Sindh Univ. Res. Jour. (Sci. Ser.) Vol. 48(1) 175-180 (2016)

SINDH UNIVERSITY RESEARCH JOURNAL (SCIENCE SERIES)

Viscoelastic Fluid Flow Relevance's in a Pipe without Porous space using with Oldroyd–B Constitutive Model: Analysis of Statics

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Received 11th August 2015 and Revised 03rd March 2016

Abstract: In this research, necessary plan is to find the symmetries using Lei- group technique of viscoelastic fluid flows without porous media by means of Oldroyd–B Constitutive Model related with viscoelastic stresses. The solution of this problem is obtained by means of the one-parameter transformation group theoretic approach which reduces the number of independent variables by one and the governing PDE system with the boundary conditions reduce to ODE system with the appropriate boundary conditions equivalent to the transformation group, so that exact invariant solutions can be obtained. Numerical results of the given system are achieved by using Mathematica solver ND–Solve on computer.

Keywords: Lie Group Analysis, Viscoelastic Flow Oldroyd–B constitutive model, analysis of viscoelastic stress

1 INTRODUCTION

Currently, the flow during a porous medium concerned with the Newtonian fluids and non-Newtonian fluids is a fascinating subject substance of engineering importance, normally engineering problems are greatly harder to undertake and complex flow phenomena, particularly in the field of process applications and present many challenges for researchers Newtonian fluids are characterizes with the supposition in more accurate technical terms, that a part of the complete stress tensor which is said to be the extra stress tensor that identify with the extensional stress and shear stress due to the flow excepting hydrostatic pressure, is a linear isotropic function of velocity gradient components, and as a result shows that stress and the strain rate are linearly connected with each other. There is simple proportionality connecting the viscosities within unlike kinds of deformation, and zero normal stress variations within simple shear flow in Newtonian fluids, which are commonly characteristics of these fluids by containing shear and time-independent viscosity, investigation are developed by Taha (2009, 2010), Owens and Phillips (2002), vanOs and Phillips (2004) and others.

Viscoelastic materials display a double nature of manners in performance signs of together elastic and viscous behavior. In a viscoelastic fluid, the stress tensor depends on the history of the deformation gradient; researched by Moran, Viscoelastic fluids have been researched due to their vast applications for some decades to know the phenomena related with it and investigated by Taha Sochi (2009, 2010)

There are several constitutive equations which predict qualitatively the behaviour of some of the material functions, requiring only a small number of free constants. Some of these constitutive equations, however, are complicated. The rheological properties of materials are of greatest interest. The rheological equations of state and material derivatives are described for viscoelastic, generalized inelastic non-Newtonian fluids and fibre suspensions. Here viscoelastic behaviour will be modelled by the Oldroyd-B Phan-thien / Tanner (PTT) differential constitutive models.

In this paper, problems related with PDE's system of viscoelastic flow without porous media in pipes associated with Darcy-Brinkman model and also using Oldroyd–B constitutive model with initial and boundary conditions may be given and our aim is to solve PDE's system by Lie-Group theory using lie-point symmetries method and obtained their exact solutions related with viscoelastic stresses. Lie-Group theory of ODE's and PDE's as a scientific branch created from efforts of the exceptional mathematician Lie of the problem is to find and use admitted Lie point symmetries algebra. These algebras are prepared use of to reduce the governing PDE's system to solvable form. The solution of the governing PDE's system is acquired analytically or numerically in the way using symmetries of the system through symmetry method. Numerical predictions of a system are resolute adopting Mathematica solver ND-Solve.

Section 2 associates with the mathematical formulations. Section 3 connected with solutions of

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viscoelastic flow in pipes without porous space; section 3.1 related with steady State Solution, 3.2 consists of Lie-point symmetries of the PDE's system (11-i and iii) of viscoelastic flow in pipes without porous space. Section 3.3 deals with invariant solutions corresponding to the operator $X_1 - \beta X_2$, section 3.4 coupled with solution of partial differential equation (11-ii). Analysis of Viscoelastic Stresses presented in section 4. As section 4.1 combined with solution of normal and shear stresses. Steady state solution discussed and graphed in section 4.2. As section 4.3 related with numerical solution of viscoelastic flow in pipes without porous space and conclusion is given in section 5.

