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Some General Properties of Stein AG-groupoids and Stein AG-Test

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Abstract: A groupoid S that satisfies the left invertive law: $ab\cdot c=cb\cdot a$ is called an AG-groupoid. We extend this concept to introduce a Stein AG-groupoid. We prove the existence of this type of AG-groupoid by providing some non-associative examples. We also explore some basic and general properties of these AG-groupoids and find their relations with other known subclasses of AG-groupoids. We present a table of enumeration for these AG-groupoids up to order 6 and further categorize into associative and non-associative. We also develop a method to test an arbitrary AG-groupoid for this new class. Further, we also characterize these AG-groupoids by the properties of their ideals.

Keywords: Stein AG-Groupoid; Stein-Groupoid; Stein AG-Test; Locally as sociativeag- Groupoid; Ideals; Nuclear Square AG-Groupoid.

1. INTRODUCTION

An AG-groupoid S is a non-associative algebraic structure that generalizes the commutative semi-groupin general, and satisfies the left invertive law: $ab\cdot c=cb\cdot a$. One can easily verify that every AG-groupoid satisfies the medial property: $ab\cdot cd=ac\cdot bd \quad \forall\ a,b,c,d\in S$. A groupoid S is called a Stein groupoid if it satisfies the Stein identity (Pelling *et al.,* 1980, Denes *et al.,* 1974): $a\cdot bc=bc\cdot a \quad \forall\ a,b,c\in S$. We adjoin this property to AG-groupoid and introduce a Stein AG-groupoid. AG-groupoids have been enumerated (Distler *et al.,* 2011)up to order 6. Using the same techniques and relevant data, wealso enumerate Stein AG-groupoids up to order 6. Table-1 provides the enumeration of Stein AG-groupoids. In this note we provide various non-associative examples for this class. It is worth mentioning that various other new classes of AG-groupoids have been recently discovered(Shah *et al.,* 2013, Shah *et al.,* 2012, Khan *et al.,* 2011) enumerated and discussed by various authors. We define and list various examples of Stein AG-groupoidin Section 3.1. Section3.2 provides procedure of testing an arbitrary AG-groupoid for a Stein AG-groupoid. In Section 3.3we discuss some relations of Stein AG-groupoids with already known classes of AG-groupoids (Rashad *et al.,* 2014, Rashad *et al.,* 2014, Yiarayong 2013) and investigate their general properties, whilein Section 3.4 we characterize Stein AG-groupoids by the properties of their ideals.

2. MATERIALS AND METHODS

In the following we list some of the already known classes of AG-groupoids with their identities that will be used frequently in the rest of this article.

Definition 1.A groupoid S is called an AG-groupoid (Pelling *et al.,* 1980) if $ab\cdot c=cb\cdot a$ for all $a,b,c\in S$.

Definition 2. An AG-groupoid (Shah 2012, Cho *et al*., 1996, Shah *et al*., 2013) S is called:

(i) a left nuclear square AG-groupoid if,
$$a^2b\cdot c=a^2\cdot bc \quad \forall\ a,b,c\in S.$$
(ii) a middle nuclear square AG-groupoid if,
$$ab^2\cdot c=a\cdot b^2c \quad \forall\ a,b,c\in S.$$
(iii) a right nuclear square AG-groupoid if,
$$ab\cdot c^2=a\cdot bc^2 \quad \forall\ a,b,c\in S.$$
(iv) a right alternative AG-groupoid if,
$$a\cdot bb=ab\cdot b \quad \forall\ a,b\in S.$$
(v) a Bol*-AG-groupoid if,
$$a(bc\cdot d)=(ab\cdot c)d \quad \forall\ a,b,c,d\in S.$$
(vi) a paramedial AG-groupoid if,
$$ab\cdot cd=db\cdot ca \quad \forall\ a,b,c,d\in S.$$

