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Analytical and Numerical Solutions of non-Newtonian fluid flow without porous media filled in channels

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Abstract: The goal of study to find the solutions of partial differential equation governing the motion of a problem related with non-Newtonian fluid flow without porous media in channels are searched numerical way and study state solution is investigated. The problem of non-Newtonian fluids flow is solved by Lie-group method and shooting method. The generator of the symmetry group is employed and the PDE decreases to an ODE with the proper conditions. ODE have been solved by shooting method, and obtained results of the problem.

Keywords: Non-Newtonian fluid Flow, Darcy-Brinkman model, Lie group method, Shooting method, Results.

1. INTRODUCTION

One of the most interesting and attractive subjects of science is fluid mechanics. Fluid participates a very fundamental role in many features of our life. In developing the equation of momentum that manages the fluid flow in channels or pipes through porous media, major development has been completed by Darcy–Brinkman model with the Darcy law. The results of PDE's leading Newtonian or non-Newtonian fluids which are the compressible or incompressible fluids are sympathetically concerned important to concentration in the literature. The partial differential equations concerning the Newtonian and non-Newtonian fluids flow contain the basic rules of continuum fluid mechanics and constitutive equation or the condition of Newtonian or non-Newtonian equation. The some imperative investigation of Newtonian and non-Newtonian fluids Flows associated with (Abel-Malek *et al.,* 2002), (Ariel *et al.,* 2006), (Chen *et al.,* 2006), (Bird *et al.,* 1981, 1983, 1987), (Fetecau and Fetecau 2005. 2006), (Rajagopal and Gupta 1984), (Rajagopal and Na 1985), and (Wafo-2005).

Lie group analysis related with the transformation groups is very significant solution technique for finding thr solution of the problems of differential equations and operators of Lie group can be established the method to simplify the problems. The partial differential equations, is solved by Lie group method and this group is generally related in different fields of mathematics, fluid mechanics and engineering and physical science. This method is used for solving the problem, in which the generator of the symmetry group decreases the number of independent variables, and founded by Birkhof (1948).. The Lie group method have developed by (Olver, 1986), (Ibragimov, 1999), Bluman and Kumei (1989), and others researchers. In this work, the problem is to find and use acknowledged symmetries algebra and operators of these algebra are used to reduce the governing PDE to solvable form of an ODE with suitable conditions. Using the shooting method the ordinary differential equation then solved analytically or numerically to compute approximated velocity value.

In this research, the Darcy Brinkman model is accepted to explain fluid flow of the non- Newtonian hydrodynamic actions. This fluid is assumed to follow the power law consideration.

In this research work, the problem is formulated in section 2.. Section 2.1 connected with solution of non-Newtonian fluid flow through channels without porous media, Section 3 deals with the symmetry analysis. This is followed by analytical and numerical approach in section 4 synthesises the results obtained.

Section 3.1 connected with solution of non-Newtonian fluid flow through channels without porous media, section 3.1.1 and 3.1.2 related with Lie point symmetries and transformation of equation (5). Section 4 deals with results obtained in other consideration, section 4.1 consists of invariant solutions corresponding to X_5 or study-state solution Section 4.2 related with invariant solution corresponding to $X = m\,X_1 + (m+1)\,X_3$ by shooting method. As section 5 associated with conclusions of the problems.

2. PROBLEM FORMULATIONS

The problem investigated is non-Newtonian laminar forced fluid flow. Such flows are an unsteady

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through porous media in a channel. The unsteadiness fluid flow has been driven due to a suddenly imposed pressure gradient and the pressure gradient is assumed to be constant. Consider a non-Newtonian fluid flow follow the power law with uniform properties and the porous medium is considered as a homogeneous and isotropic. When fluid flow is supposed to be hydro dynamically completely expanded, then velocity of fluid flow does not depend on the axial direction of the channel. The flow is stated in terms of the unaided axial velocity as a result of the continuity equation and it a unidirectional one. For inelastic generalised non-Newtonian fluid in one-dimension the governing equation is:

$$\rho \frac{\partial v}{\partial t} = \frac{\partial}{\partial y}[\mu_0 (\frac{\dot{\gamma}}{\varepsilon})^{m-1} \frac{\partial v}{\partial y}] - \varepsilon \frac{\partial p}{\partial x} - \frac{\varepsilon \mu_0}{k} v^m \quad (1)$$

Where v and y are the velocity component in axial coordinates and the transversal coordinates, p represents the isotropic liquid pressure and t is the time. As fluid parameters ρ is density, acceleration coefficient μ_0 is denoted for the zero shear viscosity and it is constant, $\dot{\gamma} = \left|\frac{\partial v}{\partial y}\right|$ is shear rate, the power law index is denoted by m, the intrinsic permeability of the porous medium, is defined by k is. In the equation (1), the co-efficient of acceleration tensor is supposed to be $1/\varepsilon$ and the porosity of the porous medium is identified with ∂ *and* is taken as constant.