2 PROBLEM SPECIFICATION

A tubular porous layer held in a pipe saturated with the incompressible laminar flow of viscoelastic fluid in radial direction is measured. A polar coordinate system is applied with radius-axis vertically upward. The main equations system of flow consists of the mass preservation and momentum preservation transport attached with the model of Oldroyd–B constitutive. The viscoelastic fluid flow in the course of porous medium is supposed to possess homogeneous and isotropic. Velocity field $\bar{u} = \left(u(r,t), 0, 0\right)$ is identified for unidirectional flow, wherever the above sense of velocity mechanically satisfies the incompressibility state. The **generaliz**ed Darcy–Brinkman model has been employed for the momentum equation and if body force is not present; continuity equation, generalised equation of momentum through porous media and the Oldroyd–B equation describes the stresses of viscoelastic in the fluid flow in vectorial form can be given as:

$$\nabla.\bar{u} = 0 \tag{1}$$

$$\frac{\rho}{\varepsilon}\frac{\partial \bar{u}}{\partial t} = \frac{1}{r}\nabla\left(\left[\frac{\mu_2}{\varepsilon} r \underline{\underline{d}}\right] + \tau\right) - \nabla p - \rho\,\bar{u}.\nabla\bar{u} - \frac{\mu}{K}\bar{u}\,, \tag{2}$$

where as t represents time, ρ and μ is the fluid density and total viscosity of viscoelastic fluid respectively, $\bar{u}$ is used for the field of velocity vector, τ is the extra stress tensor, $\underline{\underline{d}}$ is the rate–of–strain tensor, ∇ represents a spatial operator for differential, μ_2 is denoted for the Newtonian solvent viscosity, *p* is the isotropic fluid pressure, the intrinsic permeability within the porous media is identified with K and ε is porosity of porous media.

The constitutive equation of Oldroyd–B model may be written in the form as:

$$\lambda\frac{\partial \tau}{\partial t} = [2\mu_1 \underline{\underline{d}}] - \tau - \lambda\{\bar{u}.\nabla\tau - \nabla\bar{u}.\tau - (\nabla\bar{u})^T.\tau\} \tag{3}$$

Where the rest time for the fluid of viscoelastic is indicated by λ and μ_1 is used for viscoelastic solute viscosity. As total viscosity is $\mu = \mu_1 + \mu_2 = 1$ and is taken constant.

The equations are obtained by leading the unidirectional fluid flow of viscoelastic in porous pipes on the source of constitutive equation within an Oldroyd-B flow fluid. The derivation of these equations by employing the transport equation of momentum and constitutive equations of Oldroyd–B, suppose that pressure gradient is constant and body force is not present. In the dimensionless form, governing system of equations in the present problem is given as,

$$\mathrm{Re}\frac{\partial u}{\partial t} = 1 + \mu_2\frac{\partial^2 u}{\partial r^2} + \frac{\mu_2}{r}\frac{\partial u}{\partial r} + \frac{\partial \tau_{12}}{\partial r} + \frac{\tau_{12}}{r} - \frac{1}{Da}u \tag{i}$$

$$We\frac{\partial \tau_{11}}{\partial t} = 2\,We\,\tau_{12}\frac{\partial u}{\partial r} - \tau_{11} \quad \text{(ii)} \qquad We\frac{\partial \tau_{12}}{\partial t} = \mu_1\frac{\partial u}{\partial r} - \tau_{12} \quad \text{(iii)}$$

Initial and boundary conditions for completing the well posed problem are taken as

$u(0,r) = \tau_{11}(0,r) = \tau_{12}(0,r) = 0$ **(4)** When $0 < r < 1$**(5)** $u(t,1) = 0$ & $\frac{\partial u}{\partial t}(t,0) = 0$ When $t > 0$ (6)