Definition 3. An AG-groupoid (Mushtaq*et al*., 1979, Cho *et al*., 1996, Shah *et al*., 2013, Ahmad *et al*., 2013, Ahmad *et al*., 2013) S is called:

(i) an AG^{**}-groupoid if,

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$$a \cdot bc = b \cdot ac \quad \forall \quad a,b,c \in S.$$

(ii) a T_f^4 -AG-groupoid if,

$$ab = cd \Rightarrow ad = cb \quad \forall \quad a,b,c,d \in S.$$

(iii) a T_b^4 -AG-groupoid if,

$$ab = cd \Rightarrow da = cb \quad \forall \quad a,b,c,d \in S.$$

(iv) an AG-3-band if, $aa \cdot a = a \quad \forall \quad a \in S.$

(v) a cancellative AG-groupoid if for all $a,x,y \in S$; S is a left cancellative: $ax = ay \Rightarrow x = y$ and is a right cancellative: $xa = ya \Rightarrow x = y.$

(vi) a medial AG-groupoid if,

$$ab \cdot cd = ac \cdot bd \quad \forall \quad a,b,c,d \in S.$$

(vii) locally associative if $a \cdot aa = aa \cdot a \ a \quad \forall \quad a \in S.$

Table-1 Enumeration of Stein AG-groupoids up to order 6

Order	**3**	**4**	**5**	**6**
Non-associative AG-groupoid	**8**	**269**	**31467**	**4010451**
Associative AG-groupoid	**12**	**62**	**446**	**751**
Non-associative Stein AG-groupoid	**0**	**0**	**16**	**921**
Associative Stein AG-groupoid	**12**	**62**	**446**	**7503**

3. RESULTS AND DISCUSSION

3.1Stein AG-groupoid

In this section we define a Stein AG-groupoid and give a non-associative example of lowest order to show its existence.

Definition 4. An AG-groupoid S is called a Stein AG-groupoid if,

$$a \cdot bc = bc \cdot a \quad \forall \quad a,b,c \in S. \qquad (3.1)$$

Example 1. Let $S = \{1,2,3,4,5\}$. Then itis easy to verify that S with the following table is a Stein AG-groupoid.

·	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	2	2
4	1	1	2	2	3
5	1	1	2	2	3

3.2 Stein AG-test

In this section we discuss a procedure as adopted by (Proticet *al*., 1995) to check an arbitrary AG-groupoid $(G,\cdot)$ for Stein-AG-groupoid. For this we define the following binary operations.

$$a \circ b = a \cdot bx$$
$$a * b = bx \cdot a.$$

For any $x \in G$, the identity $a \cdot bx = bx \cdot a$ holds if; $a \circ b = a * b.$

To construct table of the operation '$\circ$' for any fixed $x \in G$, we multiply turn by turn elements of index column of the '$\cdot$' table from left with the elements of x -column of the '$\cdot$' table. Similarly, the table of operation '$*$' for any fixed $x \in G$ is obtained by presenting x -column of the '$\cdot$' table as index column and multiplying it with elements of index row of the '$\cdot$' table from the left. If the tables for the operation '$\circ$' and '$*$' coincides for all $x \in G$, then (2.1) holds and the AG-groupoid is a Stein AG-groupoid in this case. We illustrate this procedure in the following example.

Example2. VerifythefollowingAG-groupoidsforaSteinAG-groupoid:

(i)

·	**1**	**2**	**3**	**4**	**5**
1	**1**	**1**	**1**	**1**	**1**
2	**1**	**1**	**1**	**1**	**1**
3	**1**	**1**	**1**	**1**	**1**
4	**1**	**1**	**1**	**1**	**2**
5	**1**	**1**	**4**	**2**	**1**

(ii)

·	**1**	**2**	**3**
1	**1**	**1**	**1**
2	**1**	**1**	**1**
3	**1**	**2**	**1**

We construct '$\circ$' Table for (i) that is, $a \circ b = a \cdot bx$ as described in section 3.

