



A novel approach to minimize Peak-to-Average Power Ratio in OFDM using Tone Reservation methodology

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**Abstract:** Orthogonal Frequency Division Multiplexing (OFDM) has the drawback of having the Peak to Average Power Ratio (PAPR) because of the additive power of the various subcarriers. This paper provides an in depth analytical and graphical results of the Tone Reservation (TR) Method for minimization of PAPR in OFDM. TR involves the holding back of some subcarriers which are not used in the OFDM transmission to lower the PAPR. These subcarriers are named Peak Reduction Carriers (PRCs). The idea of TR is to determine appropriate values of these PRCs to bring down the PAPR of the transmitted signal to a minimum. In this regard, the prevalent techniques of TR for PAPR minimization work well but require that possible sequences of PRC sets are checked which increases the computational complexity.

**Keywords:** Peak-to-average power ratio (PAPR), Tone Reservation (TR), Peak Reduction carrier (PRC), Orthogonal Frequency Division Multiplexing (OFDM), Signal to Clipping Noise Ratio (SCR) and Cyclic Prefix (CP).

1. **INTRODUCTION**

Wireless digital communications is an ever increasing phenomenon thereby requiring structures that are dependable and extremely efficient. Orthogonal Frequency Division Multiplexing (OFDM), allows for efficient use of the available spectrum, tolerance in multipath delay and robustness to fading. It has therefore been used for high speed communication as well as it is a part of several standards.

As a result it has been chosen for high data rate communications, Digital Video Broadcasting (DVB), Digital Video Broadcasting Terrestrial (DVB-T) and mobile Worldwide interoperability for Microwave Access (Wi-MAX) based on OFDM access technology (Jiang., *et al.*, 2007) Digital Signal Processing has helped develop recent interest in this technique and it has found use in many international standards such as IEEE 802.11, IEEE 802.16, IEEE 802.20, European Telecommunications Standards Institute (ETSI) Broadcast Radio Access Network (BRAN) committees, and high-speed digital subscriber lines (HDSL, ADSL, and VDSL) (Gouba., and Louët, 2013).

Even with its many advantages, OFDM suffers from power issues i.e. the largest power value during an OFDM transmission can be equal N times the average power of the signal (N being the number of carriers used in the signal). This is a major drawback for its use as it results in distortions in the output. To circumvent this, one solution is to reduce the power being transmitted thus reducing the PAPR of the signal. Having a lower PAPR allows for a higher average power to be sent for a

fixed value of peak signal power. This increases the signal to noise ratio.

To achieve this, a number of methods have been given (Lim, *et al.*, 2009).. These techniques are clipping (Zhu, *et al.*, 2013) clipping and filtering (Shiragapur, *et al.*, 2013), coding (Daiming, *et al.*, 2014) tone reservation (TR) (Deumal, *et al.*, 2011), (Tao *et al.*, 2014) and active constellation extension (ACE) (Jing *et al.*, 2013) Other methods having discrete solutions such as tone injection (TI) (Damavandi *et al.*, 2013) and multiple signal representation techniques such as partial transmit sequence (PTS) (Jun 2011) selected mapping (SLM) (Chackochan., 2011) (Tao, 2013) and inter leaving (Chih-Peng, *et al.*, 2012) have also been proposed.

The results presented in this paper show that the given method improves upon the TR technique in many areas while being computationally cheap. This paper is organized as follows:

In section 2, provides an introduction to OFDM systems. In section 3, we present our motivation for the tone reservation scheme for PAPR reduction of OFDM System. In section 4 we demonstrate the performance of TR method and gain reduction. In section 5 the simulation results and analysis are shown. A brief conclusion is given in section 6.

2. **OFDM SYSTEMS**

2.1 **OFDM System Architecture**

The block diagram shown in (Fig. 1) depicts an OFDM transmission system. Starting from the

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transmitter, the input to it is a serial binary data sequence which is directly fed to the channel encoder. From here it is passed on for constellation mapping and then to the serial to parallel (S/P) converter. In here, the single signal is broken in to  $N$  parallel routes after the operation of the Inverse Fast Fourier Transform (IFFT). The  $N$  data routes are individually modulated by the orthogonal subcarriers. All these points together form a single OFDM symbol.

