method takes only 3, 1 and 1 iterations respectively in these problems giving accuracy of functional value f(x)=2×10^{-7}and R.E%=5×10^{-3} in problem 2, f(x)=-7×10^{-8}and R.E%=7×10^{-3}in problem 4 and f(x)=5×10^{-8} and R.E%=5.12×10^{-4} in problem 5 **(Fig.2)**. In figure 1 and figure 2 the graph of N.D.M is below the graph of B.M and R.F.M which clearly shows that the N.D.M not only takes fewer iterations but also gives lesser R.E% than other two methods.

Table-1

Function	Methods	No. of Iterations	x	f(x)	R.E%
$f(x) = x^5 - 8x^4 + 44x^3 - 91x^2 + 85x - 26$	B.M	18	0.557026	0.000008	0.000685
	R.F.M	20	0.557038	0.000235	0.001562
	New method	5	0.557025	0.000002	0.000214
$f(x) = 10xe^{-x^2} - 1$	B.M	20	0.101027	0.000007	0.001888
	R.F.M	6	0.101026	0.000001	0.004373
	New method	3	0.101026	0.0000002	0.005450
$f(x) = cosx - xe^x$	B.M	17	0.517754	0.000011	0.001474
	R.F.M	11	0.517754	0.000009	0.001243
	New method	4	0.517759	-0.000005	0.036908
$f(x) = xlogx - 1.2$	B.M	15	2.740631	-0.000013	0.0100
	R.F.M	4	2.740646	-0.0000001	0.007059
	New method	1	2.740646	-0.00000007	0.007383
$f(x) = sin\sqrt{x} + \cos(1 + \sqrt{x})$	B.M	13	3.445435	0.000004	0.003543
	R.F.M	3	3.445475	-0.000003	0.015763
	New method	1	3.445457	0.00000005	0.000512

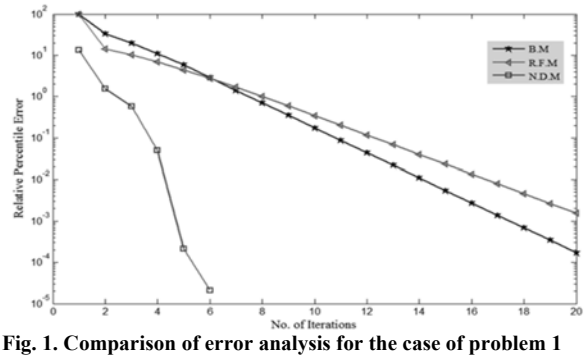


Fig. 1. Comparison of error analysis for the case of problem 1

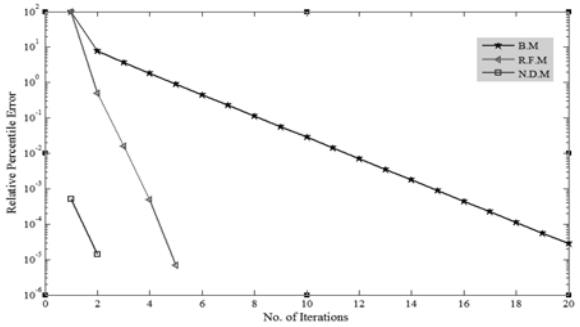


Fig.2. Comparison of error analysis for the case of problem 5

3. CONCLUSION

The new developed method is a multi-step derivative free method. It has been observed that solving a non-linear equation $f(x)=0$ by new developed method, the no. of iterations is reduced with better accuracy in contrast with bracketing methods including bisection method and regula-falsi method. Hence it is concluded that the new developed method is performing better than the mentioned bracketing methods.

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