·	1	2	3	4	5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4	1	1	1	1	2	1	1	1	2	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	4	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1

Now constructing'$*$' Table for (i) that is, $a * b = bx \cdot a$ as follows;

·	1	2	3	4	5	·	1	2	3	4	5	·	1	2	3	4	5	·	1	2	3	4	5	·	1	2	3	4	5
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	4	1	1	1	1	2	2	1	1	1	1	1	1	1	1	1	1	1

It can be seen easily that the tables for the operation '$\circ$' and '$*$' coincides for all $x \in G$. Hence (3.1) holds and the AG-groupoid in Example 2 (i) is a Stein AG-groupoid.

Now we proceed for table (ii) as follows;
'$\circ$'Table for (ii) that is, $a \circ b = a \cdot bx$

·	1	2	3	1	1	1	1	1	2	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1
3	1	2	1	1	1	1	1	1	2	1	1	1

Similarly '$*$'Table for (ii) that is, $a * b = bx \cdot a$

·	1	2	3	·	1	2	3	·	1	2	3
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	2	1	1	1	1	1	1	1

As the respective tables for the operation '$\circ$' and '$*$'are different. Hence (3.1)does not hold and consequently the AG-groupoid in (ii) is not a Stein AG-groupoid.

3.3 Characterization ofStein AG- groupoids

In this section we find some relations of Stein AG-groupoids with the other known classes of AG-groupoids. To do thiswe give some results from (Shah 2012) that will help to investigate some properties of Stein-AG-groupoids.

Lemma 1. AG**-groupoid is Bol*- AG-groupoid.
Lemma 2. Bol*-AG-groupoid is paramedial AG-groupoid.
Lemma 3. AG**-groupoid is left nuclear square AG-groupoid.

We start with the following;

Proposition.1 Let S be a Stein AG-groupoid, then S is

(i) locally associative AG-groupoid.
(ii) right alternative AG-groupoid.
(iii) AG**-groupoid.
(iv) Bol*-AG-groupoid.
(v) paramedial AG-groupoid.
(vi) left nuclear square AG-groupoid.
(vii) right nuclear square AG-groupoid.
(viii) middle nuclear square AG-groupoid.
(ix) nuclear square AG-groupoid.

Proof. Let S be a Stein AG-groupoid, and $a, b, c \in S$.

(i) Since S is a Stein AG-groupoid, then

$$a \cdot bc = bc \cdot a \quad \forall \quad a, b, c \in S.$$

Thus, by replacing b and c by a, we get $a \cdot aa = aa \cdot a$
Hence S is locally associative AG-groupoid.
(ii) Since $\forall$ $a, b \in S$ we have
$ab \cdot b = bb \cdot a = a \cdot bb \Rightarrow ab \cdot b = a \cdot bb.$ Hence S is right alternative AG-groupoid.
(iii) Since S is Stein AG-groupoid, thus for all

$$a, b, c \in S;\ a \cdot bc = bc \cdot a = ac \cdot b = b \cdot ac$$
$$\Rightarrow a \cdot bc = b \cdot ac.$$

Hence S is AG**-groupoid.
(iv)The result follows from Lemma 1.
(v) Follows from Lemma 2.
(vi)Follows from Lemma 3.
(vii)We show that S is right nuclear square AG-groupoid. Let $a, b, c \in S$. Then

$$ab \cdot c^2 = (cc \cdot b)a = (b \cdot cc)a = (bc^2)a = a(bc^2) \Rightarrow (ab)c^2 = a(bc^2).$$

Hence the result holds.
(viii) Let a, b, c ∈ S.Then

$$a(b^2c) = (bb \cdot c)a = (c \cdot bb)a = (ab^2)c \Rightarrow a(b^2c) = (ab^2)c.$$

Hence S is middle nuclear square AG-groupoid.
(ix) Follows by (vi), (vii) and (viii).