In the next step, the parallel data stream is transformed back to a serial stream and a cyclic prefix (CP) is formed. This is done by copying the last  $L$  samples of one symbol to the front. Once this is done, the sequence of symbols arrives at the transmitter once they have been converted in to analog form. Following this, radio frequency (RF) modulation is performed.

The reception of the OFDM symbols is opposite to that of the transmission. The receiver is used to recover the information within these symbols. Here, first the received signal is down sampled and then demodulated. Once the demodulation is done, the signal is sent to an analog to Digital (A/D) converter, where the output is sampled and timing estimation is used to determine the location of the first OFDM symbol.

Following this, the cyclic prefix which was added to the signal is removed and an  $N$ -point Fast Fourier Transform (FFT) is conducted on the remaining sample points to get data back in the frequency domain. The result of this operation is forwarded to the channel decoder which fetches the data back.

Subcarriers modulated by various constellation mappings, make up an OFDM symbol. The mapping operation can be brought about using Phase Shift Keying (PSK) or Quadrature Amplitude Modulation (QAM). Consider an OFDM system with  $N$  subcarriers with a high-speed binary serial stream at the input side being denoted as  $\{a_i\}$ . The serial to parallel conversion (S/P) following by the mapping by the constellation forming the parallel signal sequence  $\{d_0, d_1, \dots, d_{N-1}\}$  where  $d_i$  is a discrete complex-valued signal.

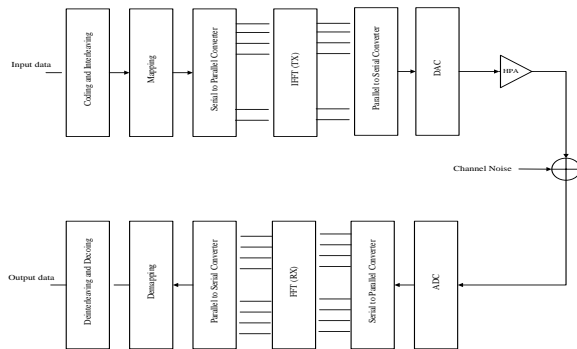


Fig. 1. Block diagram of OFDM system architecture

Each  $d_i \in \pm 1$  for a BPSK mapping. However, when a QPSK mapping is used, each member of the signal is provided to the  $N$  subcarriers with  $e^{j2\pi f_0 t}, e^{j2\pi f_1 t}, \dots, e^{j2\pi f_{N-1} t}$  being used for modulation purposes, respectively. At the end, all the modulated signals are combined to construct on symbol of the OFDM. The use of discrete Fourier transform helps in making the OFDM system less complex.

The receiver of an OFDM system comprises of a number of decoders which move various carrier frequencies to zero frequency and also integrate them over the period of one symbol. Due to the sub-carrier being orthogonal to each other, a specified subcarrier is demodulated at any time without impacting the result of the integration.

For an OFDM signal, the frequency can be written as given in equation (1).

$$f_i = f_c + i. \Delta f \quad (1)$$

where  $f_c$  stands for carrier frequency,  $\Delta f$  is the smallest interval between different sub-carrier frequencies.  $\Delta f$  can be found as given in equation (2).

$$\Delta f = \frac{1}{T} = \frac{1}{N t_s} \quad (2)$$

where  $t_s$  is time interval of symbol sequence  $\{d_0, d_1, \dots, d_i, \dots, d_{N-1}\}$ . The output signals from an OFDM system are generally specified by a complex base equivalent signal. This signal can be expressed as in (3).

$$s(t) = \sum_{i=0}^{N-1} d_i e^{j2\pi \Delta f t} = \sum_{K=0}^{N-1} d_i e^{j2\pi \frac{1}{T} t}, t \in \{0, T\}, \quad (3)$$

In this equation, the real part of the complex factor corresponds to the in phase components of the OFDM symbol and the imaginary part corresponds to the quadrature component.

