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Topological Structure of Complex Networks and its Importance in Diffusion

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Abstract: The availability of huge data, due to the tremendous storage capacity of modern computers has allowed the systematic collection, and high processing speed has permitted analysis on that data by researchers on a scale far larger than previously possible. Due to this, complex network formation has been seen and observed in many real and artificial complex systems. As these systems are very large and complex, we cannot get an understanding of these complex systems just by only examining the separate components which constitute these systems. Therefore, modeling the way these components are interconnected in a system is very important for understanding the system as a whole. Further, despite the enormous variation in their components, functions, and sizes, these networks are surprisingly similar in topology, leading to the conjecture that complex systems are governed by the ubiquitous self-organizing principle. In this research, we emphasize on the importance of heterogeneous topological structure of real-world complex networks and its importance in understanding the phenomenon of diffusion in these networks.

Keywords: Scale-free networks; Network models; Power-law,Preferential attachment

I. INTRODUCTION

We have many examples of different complex systems in the world either natural or artificial and these complex systems can be represented through complex networks. Whereas networks can be represented as nodes/vertices which show the elementary units in the systems and links/edges which express connectivity or interaction pattern in between these nodes of the networks. From the last decade in complex networks research a property of scale-free nature has been seen and observed in many complex networks ranging from social networks of friendship to covert network of terrorists and sexual contacts networks to scientific collaboration networks (Newman, 2001). Ramasco, *et al.*, 2004) (Krebs, 2002) (Liljeros, *et al.*, 2001) (Mahesar, *et al.*, 2014) Similarly in the category of technological networks Internet, World Wide Web and router networks (Faloutsos, *et al.*, 1999) (Albert, *et al.*, 1999) has been proved to be scale-free. The researchers have agreed on a point that many of the real world systems when modeled and analyzed as complex networks have similar structural properties i.e. degree distribution, topology, clustering and average path length (Albert, *et al.*, 2012) Therefore the research trend in complex networks is still continue.To model and analyze many real world systems by converting them as complex networks for their better understanding of behavior in terms of nodes and links. It is hoped that the thorough understanding of features including both structural and dynamical about these real world complex networks will greatly help us to understand and simulate the behavior of real world complex systems. Undoubtedly, the research and study on complex networks have answered regarding the immunization in networks against epidemics [9], and network tolerance to attacks (Albert, *et al.*, 2012).

A graph provide a mathematical construct to study how these complex networks are connected and this is the reason that the study of complex networks is very much related with the graph theory. A complex network can be modeled as a graph which can be described by an adjacency matrix. A network is a set of entities known as nodes and with some connection between these nodes called links. Nodes are also called vertices that make up the system and links or edges, show how these nodes are connected. The nodes represent the objects in the system like routers, web pages, people, terrorists, airports, cities and countries whereas links represent the interaction in between them like physical connection, virtual link, friendship, affiliation and flights. The advantages of representing the graph as network are generality and flexibility for representing real or artificial networks including all which undergoes dynamical topology changes in them (**Fig.1**) .

Fig.1: An example of complex network of air transportation where nodes are airports and links are direct flights. (Barrat, 2004) .

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Therefore, complex network is said to be a network with non-trivial topological features, with patterns of connections between their elements that are neither purely regular nor purely random (Soléet *et al.,* 2004):

The remaining paper is structured as follows. In section 2 we discuss two network models based on uniform degree distribution and clustering coefficient point of view. In section 3 we discuss most prevalence type of network model i.e. scale-free network model with preferential attachment concept. Finally, section 4 concludes the paper.

2. COMPLEX NETWORK MODELS

A. Random network model

In 1959, two Hungarian mathematician Pal Erdos and Alfred Renyi introduced the random graph theory and basic model of complex networks (Erdos and Renyi. 1959) They proposed the complex network model which is known as random graph model. According to them in a graph with N number of vertices and L= N (N -1) / 2 possible links, by randomly selecting *g* number of links we can have random network, GN,g. There can be $C^{g}_{\frac{N(N-1)}{2}}$ such networks which can be produced or constructed with *N* vertices and *g* edges. The probability of every network is same. Also, a related and equivalent model defined by these authors known as GN,p model is defined as in a graph G with N nodes, each of the possible N(N-1)/2 links exist with a given probability *p*. We can construct a random graph G from N nodes by selecting g = p. n (n-1)/2 links at random. As the main parameters here are the number of nodes and connection probability *p*. The GN,p model will be the combination of all graphs in which a graph with *g* links occurs with probability p_g $(1-p)^{M-g}$ here M is the total number of possible links in a network of N vertices. The degree distribution of ER model follow binomial distribution as each link occurs with fixed and independent probability and therefore,

