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Comparative Study of Existing Bracketing Methods with Modified Bracketing Algorithm for Solving Nonlinear Equations in Single Variable

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Abstract: In this paper a new bracketing algorithm has been developed to find the root of nonlinear equations. During the study it has been observed that developed algorithm takes fewer iterations than False Position and Bisection Methods to achieve required accuracy. New algorithm is faster than Bisection and False Position Method .In order to analyze the results few examples related with algebraic, trigonometric and transcendental functions are taken. The comparison of existing bracketing methods and modified bracketing algorithm have been analyzed on the basis of number of iterations and accuracy, and it has been observed that modified algorithm performs better in comparison to existing bracketing methods. MATAB is used for the results and graphical representation.

Keywords: Absolute percentage error, Bracketing algorithm, transcendental, accuracy.

1. INTRODUCTION

It is challenging task in scientific application problems to estimate the solution of nonlinear equations. Determination of roots of nonlinear equations is a very important generally in science and especially in engineering. These equations can be solved by using Numerical Methods. Development of numerical methods, to solve nonlinear equations, are very much essential because the analytic methods fail to solve the polynomial equations of the degree greater than four.

Most of the numerical methods, used to solve an equation, are based on iterative techniques. Convergence must be assured while developing any Numerical method. Numerical methods are basically approximated methods. Numerical Methods are almost invariably contaminated by errors. And Analytical methods are exact methods. Most of the analytical methods used to solve non-linear equations $f(x) = 0$ with exact root. Comparative study of various methods of locating roots has been discussed in which it is concluded that secant method is most effective method (Ehiwario, *et al,* 2014) . The root finding problem is one of the most relevant computational problems. It arises in a wide variety of practical applications in Physics, Chemistry, Biosciences, Engineering, etc. As a matter of fact, the determination of any unknown appearing implicitly in scientific or engineering formulas, gives rise to root finding problem (Biswa Nath Datta , 2012). Relevant situations in Physics where such problems are needed to be solved include finding the equilibrium position of an object, potential surface of a field and quantized energy level of confined structure (Iwetan, 2012). In recent years, many modifications in iterative methods like Bisection, Regula Falsi and Newton's method have been proposed in literature, which have either equal or better performance than these methods. Comparative study of various methods of locating roots has been discussed which it is concluded that secant method is most effective method (Ehiwario, *et al.*, 2014). (Xin *et al*, 2003) developed a new root finding with global convergence for enclosing simple zeros of nonlinear equation, which improves False Position Method.

(Tanakan, 2013) The simplest root-finding algorithm is the bisection method. It works when f is a continuous function and it requires previous knowledge of two initial guesses, a and b, such that f(a) and f(b) have opposite signs, then find the midpoint of [a,b], and then decide whether the root lies on [a,(a + b)/2] or [(a+b)/2,b]. Repeat until the interval is sufficiently small.

Regula Falsi and Newton Raphson's methods have been obtained using Taylor Interpolating Polynomials and Quadrature formulas by (Goudin, 1989), (Aslam Noor, 2007) and (Yamamoto, 2000) various iterative techniques have been designed for solving non-linear equations have been developed by the researchers, such methods are explored by using techniques of error analysis , quadrature rules or other techniques are explored by (Burden,2001), (Frontini, 2003), (Kanawar, 2004), (Mamta, 2005).

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2. METHODOLOGY

Let $[a, b]$ be the interval contains root of $f(x)$ and satisfies the condition $(a).f(b) < 0$. We divide the given interval into subintervals, The sub-intervals will be made using formula $[a + nh\,, b - mh]$ Where $m = (d-1) - n$ and n are positive integers and complementary to each other.

Sub-intervals are $[a\,,\ b-3h]$, $[a+h\,,\ b-2h]$, $[a+2h\,,\ b-h]$ and $[a+3h\,,\ b]$. Choosing any two suitable intervals. Suppose y= f(x) be the curve whose root lies in the interval $[a+nh\,, b-mh]$. The equation of chord joining the points $(a+nh\,, f(a+nh))$ and $(b-mh\ ,\ f(b-mh))$ is

$$\frac{y-f(a+nh)}{f(a+nh)-f(b-mh)} = \frac{x-(a+nh)}{(a+nh)-(b-mh)} \qquad \text{(i)}$$

To find the point of intersection , set $y = 0$ in (i) and let $(b-mh, 0)$ be such point.