Theorem 1. A Stein AG-groupoid S is a semi-group if any of the following hold:
(i) S is AG*-groupoid.
(ii) S is AG-3-band.
(iii) S is T^4-AG-groupoid.
(iv) S has a left or right cancellative element.

Proof. Let S be a Stein AG-groupoid.
Assume that S is AG*-groupoid, and $a, b, c \in S$. Then, using Definition1, 3 and 4, we get

$$ab \cdot c = b \cdot ac = ac \cdot b = bc \cdot a = a \cdot bc \Rightarrow ab \cdot c = a \cdot bc.$$

Let S be an AG-3-band, and $a, b, c \in S$. Then by Definitions 1, 3 and4, we get

$$ab \cdot c = cb \cdot a = (cb)(aa \cdot a) = (c \cdot aa)(ba)$$
$$= (ba)(c \cdot aa) = (bc)(a \cdot aa) = (bc)a$$
$$\Rightarrow ab \cdot c = a \cdot bc.$$

Let S be a T^4-AG-groupoid and $a, b, c \in S$. Then by Definition 1, 3 and 4,

$$ab \cdot c = cb \cdot a = a \cdot cb \Rightarrow ab \cdot cb = ac$$
$$\Rightarrow ac \cdot bb = ac \Rightarrow ac \cdot c = a \cdot bb = bb \cdot a = ab \cdot b$$
$$\Rightarrow ac \cdot b = ab \cdot c \Rightarrow bc \cdot a = ab \cdot c \Rightarrow a \cdot bc = ab \cdot c.$$

Let S has a left cancellative element x and $a, b, c \in S$. Then by Definitions 1, 3 and 4, itis clear that

$$x(a \cdot bc) = (a \cdot bc)x = (x \cdot bc)a = (bc \cdot x)a$$
$$= (xc \cdot b)a = ab \cdot xc = xc \cdot ab = (ab \cdot c)x = x(ab \cdot c)$$
$$\Rightarrow x(a \cdot bc) = x(ab \cdot c) \Rightarrow a \cdot bc = ab \cdot c.$$

Now, S has a right cancellative element x and $a, b, c \in S$. Then by Definitions 1, 3 and 4, we have

$$(a \cdot bc)x = (x \cdot bc)a = (bc \cdot x)a = ax \cdot bc = ab \cdot xc = xc \cdot ab = (ab \cdot c)x$$
$$\Rightarrow (a \cdot bc)x = (ab \cdot c)x \Rightarrow a \cdot bc = ab \cdot c.$$

Hence S is a semi-group.

3.4 Ideals in Stein AG-groupoids

A subset A of the AG-groupoid S is a left (resp. right) ideal of S if,

$$SA \subseteq A \text{ (resp. } AS \subseteq A) \qquad (3.2)$$

A is a two sided ideal or simply an ideal of S if it is both left and right ideal of S. Let S be an AG-groupoid and $A, B \subseteq S$, then A and B are called left connected if $SA \subseteq B$ and $SB \subseteq A$. Similarly, if S is an AG-groupoid and $A, B \subseteq S$, then A and B are called right connected sets if $AS \subseteq B$ and $BS \subseteq A$. A and B are said to be connected if A and B are both left and right connected.

Lemma 4. If S is a Stein AG-groupoid and A, B are left ideals in S, then AB and BA areboth left and right connected sets.

Proof. Let S be a Stein AG-groupoid. Then by Definitions 1 and 4, we get

$$S(AB) = (AB)S = (SB)A \subseteq BA \Rightarrow S(AB) = BA.$$

Similarly,

$$S(BA) = (BA)S = (SA)B \subseteq AB \Rightarrow S(BA) \subseteq AB.$$

Hence AB and BA are left connected sets. Now using Definition 1, we have

$$(AB)S = (SB)A \subseteq BA \Rightarrow (AB)S = BA.$$

Similarly,

$$(BA)S = (SA)B \subseteq AB \Rightarrow (BA)S = AB.$$

Hence AB and BA are right connected sets.