$$p_g = \binom{n}{g} p^g (1-p)^{n-g} \qquad (1)$$

Let us suppose, the mean degree of the network is λ= (n-1) p. Now if we take the limit n→∞ while keep the average degree fixed, then equation 1 tends to Poisson distribution,

$$p_g = \lambda^g e^{-\lambda} / g! \qquad (2)$$

Therefore, this model is also known as Poisson random graph. In this model the structure of graph generated totally depends on the value of probability *p*. If the value is low there will be very few links between nodes and when the value becomes closer to one almost all nodes will be connected together. The Poisson distribution implies that in particular the number of nodes with degree *g* decays very rapidly around the average degree, and therefore all nodes have nearly the same degree. In this way, the network will be homogeneously connected with average-path length and low clustering coefficient. **(Fig.2)** shows the process of different connection probabilities in ER model. As the degree distribution follow uniform or bell shape curve, the diffusion in this type of networks takes place very slowly.

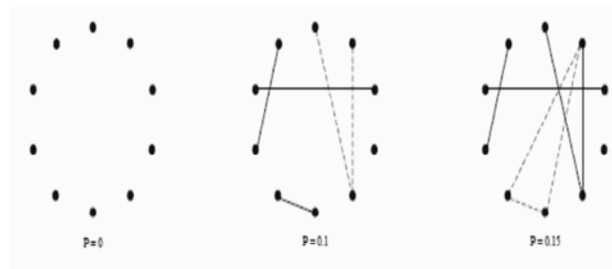

Fig. 2: The process with different probabilities in ER network model.

This model remained very popular from late fifties to late nineties and, the modeling of almost all complex networks was based on random graph theory, given by ER. Only before the last decade due to the advancement in data processing and storage capability of modern computers, researchers observed that many real-world complex networks are not completely random.

B. WS Small world network model

In 1998, Watts and Strogatz introduced the first generic model for complex networks (Watts and Strogatz, (1998) According to this model, if we start using a ring topology with *n* nodes and each node is connected with its *m* next neighbors for a given *m* on each side. Then, if every link of the ring is rewired with some probability p (by excluding self-loops and duplicate links) and by selecting randomly nodes, we get small world network as shown in **(Fig. 3).**

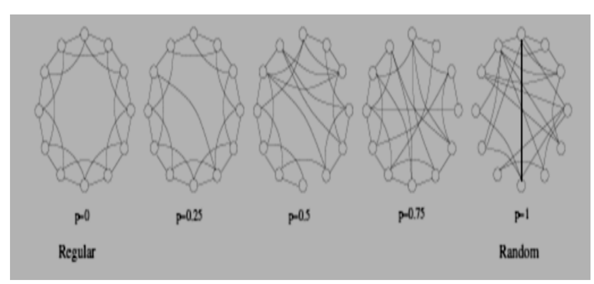

Fig. 3: The random rewiring process of SW model.

They concluded that, if the value of connection probability *p* is very small then the graph is almost a ring due to a few edges is rewired. But in that case clustering is high since every node is connected to its nearest neighbors, and these neighbors are connected together. But, if the value of *p* is high, then most of the links are rewired, and therefore the graph becomes similar to a random graph: with low average distance as well as low clustering coefficient. As for as medium or average values of probability *p* are concerned, the graph become highly clustered with a small average distance in between nodes. This model emphasizes the occurrence of small distance between real-world networks which makes the diffusion of information or traffic in complex network fastest as compared to ER model. The main drawback of this model is the constant number of nodes in the network. On the other hands real-world network evolve with dynamic nature.

3. RESULT AND DISCUSSION

SCALE-FREE NETWORK MODEL

In both models of WS (Watts and Strogatz) and ER (Erdos and Renyi) the node degree distribution shows very different behavior as compared to complex networks of the world. In 1999, Barabasi and Albert analyzed the network of World Wide Web and found that the node degree of WWW does not follow random graph connections rather, it is scale-free graph and its degree distributions follow power-law form as given in the following equation:

$$P(k) \sim k^{-\gamma} \quad (3)$$

where P(*k*) is the probability of node degree distribution and γ is a scaling exponent which is a numerical parameter called connectivity distribution exponent. In fact, γ is a scale-free parameter in the sense that it does not depend on a characteristic scale of the network. This exponent gamma (γ) has been measured as well as confirmed in a number of research studies to be approximately 2.1 as decreasing slope on log-log scale in between nodes and number of links of the nodes in network.