Employing the dimensionless parameters $v^* v_c = v$, $y^* L = y$ and $t^* t_c = t$, where non-dimensional velocity is taken as v^* and dimensionless transversal direction is identified with y^* respectively. As k^* is denoted for dimensionless permeability of the porous media and s the-dimensionless time represents by t^*. As L represents the characteristic length occupied as half width in the channel, the feature velocity is employed for v_c and is supposed as reference axial velocity $v_c = \sqrt[m]{(-\frac{dp}{dx})\varepsilon^m (\frac{L^{m+1}}{\mu_0})}$ and t_c is the attribute time assumed as reference time $t_c = \frac{\rho \varepsilon^{m-1} L^{m+1}}{\mu_0 v_c^{m-1}}$, then after reducing asterisk from given variables for brevity, as the dimensionless equation is given

$$\frac{\partial v}{\partial t} = 1 + \frac{\partial}{\partial y}[\left|\frac{\partial v}{\partial y}\right|^{m-1} \frac{\partial v}{\partial y}] - \frac{1}{Da} v^{m\,2}, \quad (2)$$

Where dimensionless Darcy's number D_a is defined as $Da = \frac{k^*}{\varepsilon^m L^{m+1}}$.

For equation (2), we define initial and boundary conditions in order to make problem well posed.

$$v(0, y) = 0, \quad \text{When} \quad -1 \le y \le 1 \quad (3)$$

$$v(t,-1) = v(t,1) = 0. \quad \text{Where } t > 0 \quad (4)$$

2.1 As $D_a \to \infty$ or it vanishes, so the equation (2) is called non-Newtonian fluid flow filled in channel without porous media and Equation (2) take the form as

$$\frac{\partial v}{\partial t} = 1 + \frac{\partial}{\partial y}[\left|\frac{\partial v}{\partial y}\right|^{m-1} \frac{\partial v}{\partial y}], \quad (5)$$

Related with same conditions (3) and (4)

3. Symmetries analysis and one- parameter group of transformation.

Lie group method is great method in finding the results of differential equations analytically. Once generator in Lie algebra of the differential equation is identified, this generator can be used in the search of transformations **so** that it will reduce the given equation to solvable form. Hence method is used for finding generators called the Lie point symmetries of the differential equations (5) are introduced.. Our method of solution depends on transformation group of one-parameter of to the PDE (5). Under this transformation, the independent variables will be reduced by one from two, and the PDE transform into ODE.

Method is used for discovering the Lie point symmetries of the partial differential equation (5) and symmetry conditions are introduced. Under the symmetries , then one parameter Lie point transformations is given by"

$$T(t,y,v,\varepsilon) = t^* = t + \varepsilon\varphi(t,y,v) + O(\varepsilon^2), \quad (6\text{-a}), \qquad Y(t,y,v,\varepsilon) = y^* = y + \varepsilon\xi(t,y,v) + O(\varepsilon^2) \quad (6\text{-b})$$

$$v^* = V(t,y,v,\varepsilon) = v + \varepsilon\eta(t,y,v) + O(\varepsilon^2) \quad (6\text{-c})$$

$$V_i(t,y,v,v_1,v_2,\varepsilon) = v_i^* = v_i + \varepsilon\xi_i^{(1)}(t,y,v,v_1,v_2) + O(\varepsilon^2) \quad (6\text{-d})$$

$$V_{ij}(t,y,v,v_1,v_2,v_{11},v_{12},v_{22},\varepsilon) = v_{ij}^* = v_{ij} + \varepsilon\xi_i^{(1)}(t,y,v,v_1,v_2,v_{11},v_{12},v_{22}) + O(\varepsilon^2) \quad (6\text{-e})$$

where i , j = 1, 2, ε is a small parameter, v_1 is differentiate with respect to t, v_2 is differentiate with respect to y etc and

$$\varphi(t,y,v)=\frac{dt^*}{d\varepsilon},\qquad \xi(t,y,v)=\frac{dy^*}{d\varepsilon},\qquad \eta(t,y,v)=\frac{dv^*}{d\varepsilon} \tag{7}$$

using the initial conditions (8), the system of ODE's (7) is solved

$$t^*\Big|_{\varepsilon=0}=t,\qquad y^*\Big|_{\varepsilon=0}=y,\qquad v^*\Big|_{\varepsilon=0}=v \tag{8}$$