When the subcarriers reach the receiver, these are fed to the input in order to demodulate them. The method for demodulation is the product of the output signal and  $k$ -th demodulation carrier expression  $exp(-j\pi(2K - N) t/T)$  is calculated, this result is then integrated over the time period of one OFDM symbol  $T$ . After this process is completed, the resulting signal is the signal to be transmitted for each of the  $k$ -th subcarriers. As is understood, the IFFT and FFT can be used for modulation and demodulation of OFDM signals respectively. During these processes, data in the “frequency domain” is converted in to “time domain” through the  $N$ -point IFFT process. These are then transmitted using the wireless channel after appropriate modulation.

### 2.2PAPR of the OFDM Signal

For computing thePAPR of a signal  $s(t)$  over the time interval  $[0, T]$ , we can use the following equation:

$$PAPR = \frac{\max |s(t)|^2, 0 \leq t \leq T}{1/T \int_0^T |s(t)|^2 dt} \quad (4)$$

Being the most popular parameter for determining the dynamic range of the OFDM signals in the time domain as well as the variation in its envelope also called the crest factor (CF), PAPR can be used for this purpose as

Crest Factor,  $C.F = \sqrt{PAPR}$  where  $PAPR = CF^2$ (5) Another parameter which is usually utilized is the crest factor and is given by the square root of the PAPR.

$$PAPR_{max} = 10 \log(N)db \quad (6)$$

Take for example, the  $N$  length data sequence given in (2) has a time interval of  $[0,T]$  and is taken to be modulated data for the multicarrier OFDM system.

$$A = (A_0, A_1, \dots, A(N-1)) \quad (7)$$

The central limit theorem allows us to consider an OFDM signal with many components to be taken as Gaussian, therefore, the in b phase (I) component and the Quadrature (Q) component is also Gaussian. This infers that the signals envelope is subject to chi-square distribution having two degrees of freedom. Consequently, the PAPR of this continuous signal tends to approximate that of a discrete version of this signals as the frequency of sampling is increased. Following this logic, the PAPR of a signal oversampled by a factor of four can be taken as being approximately equal to the PAPR of the continuous signals. This makes the Complementary Cumulative Distribution Function (CCDF)

$$(CCDF(\delta) = P_0\{PAPR > \delta\}) \quad (8)$$

To be defined as the  $4N$  chi-square distributed variables that do not exceed the given level  $\delta_0$ :

$$P(PAPR > \delta_0) = 1 - \{1 - \exp(-\delta_0)\}^{4N} \quad (9)$$

### 3. MOTIVATION FOR USING TONE RESEVATION

There have been many methods proposed for decreasing the PAPR values during an OFDM transmission. Of all the presented techniques, Tone Reservation is considered to be an effective technique to achieve this task. Each method of TR has its own benefit and suitable for use in a certain scenario.

All the approaches used here to apply the tone reservation technique have their own benefits, so that each can be used in different condition. If the complexity is not an issue then Signal to Clipping Noise Ratio (SCR) Gradient TR iterative algorithm or Adaptive Scaling TR algorithm can be used that give reasonable PAPR reduction. On other hand if system cannot be offered much complexity, then other algorithm like Gaussian pulse based TR is suitable. Active set TR Algorithm can be used where there is a need to attack only high peaks, the complexity is bit higher but techniques is reasonable as not all the symbols samples are checking.

The simplest of the methods for the reduction of Peak-to-Average Power Ratio (PAPR) are Clipping and Companding, which rely on literally clipping the amplitude of the multicarrier signal. These procedures have a number of shortfalls such as in-band distortions and noise amplification which result in Bit Error Rate (BER) degradation. Another class of PAPR reduction techniques includes multiple signal representation methods such as Partial Transmit Sequences (PTS) and Selective Mapping (SLM). These techniques do not degrade the BER performance but are computationally quite expensive. BER performance is improved due to Tone Reservation (TR). Some subsets of subcarriers are exploited by the TR for controlling PAPR.

No internal separation method is applied in TR to eventually calculate effective PAPR. All of the measurements are computed at Transmitter side without involvement of the receiver. Recent counterpart communication systems comprising 3G and 4G standards like LTE-Advanced its predecessor, and Wireless Interoperability for Microwave Access (WiMAX) utilize TR for this purpose. Distinguished sided quality of TR is industrious BER enhancement i.e. since TR does not control information subcarriers accordingly it doesn't outcome in any omission to it.