Proposition 2. If L is a left ideal ofa Stein AG-groupoid S and then L^2 is an ideal of S.

Proof. Let S be Stein-AG-groupoid and L be a left ideal of S then,

$$S(L^2) = (LL)S = (SL)L \subseteq LL = L^2 \Rightarrow S(L^2) \subseteq L^2$$

and

$$(L^2)S = SL \cdot L \subseteq LL = L^2.$$

Hence L^2 is an ideal.

Theorem 2. If S is a Stein AG-groupoid. Then for any fixed a in S

(i) aS is an ideal of S.
(ii) $a(Sa)$ is minimal ideal of S.
(iii) $(aS)a$ is an ideal of S.

Proof. Let S be a Stein AG-groupoid. Let a be any fixed element of S then,

(i)

$$S(aS) = \bigcup_{x,y \in S} x(ay) = \bigcup_{x,y \in S} (ay)x = \bigcup_{x,y \in S} (xy)a$$
$$= \bigcup_{x,y \in S} a(xy) \subseteq aS \Rightarrow S(aS) \subseteq aS.$$

Thus aS is a left ideal of S. Similarly,

$$(aS)S = \bigcup_{x,y \in S} (ax)y = \bigcup_{x,y \in S} (yx)a = \bigcup_{x,y \in S} a(xy)$$
$$= \bigcup_{x,y \in S} a(xy) \subseteq aS \Rightarrow (aS)S \subseteq aS.$$

Thus aS is a right ideal of S. Equivalently aS is an ideal.

(ii) Again by Definitions 1, 4 and Proposition 1(c), we get

$$S(a(Sa)) = \bigcup_{x,y \in S} x(a \cdot ya) = \bigcup_{x,y \in S} x(ya \cdot a)$$
$$= \bigcup_{x,y \in S} x(aa \cdot y) = \bigcup_{x,y \in S} (aa \cdot y)x = \bigcup_{x,y \in S} xy \cdot aa$$
$$= \bigcup_{x,y \in S} a(xy \cdot a) \subseteq a(Sa) \Rightarrow S(a(Sa)) \subseteq a(Sa).$$

Thus $a(Sa)$ is a left ideal of S. Similarly, by Definition1, 4 and Proposition1 (c), we have

$$(a(Sa))S = \bigcup_{x,y \in S} (a \cdot xa)y = \bigcup_{x,y \in S} (xa \cdot a)y = \bigcup_{x,y \in S} (aa \cdot x)y$$
$$= \bigcup_{x,y \in S} yx \cdot aa = \bigcup_{x,y \in S} a(yx \cdot a) \subseteq a(Sa) \Rightarrow (a(Sa))S \subseteq a(Sa).$$

Thus $a(Sa)$ is a right ideal of S and hence is an ideal of S. Now let L be an ideal of S such that $a \in L$. Then

$$a(Sa) \subseteq L(SL) \subseteq LL \subseteq L.$$

Hence $a(Sa)$ is minimal ideal of S.

(iv) NowusingDefinitions1and4,wehave

$$S((aS)a) = \bigcup_{x,y \in S} x(ay \cdot a) = \bigcup_{x,y \in S} x(a \cdot ay)$$
$$= \bigcup_{x,y \in S} (a \cdot ay)x = \bigcup_{x,y \in S} (x \cdot ay)a = \bigcup_{x,y \in S} (ay \cdot x)a = \bigcup_{x,y \in S} (xy \cdot a)a$$
$$= \bigcup_{x,y \in S} (a \cdot xy)a \subseteq (aS)a \Rightarrow S((aS)a) \subseteq (Sa)a.$$

So $(aS)a$ is a left ideal of S. Similarly, byDefinition4,wehave

$$((aS)a)S = \bigcup_{x,y\in S}(ax\cdot a)y = \bigcup_{x,y\in S}(a\cdot ax)y = \bigcup_{x,y\in S}(y\cdot ax)a$$
$$= \bigcup_{x,y\in S}(ax\cdot y)a = \bigcup_{x,y\in S}(yx\cdot a)a = \bigcup_{x,y\in S}(a\cdot yx)a$$
$$\subseteq (aS)a \Rightarrow ((aS)a)S \subseteq (Sa)a.$$

Thus $(aS)a$ is rightideal and hence is a nideal of S.