The corresponding following infinitesimal generator is given by

$$X=\varphi(t,y,v)\frac{\partial}{\partial t}+\xi(t,y,v)\frac{\partial}{\partial y}+\eta(t,y,v)\frac{\partial}{\partial v}$$

In order to state the s generator of the PDE, the following second order prolongation of the infinitesimal generator is required, i.e.

$$X^{(2)}=\varphi(t,y,v)\frac{\partial}{\partial t}+\xi(t,y,v)\frac{\partial}{\partial y}+\eta(t,y,v)\frac{\partial}{\partial v}+\eta_i^{(1)}(t,y,v,v_i)\frac{\partial}{\partial v_i}+\eta_{ij}^{(2)}(t,y,v,v_i,v_{ij})\frac{\partial}{\partial v_{ij}},$$

In which the additional coefficient functions, $\eta_i^{(1)}(t,y,v,v_i)$ and $\eta_{ij}^{(2)}(t,y,v,v_i,v_{ij})$ satisfy as:

$$\eta_1^{[1]}=D_t(\eta)-v_t\,D_t(\phi)-v_y\,D_t(\xi),\qquad \eta_2^{[1]}=D_y(\eta)-v_t\,D_y(\phi)-v_y\,D_y(\xi),$$
$$\eta_{11}^{[2]}=D_t(\eta_1^{[1]})-v_{tt}\,D_t(\phi)-v_{ty}\,D_t(\xi)$$
$$\eta_{22}^{[2]}=D_y(\eta_2^{[1]})-v_{ty}\,D_y(\phi)-v_{yy}\,D_y(\xi) \tag{9}$$

With total derivative operators given by

$$D_t=\frac{\partial}{\partial t}+v_t\frac{\partial}{\partial v}+v_{tt}\frac{\partial}{\partial v_t}+v_{ty}\frac{\partial}{\partial v_y}+\dots,\qquad D_y=\frac{\partial}{\partial y}+v_y\frac{\partial}{\partial v}+v_{ty}\frac{\partial}{\partial v_t}+v_{yy}\frac{\partial}{\partial v_y}+\dots,$$

Where $\frac{\partial v}{\partial t}=v_t$, $\frac{\partial v}{\partial y}=v_y$, $\frac{\partial^2 v}{\partial t\partial y}=v_{ty}$ and $\frac{\partial^2 v}{\partial y^2}=v_{yy}$ etc.

Now, find the lie point symmetries of the given equation with the following consideration.

3.1 Solution of non-Newtonian fluid flow through channels without porous media

As Equation (5) can also be written in the following form

$$\frac{\partial v}{\partial t}=1+\frac{\partial}{\partial y}\left[\left|\frac{\partial v}{\partial y}\right|^{m-1}\frac{\partial v}{\partial y}\right]=1+\frac{\partial}{\partial y}\left[\left(\sqrt{\left(\frac{\partial v}{\partial y}\right)^2}\right)^{m-1}\frac{\partial v}{\partial y}\right]=1+\frac{\partial}{\partial y}\left[\left(\left(\frac{\partial v}{\partial y}\right)^2\right)^{\frac{m-1}{2}}\frac{\partial v}{\partial y}\right]$$

$$\Rightarrow\ \frac{\partial v}{\partial t}=1+m\left|\frac{\partial v}{\partial y}\right|^{m-1}\frac{\partial^2 v}{\partial y^2} \tag{10}$$

Subject to

$$v(0,y)=0\qquad -1\le y\le 1 \tag{11}$$
$$v(t,-1)=0,\ v(t,1)=0\qquad t>0 \tag{12}$$

3.1.1 Lie point symmetries and transformation

In (7), the functions φ, ξ and η are unknown and are independent of v and its derivatives. Then symmetry group of transformations is admitted by PDE (10) iff

$$X^{(2)}\left(1+m\left|\frac{\partial v}{\partial y}\right|^{m-1}\frac{\partial^2 v}{\partial y^2}-\frac{\partial v}{\partial t}\right)\Bigg|_{(10)}=0\quad\Rightarrow -\eta_1^{[1]}+m(m-1)\left|\frac{\partial v}{\partial y}\right|^{m-3}\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial y^2}\eta_2^{[1]}+m\left|\frac{\partial v}{\partial y}\right|^{m-1}\eta_{22}^{[2]}\Bigg|_{(10)}=0 \tag{13}$$