### 4. PERFORMANCE OF TONE RESERVATION METHOD AND PAPR REDUCTION GAIN

#### 4.1 Tone Reservation Method

The technique of Tone Reservation is based on combining PAPR reduction signals in the time domain with the time data signals so as to decrease its peak value. The generation of these PAPR reduction signals requires that there should be an agreement on the subset of tones used in the generation of these signals between the transmitter and receiver since these are not used during the signal transmission process. In the TR technique, a signal  $m$  is added to the original signal  $r$  in the time domain so as to decrease the PAPR.  $\{m = m_n | n = 0, 1, \dots, N-1\}$  is used to denote complex symbols during the tone reservation process at reserved

tones. The resulting vector of data thus become  $sr + m$  after this process and new modulating signals are generated as

$$\tilde{R} = IFFT(r + m) = R + M \quad (10)$$

Original time domain signal =  $r$ . Both transmitter and receiver reserve a subset of tones for PAPR reduction signal =  $m$  Signal after TR processing =  $r + m$ . Signal with low PAPR =  $\hat{r}$  But here IFFT ( $m$ ) =  $M$  and IFFT ( $r$ ) =  $R$

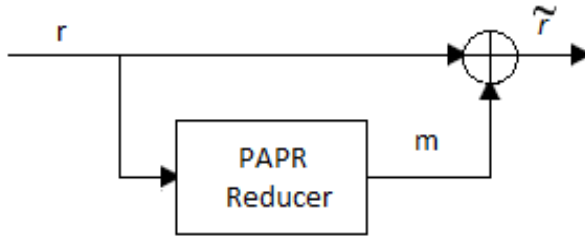


Fig. 2. Block diagram of TR approaches for PAPR reduction

where  $M = IFFT(m)$ . As is apparent, the prime aim of TR is to determine an appropriate  $m$  so as to make the vector  $\tilde{R}$  have a low value of PAPR. Here it is necessary to mention that the value of  $m$  can be determined by solving a convex optimization problem. This process is of high computational complexity. Consider a signal  $m[t]$  is added to the data signal  $r[t]$  to decrease the PAPR.

In order to achieve this, in Tone Reservation, the signal is given to subcarriers which are reserved and not used for transmission of data or already preserved as in (11).

$$M_k + R_k \begin{cases} M_k & k \in S \\ R_k & k \in S \end{cases} \quad (11)$$

Where the data symbol  $M_k$  and the symbol  $R_k$  used to reduce PAPR are located exclusively, the index of the subcarrier is  $S$  for the data transmission. An advantage of the Tone Reservation technique is that it does not require any side information and any receiver operation.

#### 4.2 PAPR Reduction Gain

The reduction gain of the PAPR is found by subtracting PAPR with PAPR reduction from PAPR without PAPR reduction. Therefore  $G_{PAPR}$  means the amount of peak reduction in dB.

$$G_{PAPR} = \frac{\max_{0 \leq t \leq N-1} [|r[t]|^2]}{\max_{0 \leq t \leq N-1} [|r[t] + m[t]|^2]} \quad (12)$$

### 5. SIMULATION RESULTS

In order to validate our scheme proposed in this paper we have conducted few simulation experiments, having different no of carriers for PRC observations. To

check flat signal observation we have considered 12 subcarriers, consequently we decide  $+r1-r2+r3$  and  $+r4$  as a PRC and transmit on  $x1: x2$ , that results  $x1: x2$  summation to create a peak, while  $r4$  creates an anti-peak, which concludes that the exact ant peak or results output to be a flat signal from which data cannot be retrieved.

Similarly (Fig. 3 through Fig. 18) compares the TR OFDM signal with 12 subcarrier and 4 peak cancellation simulation results. The blue line represents the original value of PAPR in dB, whereas red line indicates the result after PAPR reduction in dB. Table 1 presents the summary of the results presented in figs. 3 to 18, depicting the significant reduction in PAPR.