Thus

$$x_2 = (a+nh) - \frac{f(a+nh)}{f(a+nh)-f(b-mh)}\,((b-mh) - (a+nh))$$

$$x_2 = \frac{(a+nh)\{f(b-mh)-f(a+nh)\}-f(a+nh)((b-mh)-(a+nh))}{f(b-mh)-f(a+nh)}$$

$$x_2 = \frac{(a+nh)f(b-mh)-(a+nh)f(a+nh)-(b-mh)\,f(a+nh)+(a+nh)f(a+nh)}{f(b-mh)-f(a+nh)}$$

$$x_2 = \frac{(a+nh)f(b-mh)-(b-mh)\,f(a+nh)}{f(b-mh)-f(a+nh)}$$

If $f(x_2)$ and $f(b-mh)$ are of opposite sign then the root lies between $(b-mh)$ and x_2 . and the iterative formula for Modified Bracketing Algorithm in the interval $[a+nh\,, b-mh]$ is

$$x_{k+1} = \frac{(a+nh)f(b-mh)-(b-mh)\,f(a+nh)}{f(b-mh)-f(a+nh)} \qquad \text{(ii)}$$

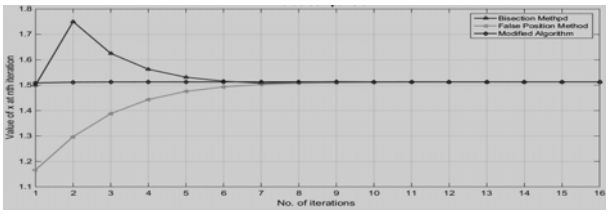
Where k positive integer .The procedure is is repeated and we get new interval in every iteration. Continue till the root is obtained to the required accuracy.

3. RESULTS AND DISCUSSIONS

To justify the modified bracketing algorithm, few examples are tested on developed/modified algorithm and existing bracketing methods .From the above results, it has been observed that existing bracketing methods i-e bisection and False Position methods take 11 number of iterations for the algebraic function , whereas modified algorithm takes only 3 number of iterations up to accuracy of A.E%=0.0869% **(Table.1, Fig.1(a) and Fig.1(b))**.Similarly for trigonometric function, Bisection and False Position methods take 12 and 3 number of iterations respectively, whereas modified algorithm takes only 2 iterations up to accuracy of A.E%=0.4501% **(Table.2, Fig.2(a) and Fig.2(b))** and almost same trend is observed for transcendental function with accuracy of A.E%=0.043% **(Table.3, Fig.3(a) and Fig.3(b))**,which guarantees for the best performance of modified bracketing algorithm

Example 1: (Algebraic Function): $x^4 - x^3 - x^2 + x - 1 = 0$ **(Table.1)**

Iteration	Bisection Method		False position Method		Modified Algorithm	
	x_n	A.E%	x_n	A.E%	x_n	A.E%
1st	1.5		1.166666667		1.50883	
2nd	1.75	25	1.297332466	13.0665799	1.511613	0.27832492
3rd	1.625	12.5	1.387548168	9.0215702	1.512483	0.086965103
4th	1.5625	6.25	1.443471793	5.59236251	1.512754	0.027098404
5th	1.53125	3.125	1.475600535	3.21287417	1.512838	0.00843664
6th	1.515625	1.5625	1.493207231	1.76066968	1.512865	0.002625906
7th	1.5078125	0.78125	1.502597991	0.93907592	1.512873	0.000817246
8th	1.51171875	0.390625	1.507533047	0.49350567	1.512875	0.00025434
9th	1.513671875	0.195313	1.510106157	0.25731096	1.512876	7.91541e-05
10th	1.512695313	0.097656	1.511442217	0.13360598	1.512876	2.46338e-05
11th	1.513183594	0.048828	1.512134456	0.06922391	1.512876	7.66635e-06
12th	1.512939453	0.024414	1.512492717	0.03582612	1.512876	2.38586e-06
13th	1.512817383	0.012207	1.512678024	0.01853068	1.512876	7.42511e-07
14th	1.512878418	0.006104	1.512773843	0.00958192	1.512876	2.31079e-07
15th	1.5128479	0.003052	1.512823382	0.00495389	1.512876	7.19147e-08
16th	1.512863159	0.001526	1.512848992	0.00256097	1.512876	2.23808e-08



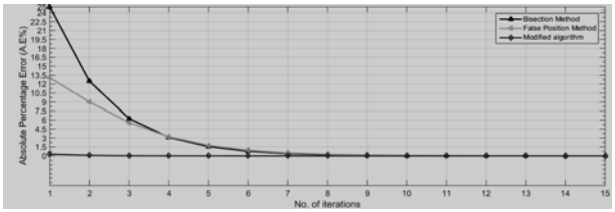
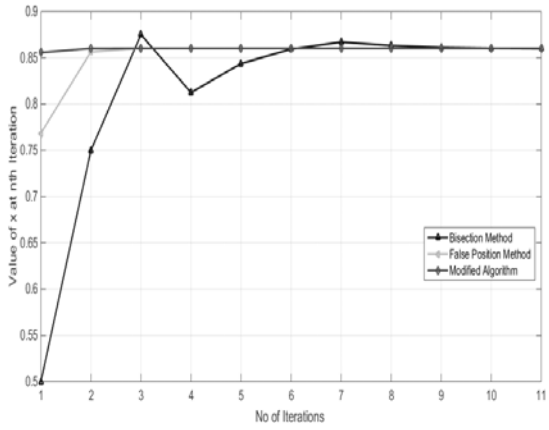