After substituting the values of $\eta_1^{[1]}$, $\eta_2^{[1]}$ and $\eta_{22}^{[2]}$ from equation (9) and substituting the value of u_{yy} into (13) from equation (10), then disconnect this equation with respect to powers of derivatives and after simplifying the following over resolute system of linear differential equations to obtain the result. System is

$$\frac{\partial\eta}{\partial y}=0,\ \frac{\partial\xi}{\partial t}=0,\ \frac{\partial\varphi}{\partial y}=0,\ \frac{\partial\xi}{\partial v}=0,\ \frac{\partial\varphi}{\partial v}=0,\ \frac{\partial^2\xi}{\partial y^2}=0,\ \frac{\partial^2\eta}{\partial v^2}=0,\ -\frac{\partial\eta}{\partial t}-m\frac{\partial\eta}{\partial v}+(m+1)\frac{\partial\xi}{\partial y}=0$$

and

$$\frac{\partial\varphi}{\partial t}+(m-1)\frac{\partial\eta}{\partial v}-(m+1)\frac{\partial\xi}{\partial y}=0 \tag{14}$$

Solve the (14) to obtain result in the following form:

$$\varphi=\{(m+1)c_1-(m-1)c_3\}t+c_5\ \text{(i)}\quad \xi=c_1y+c_2\ \text{(ii)}\quad \eta=c_3v+\{(m+1)c_1-mc_3\}t+c_4\ \text{(iii)} \tag{15}$$

From the integration of these equations, we obtain five arbitrary constants, so we have given five different symmetry generators. According to these generators, we have five-dimensional Lie algebra of equation (11) and known by the following operators; $X_1 = (m+1)t\frac{\partial}{\partial t} + y\frac{\partial}{\partial y} + (m+1)t\frac{\partial}{\partial v}$, $X_2 = \frac{\partial}{\partial y}$,

$$X_3 = (1-m)t\frac{\partial}{\partial t} + (v - mt)\frac{\partial}{\partial v},$$

$$X_4 = \frac{\partial}{\partial v} \quad \& \quad X_5 = \frac{\partial}{\partial t} \tag{16}$$

The commutators which are non- zero are given as under.

$[X_1, X_2] = -X_2$, $[X_1, X_5] = -(m+1)(X_4 + X_5)$, $[X_3, X_4] = -X_4$,

$[X_3, X_5] = m X_4 + (m+1) X_5$.

3.1.2 Transformations

The finite transformation is obtained by solving the equations (8) and (9) which is related to infinitesimal generator is given as

$$\varphi(t,y,v) = \frac{dt^*}{d\varepsilon} \qquad \xi(t,y,v) = \frac{dy^*}{d\varepsilon} \qquad \eta(t,y,v) = \frac{dv^*}{d\varepsilon}$$

Which are connected with initial conditions

$$t^*\big|_{\varepsilon=0} = t \quad y^*\big|_{\varepsilon=0} = y \qquad v^*\big|_{\varepsilon=0} = v$$

Calculations of these equations show the finite transformations equivalent to X_1 -to- X_5 respectively.

Here $\varepsilon_1 - to - \varepsilon_5$ are group parameters.

4. RESULTS

Invariant physical solutions: A result $v = G(t,y)$ of (5) is invariant under the opertor

$$X = \varphi(t,y,v)\frac{\partial}{\partial t} + \xi(t,y,v)\frac{\partial}{\partial y} + \eta(t,y,v)\frac{\partial}{\partial v} \text{ iff } X[v - F(t,y)]\big|_{v-F(t,y)} = 0 \quad \text{i.e.}$$

$$\varphi(t,y,v)\frac{\partial F}{\partial t} + \xi(t,y,v)\frac{\partial F}{\partial y} = \eta(t,y,v) \cdot \tag{17}$$

The solution of first-order PDE (17) gives the invariant solution form. This form is replacement into (11) to (13).