Theorem3. Let L be a left ideal and R be a right ideal of a Stein-AG- groupoid S, such that $x = x^2$ and $y = y^2$ for some $x, y \in S$. Then

(i) $xL = L \cap xS$
(ii) $Rx = Sx \cap R$
(iii) $Sy \cap xS = x(Sy)$

Proof: Let L be a left ideal and R be a right ideal of S such that $x = x^2$ and $y = y^2$ for some $x, y \in S$. Then

(i) Let $t \in xL$, so $t = xl$ for some $l \in L$. Since $l \in L$ and $x \in S$, then $xl \in L$, so that $t \in L$. But $l \in L \subseteq S$, therefore $l \in S$, and so $xl = t \in xS$. Hence $t \in L \cap xS$, that is

$$xL \subseteq L \cap xS. \qquad (3.3)$$

Conversely, let $u \in L \cap xS$, then $u \in L$ and $u \in xS$, so $u = xs$ for some $s \in S$. Now using Definition 4, Proposition 1(c) and by the assumption, we get

$$u = xs = (xx)s = s(xx) = x(sx) = x(s\cdot xx)$$
$$= x(xx\cdot s) = x(xs) = xu \in xL \Rightarrow u \in xL.$$

Therefore,

$$L \cap xS \subseteq xL. \qquad (3.4)$$

Hence, from (3.3) and (3.4), we have $xL = L \cap xS$.

(ii) Let $b \in Rx$, so $b = rx$, for some $r \in R$. Since $r \in R$ and $x \in S$, then $rx \in R$, so that $b \in R$. But $r \in R \subseteq S$, then $r \in S \Rightarrow rx = b \in Sx$, hence $b \in R \cap Sx$, this implies that

$$Rx \subseteq R \cap Sx. \qquad (3.5)$$

Conversely, let $t \in R \cap Sx$, then $t \in R$ and $t \in Sx$. Since $t \in Sx$, so $t = sx$, forsome $s \in S$. Now by assumption and Definition 1, 4, we have

$$tsx = s(xx) = (xx)s = (sx)x = tx \in Rx \Rightarrow t \in Rx.$$

Therefore,

$$R \cap Sx \subseteq Rx. \qquad (3.6)$$

Hence by equations (3.5) and (3.6), we have

$$Sx \cap R = Rx.$$

(iii) If $a \in Sy \cap xS$, then for some $s, s' \in S$, we have $a = sy$ and $a = xs'$. Now by Definition 1, 4 and assumption, we have $ay = sy\cdot y = yy\cdot s = s\cdot yy = sy = a$.

$$a = xs' = xx\cdot s' = s'x\cdot x = s'x\cdot xx$$
$$= x(s'x\cdot x) = x(xx\cdot s') = x\cdot xs' = xa.$$

Therefore, $ay = xa = a$, and again by Definitions 1, 4 and assumption, we have

$$x(ay) = x(xa) = xx\cdot xa = xa\cdot xx = xx\cdot ax$$
$$= x\cdot ax = ax\cdot x = xx\cdot a = xa = a.$$