Where u and τ are dimensionless velocity and dimensionless stress tensor, r is radial coordinates, t is the time using for non-dimensional and the dimensionless Reynolds number (*Re*), Weissenberg number (*We*) and Darcy's number (*Da*) are defined as

$$\mathrm{Re} = \frac{R\,\rho\,Vc}{\mu}, \quad We = \frac{\lambda\,Vc}{R}, \quad Da = \frac{K}{\varepsilon R^2}.$$

Hence *K* is the adapted permeability concern with the porous medium using for non-dimensional. As *R* is a radius of the pipe and V_C is used for the feature velocity supposed since reference redial velocity $Vc = \varepsilon R^2\left(-\frac{\partial p}{\partial z}\right)\Big/\mu$

3 SOLUTIONS OF VISCOELASTIC FLOW IN PIPES WITHOUT POROUS SPACE

When the last Darcy's term *Da* vanishes (i.e. $Da \to \infty$), then the system (4) is called viscoelastic flow in pipes without porous space i.e.

$$\mathrm{Re}\frac{\partial u}{\partial t} = 1 + \mu_2\frac{\partial^2 u}{\partial r^2} + \frac{\mu_2}{r}\frac{\partial u}{\partial r} + \frac{\partial \tau_{12}}{\partial r} + \frac{\tau_{12}}{r} \qquad \text{(i)}$$

$$We\frac{\partial \tau_{11}}{\partial t} = 2\,We\,\tau_{12}\frac{\partial u}{\partial r} - \tau_{11} \qquad \text{(ii)} \qquad We\frac{\partial \tau_{12}}{\partial t} = \mu_1\frac{\partial u}{\partial r} - \tau_{12} \qquad \text{(iii)} \qquad (7)$$

Subject to same initial and boundary conditions (5) and (6)

$$u(0,r) = \tau_{11}(0,r) = \tau_{12}(0,r) = 0 \quad \text{When } 0 < r < 1 \quad u(t,1) = 0 \;\;\&\;\; \frac{\partial u}{\partial t}(t,0) = 0 \quad \text{When } t > 0$$

3.1 Steady State Solution

Some problems involving non–homogeneous equations can be solved by means of a change of dependent variable and to find the steady state solution, so for this consider

$$u(t,r) = v_1(t,r) + \varphi_1(r), \quad \tau_{11}(t,r) = v_2(t,r) + \varphi_2(r) \;\text{ and }\; \tau_{12}(t,r) = v_3(t,r) + \varphi_3(r) \qquad (8)$$

Substituting above values in Equations (7-i, ii, and iii) and separating the like terms of variables, gives the two systems of equations. The first system is

$$\mu_2\varphi_1''(r) + \frac{\mu_2}{r}\varphi_1'(r) + \varphi_3'(r) + \frac{\varphi_3(r)}{r} = 0, \quad \varphi_2(r) = 2We\,\varphi_3(r)\varphi_1'(r) \quad \varphi_3(r) = \mu_1\,\varphi_1'(r)$$

Subject to boundary condition: $\varphi_1(1) = 0$ and $\varphi_1'(0) = 0$ and integrating the above equations system and applying the boundary conditions then following solution is obtained

$$\varphi_1(r) = \tfrac{1}{4}\left(1 - r^2\right) \quad \text{(9-i)} \qquad \varphi_2(r) = \tfrac{1}{2}\,We\mu_1 r^2 \quad \text{(9-ii) and} \qquad \varphi_3(r) = -\tfrac{1}{2}\,\mu_1 r \quad \text{(9-iii)}$$

Therefore Equation (8) is written as

$$u(t,r) = v_1(t,r) + \tfrac{1}{4}\left(1 - r^2\right), \; \tau_{11}(t,r) = v_2(t,r) + \tfrac{1}{2}\,We\,\mu_1 r^2 \;\text{ and }\; \tau_{12}(t,r) = v_3(t,r) - \tfrac{1}{2}\,\mu_1 r \qquad (10)$$