4.1. Invariant solutions corresponding to *X₅* or study-state solution

The solution connected with X_5 is agreed in the study-state solution function $v(t,y) = \psi(y)$ (18)

Substituting this function into partial differential equation (11) yields ODE for $\psi(y)$,

$$1 + m\,|\psi'(y)|^{m-1}\psi''(y) = 0 \tag{19}$$

Related with the conditions $\psi(-1) = 0 \; \psi(1) = 0$ (20)

Where prime appears for derivative with respect to y, as solution of ODEs (19) depending on the boundary conditions (20) for non-Newtonian fluid is solved analytically, and result under these statements is obtained as

$$\psi(y) = \frac{m}{m+1}(1 - |y|^{\frac{m+1}{m}})$$

Substituting the value of $\psi(y)$ into (18), the result is

$$v(t,y) = \psi(y) = \frac{m}{m+1}(1 - |y|^{\frac{m+1}{m}}) \tag{21}$$

4.2 Invariant solutions corresponding to $X = m X_1 + (m+1) X_3$

Hence $X = (m+1)\,t\frac{\partial}{\partial t} + m\,y\frac{\partial}{\partial y} + (m+1)v\frac{\partial}{\partial v}$

The invariant result or solution of equation (11) for the symmetry operator $X = mX_1 + (m+1)X_3$ yields

$$v(t,y) = t\varphi(\lambda), \qquad \text{Where } \lambda = y\, t^{\frac{-m}{(m+1)}} \tag{22}$$

The partial differential equation (10) can be transformed in the next form given as.

As $\frac{\partial v}{\partial t} = \varphi(\lambda) - \frac{m}{m+1}\lambda\ \varphi'(\lambda)$ and $\frac{\partial v}{\partial y} = \frac{t}{y}\ \lambda\varphi'(\lambda) \Rightarrow \varphi'(\lambda) = \frac{y}{t\ \lambda}\frac{\partial v}{\partial y}$ So $\frac{\partial v}{\partial t} = \frac{v}{t} - \frac{m\ y}{(m+1)\,t}\ \frac{\partial v}{\partial y}$ (23)

By putting this value of $\frac{\partial v}{\partial t}$, then equation (11) takes the form

$$1 + m\left|\frac{dv}{dy}\right|^{m-1}\frac{d^2v}{dy^2} + \frac{m\ y}{(m+1)\,t}\ \frac{dv}{dy} - \frac{v}{t} = 0 \text{ or } 1 + m\left|v'(y)\right|^{m-1} v''(y) + \frac{m\ y}{(m+1)\,t}\ v'(y) - \frac{v(y)}{t} = 0 \tag{24}$$

Subject to $v(-1) = 0$ *and* $v(1) = 0$ (25)

Since the equation (24) is highly nonlinear, a numerical treatment would be more appropriate. The transformed equations (24) linked with the conditions (25) are numerically resolved by means of Shooting techniques with a guessing of $v'(-1) = s$. The procedure is repeated until find the value of s when 2nd boundary condition is satisfied or its solutions are up to the aspiration degree of accuracy, namely 10^6 or its above, A code is written in MATHEMATICA box up and results are presented graphically. For numerical solution, the solution of equation (24) subject to boundary conditions (25) at $m = 0.5,\ m = 1.5$ and at different values of t is determined in MATHEMATICA using ND Solve by shooting method,

Plot of the numerical solution of equation (24) subject to (25) at $m = 0.5,\ m = 1.5$ and at different values of times. The result of numerical solutions by shooting method for channel velocity are displayed in figures–3 and 4 at $m = 0.5,\ m = 1.5$ respectively. Both figures illustrate that the velocity of channel enlarges as the time proceeds from rest and and reached at the value of an upper limit from transition to condition of steady. Steady state value at $m = 0.5$ is $v = 0.332$ and there further change is not available in the velocity. Similarly at $m = 1.5$, velocity expands at an increased for time values and there appears an upper limit for given increase and got to a greatest value of channel velocity when it arrives at the steady state whose highest value of velocity is equal to $v = 0.6$.

5. CONCLUSION

As Symmetry method Lie is used to find the solutions of the problems of PDE connected with conditions (initial and boundary) which are associated with the model of Darcy Brinkman to describe the hydrodynamic performance of flow of Non -Newtonian fluid with not related porous medium. In this paper, For the solutions of partial differential equation, we have found the symmetries. By using Lie group method and using the suitable symmetry, we reduce PDE in to ODE. We have been solved this ODE using proper conditions of boundary corresponding to the Lie-group, so that solve it numerically at $m = 0.5$ *and* $m = 1.5$ by shooting method. This method can provide several practical insights attention in comprise of results and may support for formative results in some cases.. The results are not only important in their own right but also provide support to the accuracy of analytical solution and vice versa.

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