Thus $a = x(ay) \Rightarrow a \in x(Sy)$. Therefore,

$$Sy \cap xS \subseteq x(Sy) \qquad (3.7)$$

Conversely, if $a \in x(Sy)$, then by Definitions 1, 4 and assumption, we have

$$a = x(sy) = x(s\cdot yy) = x(yy\cdot s) = x(sy\cdot y)$$
$$= x(y\cdot sy) = (y\cdot sy)x = (x\cdot sy)y = ay,$$

and

$$a = x(sy) = xx\cdot sy = (sy\cdot x)x = (x\cdot sy)x$$
$$= x(x\cdot sy) = x(x\cdot sy) = xa.$$

That is $a \in Sy \cap xS$. Therefore,

$$x(Sy) \subseteq Sy \cap xS. \qquad (3.8)$$

Hence, by(3.7) and (3.8),we have $x(Sy) = Sy \cap xS$.

4. CONCLUSIONS

The concept of Stein groupoids satisfying the identity, $a\cdot bc = bc\cdot a \quad \forall\, a,b,c \in S$ have been extended to introduce Stein AG-groupoids. Sufficient finite examples of these AG-groupoids are produced using GAP: Groups Algorithm and Programming. Further, these AG-groupoids are enumerated up to order 6. Various basic and general properties of these AG-groupoids are explored and various relations of these AG-groupoids with other already known subclasses of AG-groupoids have been investigated. A simple method, Stein AG-test, to verify an arbitrary AG-groupoid for this new class is developed. Moreover, these AG-groupoids are characterized by the properties of their ideals.

REFERENCES:

Ahmad, I., M. Rashad, M. Shah, (2013) "Some new result on T^1, T^2 and T^4-AG-groupoids" Research Journal of Recent Sciences, Vol. 2(3), 64-66.

Ahmad, I., M. Rashad M. Shah, (2013) "Some Properties of AG^* -groupoids" Research Journal of Recent Sciences, Vol. 2(4), 91-93.

Cho J. R., J. Jezek T. Kepka, (1999) "Paramedial Groupoids" Czechoslovak Mathematical Journal, 49 (124)277-290.

Denes, J., A. D. Keedwell, (1974) "Latin Squares and their applications "Academic Press New York and London.

Distler, A., M. Shah, V. Sorge, (2011) "Enumeration of AG-groupoids" Lecture Notes in Computer Science, Volume 6824/2011, 1-14.

Khan, M., F. V. Amjad, (2011) "On some classes of Abel-Grassmann's groupoids" J. Adv. Res. Pure Math. 3(4), 109–119.

Mushtaq, Q., S. M. Yusuf, (1979)"On Locally Associative LA-Semigroup" J. Nat. Sci. Math., XIX(1), 57–62.

Pelling, M. J., D. G. Rogers, (1980) "Some (2, n) combinatorial Stein groupoids" Equations Mathematic ae, Volume 21 (1), 225-235.

Protic, P. V., N. Stevanovic, (1995) "AG-test and some general properties of Abel-Grassmann's groupoids", Pure. Math. Appli, 6 (4), 371 - 383.

Rashad, M., I. Ahmad, M. Shah, A. B. Saeid, (2013) "Enumeration of Bi-commutative AG-groupoids" www.arXiv.org.

Rashad, M., I. Ahmad, M. Shah, (2014) "Left transitive AG-groupoids" Sindh Univ. Res. Jour. (Sci. Ser.) Vol.46 (4):547-552 (2014).

Shah, M., (2012) "A theoretical and computational investigation of AG-groups" PhD thesis, Quaid-i-Azam University Islamabad.

Shah, M., I. Ahmad, A. Ali, (2012) "Discovery of new classes of AG-groupoids" Res. J. Recent Sci., Vol. 1(11), 47-49.

Shah, M., I. Ahmad, A. Ali, (2013) "On introduction of new classes of AG-groupoids" Res. J. Recent Sci., Vol. 2(1), 67-70.

Yiarayong, P. (2013) "On Semi-prime and Quasi Semi-prime ideals in AG-groupoids" KKU Res. 18(6), 893-896.