Fig.1(a) **Fig.1(b)**

Example 2 : (Trignometric Function) $xsinx - cosx = 0$ **(Table.2)**

Iterations	Bisection Method		False position Method		Modified Algorithm	
	x_n	A.E%	x_n	A.E%	x_n	A.E%
1st	0.5		0.768539864		0.855659396	
2nd	0.75	25	0.856535358	8.799549	0.860160991	0.45016
3rd	0.875	12.5	0.860193501	0.365814	0.860327254	0.016626
4th	0.8125	6.25	0.860328448	0.013495	0.860333357	0.00061
5th	0.84375	3.125	0.8603334	0.000495	0.86033358	2.24e-05
6th	0.859375	1.5625	0.860333582	1.82e-05	0.860333589	8.22e-07
7th	0.867188	0.78125	0.860333589	6.67e-07	0.860333589	3.02e-08
8th	0.863281	0.390625	0.860333589	2.45e-08	0.860333589	1.11e-09
9th	0.861328	0.1953125	0.860333589	8.98e-10	0.860333589	4.06e-11
10th	0.860352	0.09765625	0.860333589	3.3e-11	0.860333589	1.49e-12
11th	0.859863	0.04882813	0.860333589	1.2e-12	0.860333589	5.55e-14



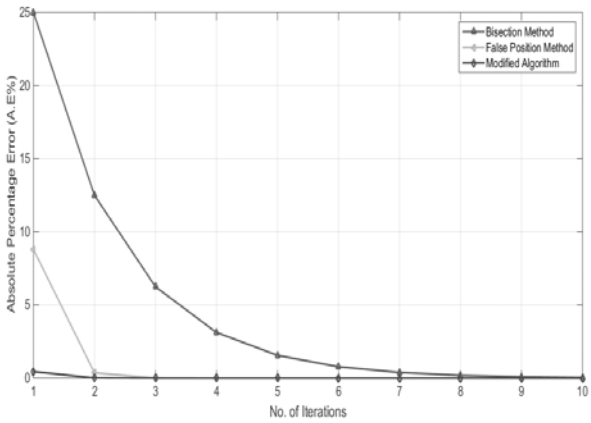


Fig.2(a) **Fig.2(b)**

Example 3 : (Transcendental Function) $e^x - 5x = 0$ **(Table.3)**

Iterations	Bisection Method		False position Method		Modified Algorithm	
	x_n	A.E%	x_n	A.E%	x_n	A.E%
1st	0.5		0.304718427		0.259608398	
2nd	0.25	25	0.26103454	4.368388707	0.2591718	0.04366
3rd	0.375	12.5	0.259245695	0.178884553	0.259171103	6.97e-05
4th	0.3125	6.25	0.259174085	0.007160957	0.259171102	1.11e-07
5th	0.28125	3.125	0.259171221	0.000286398	0.259171102	1.78e-10
6th	0.265625	1.5625	0.259171107	1.14539e-05	0.259171102	2.78e-13
7th	0.2578125	0.78125	0.259171102	4.58072e-07	0.259171102	2.78e-15

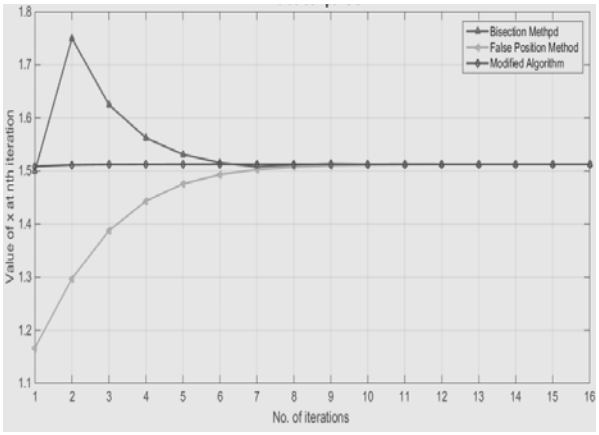


Fig. 3 (a)

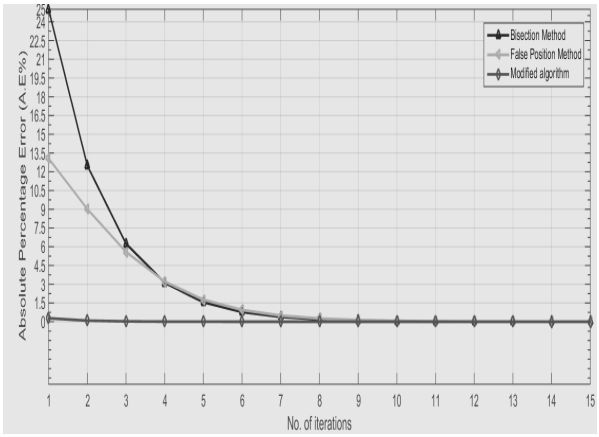


Fig. 3 (b)

4. CONCLUSION

Finally, it has been concluded that modified /developed algorithm performs better than existing bracketing methods i-e Bisection and False Position Method from accuracy as well as iteration point of view, hence the developed/modified bracketing algorithm is a good achievement to find root of nonlinear equations in one variable.

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