Thus to determine $v_1(t,r)$, $v_2(t,r)$ and $v_3(t,r)$, the new initial and boundary value problem is given as

$$\mathrm{Re}\,\frac{\partial v_1}{\partial t} = \mu_2\frac{\partial^2 v_1}{\partial r^2} + \frac{\mu_2}{r}\frac{\partial v_1}{\partial r} + \frac{\partial v_3}{\partial r} + \frac{v_3}{r} \quad \text{(11-i)} \qquad We\,\frac{\partial v_2}{\partial t} = 2We\left(v_3 - \tfrac{1}{2}\,\mu_1 r\right)\frac{\partial v_1}{\partial r} - r\,We\,v_3 - v_2 \quad \text{(11-ii)}$$

$$We\,\frac{\partial v_3}{\partial t} = \mu_1\frac{\partial v_1}{\partial r} - v_3 \qquad \text{(11-iii)}$$

Subject to initial and boundary conditions are,

$$v_1(t,1) = 0 \quad \text{(12-i)}, \qquad \frac{\partial v_1(t,0)}{\partial r} = 0 \quad \text{(12-ii)} \quad t > 0 \;\text{ and}$$

$$v_1(0,r) = -\tfrac{1}{4}\left(1 - r^2\right) \;\text{(13-i)} \quad v_2(0,r) = -\tfrac{1}{2}\,We\,\mu_1 r^2 \;\text{(13-ii)} \quad v_3(0,r) = \tfrac{1}{2}\,\mu_1 \qquad \text{(13-iii)}$$

3.2 Lie-point symmetries of the PDE's system (11-i and-iii) of viscoelastic flow in pipes without porous space

In this section, method for finding the Lie point symmetries of the governed system of two PDE (11-i) and (11-iii) (because derivatives of PDE's are linked each other) and symmetry conditions are introduced. Let us consider the one parameter Lie point transformations of (t, r, v_1, v_3) is given by

$$\begin{aligned} t^* &= t + \delta\,\phi(t,r,v_1,v_3) + \dots\dots\dots, & r^* &= r + \delta\,\xi(t,r,v_1,v_3) + \dots\dots\dots, \\ v_1^* &= v_1 + \delta\,\eta^1(t,r,v_1,v_3) + \dots\dots\dots, & v_3^* &= v_3 + \delta\,\eta^2(t,r,v_1,v_3) + \dots\dots\dots, \end{aligned} \qquad (14)$$

The governed system of PDE's (11-i and iii) is invariant under the transformation (14) if it is invariant under the generator

$$X=\phi(t,r,v_1,v_3)\frac{\partial}{\partial t}+\xi(t,r,v_1,v_3)\frac{\partial}{\partial r}+\eta^1(t,r,v_1,v_3)\frac{\partial}{\partial v_1}+\eta^2(t,r,v_1,v_3)\frac{\partial}{\partial v_3} \qquad (15)$$

The unknown functions ϕ, ξ, η^1, η^2 are found from the determining equations derived from the invariance condition. As

$$X^{[1]}=X+\eta_t^{1[1]}\frac{\partial}{\partial u_t}+\eta_r^{1[1]}\frac{\partial}{\partial v_{1r}}++\eta_t^{2[1]}\frac{\partial}{\partial v_{3t}}+\eta_r^{2[1]}\frac{\partial}{\partial v_{3r}}, \qquad X^{[2]}=X^{[1]}+\eta_{rr}^{1[2]}\frac{\partial}{\partial v_{1rr}}+\ldots. \qquad (16)$$

be the corresponding first and second extended infinitesimal generator for the governed system of PDEs (13-i) and (13-iii).