It means, the node degree k and the number of links a node can have, follows the power-law distribution relation. Thus, power-law implies that few nodes in the network can have large number of links whereas majority of nodes have very small number of connections. This result was very generic and further it was observed that many large complex networks follow this power-law distribution. It means that, the probability for any vertex has k edges decreases rapidly if the considered value of k is very large. This finding provided another direction of modeling complex networks, which previously were focused in one dimension of uniform degree distribution. Because the results of research showed the network of World Wide Web have highly right skewed degree distribution, in which web pages were modeled as nodes and hyperlinks were modeled as edges of the network, as compared to Poisson distribution. Moreover, networks formed by scale-free behavior create small average path length in between nodes and have high clustering coefficient.

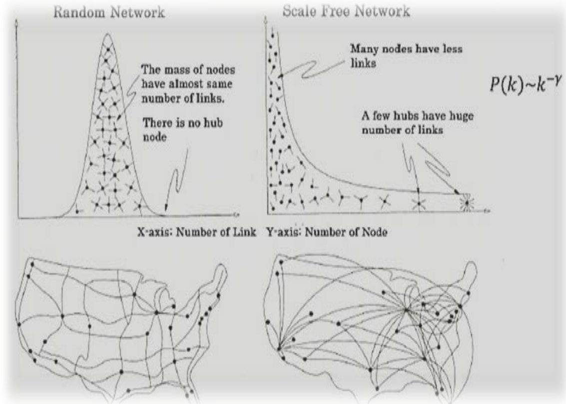

Fig. 4.The degree distribution in scale free and random networks.

According to BA model of preferential attachment, there are two main steps (Barabasi , Albert, (1999)

1) *Growth:* Starting with a small number (m_0) of nodes, at every time step, we add a new node with m ≤ (m_0) edges that link the new node to m different nodes already present in the system.
2) *Preferential attachment:* When choosing the nodes to which the new node connects, we assume that the probability that a new node will be connected to node *i* depends on the degree Ki of node *i*, such that

$$\prod_i = \frac{k_i}{\sum_j k_j} \quad (4)$$

where k_i is the degree of node *i* and k_j is the sum of the degrees of all nodes in the network.

After t time steps, there are

$$N(t) = \lim_{t\to\infty}(t + m_0) \quad \text{nodes} \quad (5)$$

And

$$L(t) = \lim_{t\to\infty}(mt) \quad \text{links} \quad (6)$$

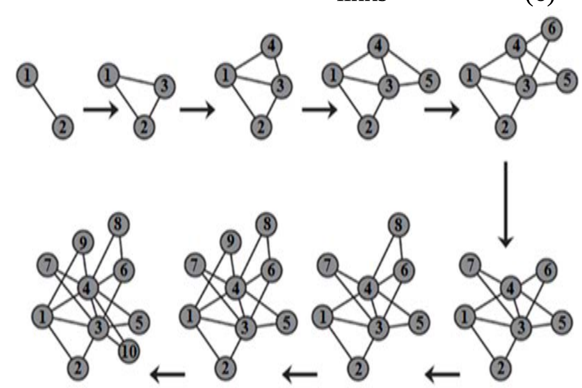

Fig. 5: The formation of a scale-free network as an effect of the preferential attachment.

The BA model became very popular in the field of complex networks since its publication, and from last decade a number of researchers improved and extended the model to better understand the behavior of complex networks. According to BA model, as network grows new node try to connect with established nodes present in the network. This feature is called as preferential node attachment and has been observed in many growing networks like technological, social, biological, transportation and many fields or domains. The concept of preferential attachment is shown graphically in **(Fig. 5)**. This feature gives rise to the power- law or "rich get richer" phenomenon which means that already established attract more and more connections in the growth process of network. Due to this preferential attachment few giant nodes appear in the topology of networks and they make the diffusion of information very efficient as compared to ER and WS (Small World) models. Few real-world networks take advantage of this phenomenon and diffuse the information in networks efficiently by using giant nodes. Similarly, this feature make the few networks very vulnerable under targeted attacks for instance, the spreading of computer virus and dieses epidemically.

4. CONCLUSION

Many real-world systems and phenomenon shows various different patterns of complex interactions among underlying components. The interaction and communication of different components in these networks depends mainly on the underlying topology of the structures. In this research we have analyzed three different models of complex networks which have remained very popular in the field of complex networks.

The results of comparison between these models show that the BA model depicts very close behavior of real-world growing networks due to the preferential attachment concept. Whereas, in other two models the connection probability shows homogeneous behavior which can be the reason of less robust and less efficient diffusion of information in complex networks. The obvious reason of this deficiency is the absence of "hubs" nodes in networks due to homogenous degree distribution. In future, the extended models of BA and the concept of fit get better and winners take all can be analyzed from the WS and ER models.

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