Where $\eta_t^{1[1]}$, $\eta_r^{1[1]}$, $\eta_t^{2[1]}$, $\eta_r^{2[1]}$, $\eta_{rr}^{1[2]}$ are given by

$$\eta_t^{1[1]}=D_t\eta^1-v_{1t}D_t\phi-v_{1r}D_t\xi\,;\eta_r^{1[1]}=D_r\eta^1-v_{1t}D_r\phi-v_{1r}D_r\xi\,;\quad \eta_t^{2[1]}=D_t\eta^2-v_{3t}D_t\phi-v_{3r}D_t\xi\,;$$

$$\eta_r^{2[1]}=D_r\eta^2-v_{3t}D_r\phi-v_{3r}D_r\xi\,;\quad \eta_{rr}^{1[2]}=D_r\eta_r^{1[1]}-v_{1tr}D_r\phi-v_{1rr}D_r\xi\,. \qquad (17)$$

Where D_t and D_r are the total derivative operators given as

$$D_t=\frac{\partial}{\partial t}+v_{1t}\frac{\partial}{\partial v_1}+v_{1tr}\frac{\partial}{\partial v_{1t}}+v_{1tt}\frac{\partial}{\partial v_{1t}}+v_{3t}\frac{\partial}{\partial v_3}+v_{3r}\frac{\partial}{\partial v_{3tr}}+v_{3tt}\frac{\partial}{\partial v_{3t}}+v_{3tr}\frac{\partial}{\partial v_{3r}}+\ldots\ldots\ldots\ldots,$$

$$D_r=\frac{\partial}{\partial r}+v_{1r}\frac{\partial}{\partial v_1}+v_{1tr}\frac{\partial}{\partial v_{1t}}+v_{1rr}\frac{\partial}{\partial v_{1r}}+v_{3r}\frac{\partial}{\partial v_3}+v_{3t}\frac{\partial}{\partial v_{3tr}}+v_{3rr}\frac{\partial}{\partial v_{3r}}+v_{3tr}\frac{\partial}{\partial v_{3t}}+\ldots\ldots\ldots\ldots, \qquad (18)$$

Therefore one parameter Lie group of transformations (14 to 18) is admitted by the governed PDE's system (11-i and11-iii).iffy

$$X^{[2]}\left(\mu_2 v_{1rr}+\frac{\mu_2}{r}v_{1r}+v_{3r}+\frac{v_3}{r}-\mathrm{Re}\,v_{1t}\right)\Big|_{(11-i\&iii)}=0$$

$$\Rightarrow-\frac{1}{r^2}(\mu_2 v_{1r}+v_3)\xi+\frac{1}{r}\eta^2-\mathrm{Re}\,\eta_t^{1[1]}+\frac{\mu_2}{r}\eta_r^{1[1]}+\eta_r^{2[1]}+\mu_2\eta_{rr}^{1[2]}\Big|_{(11-i\&iii)}=0 \qquad (19)$$

$$X^{[1]}(We\,v_{3t}-\mu_1 v_{1r}+v_3)\Big|_{(17-i\&iii)}=0\Rightarrow\eta^2-\mu_1\eta_r^{1[1]}+We\,\eta_t^{2[1]}\Big|_{(17-i\&iii)}=0 \qquad (20)$$

Where v_{1t}, v_{1r}, v_{1rr}, v_{3t}, v_{3r}, etc, .are partial derivatives.

Where $X^{[1]}$, $X^{[2]}$ and ($\eta_t^{1[1]}$, $\eta_r^{1[1]}$, $\eta_{rr}^{1[2]}$, $\eta_t^{2[1]}$, $\eta_r^{2[1]}$) are defined in the relations (16 and 18). In the above equations, the unidentified functions ϕ, ξ, η^1 and η^2 are independent for the differentials of v_1 and v_3. Thus separating w. r. to the differentials of v_1 and v_3 and powers of the differentials of v_1 and v_3 leads to the two simplified over determined systems of PDE's and after solving these two over determined systems of linear PDE's, the values of the unknown functions ϕ, ξ, η^1 and η^2 are obtained as $\phi=c_1$, $\xi=0$, $\eta^1=c_2 v_1+g_1(t,r)$ and $\eta^2=c_2 v_3+g_2(t,r)$ (21)

Where c_1 to c_3 are arbitrary constants of integration and $g_1(t,r)$, $g_2(t,r)$ are an arbitrary functions of the PDE's of the form

$$\mathrm{Re}\frac{\partial g_1}{\partial t}=\mu_2\frac{\partial^2 g_1}{\partial r^2}+\frac{\mu_2}{r}\frac{\partial g_1}{\partial r}+\frac{\partial g_2}{\partial r}+\frac{g_2}{r} \quad \text{and} \quad We\frac{\partial g_2}{\partial t}=\mu_1\frac{\partial g_1}{\partial r}-g_2 \qquad (22)$$

Thus the symmetry Lie algebra of the system of equations (11-i and iii) is two–dimensional and defined by

$$X_1=\frac{\partial}{\partial t}, \qquad X_2=v_1\frac{\partial}{\partial v_1}+v_3\frac{\partial}{\partial v_3} \quad \text{and} \quad X_m=g_2(t,r)\frac{\partial}{\partial v_1}+g_3(t,r)\frac{\partial}{\partial v_3} \qquad (23)$$

Where 'm' is non-negative integer

The finite transformations related to X_1 to X_2 be written as

$$\begin{array}{lllll} X_1: & t^* = t+\delta_1 & r^* = r & v_1^* = v_1 & v_3^* = v_3 \\ X_2: & t^* = t & r^* = r & v_1^* = v_1\, e^{\delta_2} & v_3^* = v_3\, e^{\delta_2} \\ X_m: & t^* = t & r^* = r & v_1^* = v_1 + h_1(t,r)\,\delta_m & v_3^* = v_3 + h_2(t,r)\,\delta_m \end{array} \quad (24)$$

Where $\delta_1 - to - \delta_2$ are parameters of group.

3.3 Invariant solutions corresponding to the operator X_1 - β X_2

$$X = X_1 - \beta X_2 = \partial/\partial t - \beta v_1\, \partial/\partial v_1 - \beta\, v_3\, \partial/\partial v_3$$

The invariant solutions admitted by the operator X are achieved by solving the characteristic system $\frac{dt}{1} = -\frac{dv_1}{\beta v_1} = -\frac{dv_3}{\beta v_3}$ are given by

$$\left. v_1(t,r) = e^{-\beta t}\,\psi_1(r), \qquad v_3(t,r) = e^{-\beta t}\,\psi_3(r) \right\} \quad (25)$$

Set the above relations (25) into governing equations (11-i) and (11-iii).indicates ODE's of the functions $\psi_1(r)$ and $\psi_3(r)$.

$$\mu_2\,\psi_1''(r) + \frac{\mu_2}{r}\psi_1'(r) + \psi_3'(r) + \frac{1}{r}\psi_3(r) + \beta\,\mathrm{Re}\,\psi_1(r) = 0 \ \text{(i)} \qquad \psi_3(r) = \frac{\mu_1}{(1-\beta We)}\psi_1'(r) \ \text{(ii)} \quad (26)$$

Now, putting $\psi_3(r)$ from (26-ii) into (26-i), the Bessel's differential equation is obtained whose order is zero which is.

$$\psi_1''(r) + \frac{1}{r}\psi_1'(r) + \alpha^2\,\psi_1(r) = 0 \quad (27)$$

Where $\alpha^2 = \dfrac{\beta\,\mathrm{Re}\,(1-\beta\,We)}{(1-\mu_2\,\beta\,We)}$

General result of the Bessel's differential equation (27) is agreed as $\psi_1(r) = A\,J_0(\alpha\,r) + B\,Y_0(\alpha\,r)$ (28)

Where $J_0(\alpha\,r)$ and $Y_0(\alpha\,r)$ are Bessel function of order zero of first and second kind respectively. Of course, equation (27) is singular or when $r = 0$, then $Y_0(\alpha\,r) \to -\infty$. Physically significant solution must be twice continuously differentiable in $0 \le r \le 1$. We must take $B = 0$ and equation (28) has only one bounded solution, i.e. $\psi_1(r) = A\,J_0(\alpha\,r)$

Substitute this value of $\psi_1(r)$ in equation (26-ii), then $\psi_3(r) = \dfrac{-\mu_1\,\alpha}{(1-\beta We)} A\,J_1(\alpha\,r)$ is obtained and after setting the values of functions $\psi_1(r)$ and $\psi_3(r)$, general solutions of the governed PDE's (11-i) and (11-iii) are given as under

$$\left. v_1(t,r) = A\,e^{-\beta t}\,J_0(\alpha\,r) \quad \text{and} \quad v_3(t,r) = \frac{-A\,\mu_1\,\alpha\,e^{-\beta t}}{(1-\beta\,We)}\,J_1(\alpha\,r) \right\} \quad (29)$$

4. CONCLUSION

The point of this paper was to develop mathematical models and to find the analytical solutions of the problems arising in the study of viscoelastic fluid flow in pipes without porous media using with Oldroyd–B Constitutive Model. Lie group method is used to find the analytical solutions of the problems and most imperative research of this paper is to find the accurate results of normal and shear stress components. By using Lie group method and using the suitable symmetry, we reduce system of PDE's in to an ODE's system. We have solved this system of ODE's using proper conditions of boundary corresponding to the Lie-group, so that exact invariant solutions have been establish. Also, the numerical results of the problem of PDE's system are achieved taking on computer by ND–Solver in Mathematica which were compared with analytical solutions and were in best agreement with analytical solutions with porous media in pipes. Lie-group technique can give some practical insights interested in include of results and may support for determining exacting results in several cases. Graph of solutions were presented and talked corresponding to these results. The accuracy of the numerical results may be compared through the developed analytical solutions for real problems of the field. We hope that the results may be helpful for other personnel in the field.

REFERNCES:

Arada N. and A. Sequeira (2003) 'Strong Steady Solutions for a Generalized Oldroyd-B Model with Shear Dependent Viscosity in a Bounded Domain', Mathematical Models in Applied Sciences, World Scientific Publishing Company, Vol. 13, No. 9, 567-672

Abdel-Malek, M. B., N. A. Badran and H. S. Hassan, (2002), Solution of the Rayleigh problem for a power law non-Newtonian conducting fluid via group method, Int. J. Eng. Sci. 40, 1599-1609.

Kwang S. P. and D, K. Young (2009), 'Numerical description of start-up viscoelastic plane Poiseuille flow',Korea Australia Rheology Journal, vol. 21, No. 1, 47-58.

Larson, R. G., (2010), 'The Structure and Rheology of Complex Fluids', Oxford University Press.

Owens. R. G. and T. N. Phillips (2002), 'Computational Rheology' Imperial College Press, 1, 7, 10, 13, 16, 29, 86, 91, 116-122.

Taha S., (2009), 'Modeling the Flow of a Bautista-Manero Fluid in Porous Media', 121, 32-33.

Taha S., (2010). 'Flow of non-Newtonian Fluid in Porous Media, Journal of Polymer Science Part B polymer Physics vol. 446-454

Tan W.C. and T. Masuoka, (2006) Stokes' Velocity overshoot of start-up flow for a Maxwell fluid in a porous half space, Chinese Physics, vol. 15, No. 11, 2644-2650,

Tan W.C and T. Masuoka (2007). 'Stability analysis of a Maxwell fluid in a porous medium heated from below', Physics Letters A, Vol. 360, No.3, 454-460

Vafai, K., (2002), 'Handbook of Porous Media', Marcel Dekker, New York

Van R.G., and T. N. Phillips (2004) 'Spectral element methods for transient viscoelastic flow problems' Journal of Computational Physics', 201 (1) 286 – 314.

Wafo S. C. (2005), 'Invariant solutions of the unidirectional flow of an electrically charged power law non-Newtonian fluid over a flat plate in presence of a transverse magnetic field', Comm. in Nonlinear Science and Numerical Simulation, 10, 537-548.