Table.1 shows the different PRC Sum Variable Combinations

Figure #	Peak Cancellation Sum Variable	Carrier Sum Calculation	PAPR level (dB)
Fig. 3	sum1	-r1-r2-r3-r4	2.45
Fig. 4	sum2	-r1-r2-r3-r4	1.4
Fig. 5	sum3	+r1+r2-r3-r4	1.1dB
Fig. 6	sum4	-r1-r2+r3+r4	0.7
Fig. 7	sum5	-r1-r2-r3+r4	0.7
Fig. 8	sum6	-r1+r2+r3+r4	0.8
Fig. 9	sum7	+r1-r2-r3-r4	0.4
Fig. 10	sum8	+r1-r2-r3+r4	0.7
Fig. 11	sum9	+r1+r2+r3-r4	1.1
Fig. 12	sum10	-r1+r2-r3+r4	0.9
Fig. 13	sum11	+r1+r2+r3-r4	1.1
Fig. 14	sum12	+r1-r2+r3+r4	1.3
Fig. 15	sum13	+r1+r2-r3+r4	1
Fig. 16	sum14	-r1+r2-r3-r4	0.25
Fig. 17	sum15	-r1-r2+r3-r4	0.2
Fig. 18	sum16	-r1+r2+r3-r4	0.3

(Table-1) eventually provides use of TR method results to accomplish that PRC-4 with combination  $+r1+r2+r3$  and  $r4$  provides the greatest reduction of PAPR to be 2.45 dB, from all experimented combinations.

### 6. CONCLUSIONS

This paper presents simulation results of PAPR reduction using the TR methodology. The proposed methodology can be utilized for the OFDM based system. Furthermore, it makes no BER degradation, no interference to different subcarriers and does not require any extra processing at the receiver. Even though, only one subcarrier is reserved in the proposed method, and it reduced the maximum PAPR gain to be 2.45 dB. It can be seen that the proposed technique is very useful and offers advantages over the other techniques to reduce the PAPR efficiently.

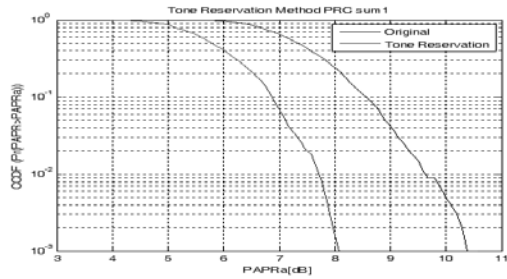


Fig. 3 TR-OFDM signal for -r1-r2-r3-r4 combination

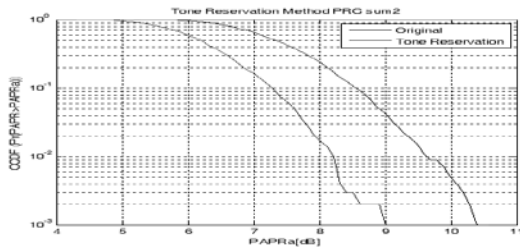


Fig. 4 TR-OFDM signal for -r1-r2-r3-r4 combination

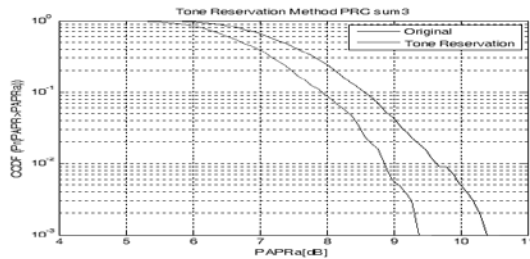


Fig. 5 TR-OFDM signal for +r1+r2-r3-r4 combination

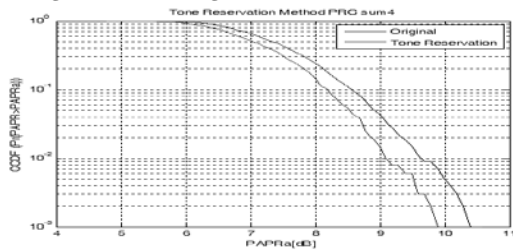


Fig. 6 TR-OFDM signal for -r1-r2+r3+r4 combination

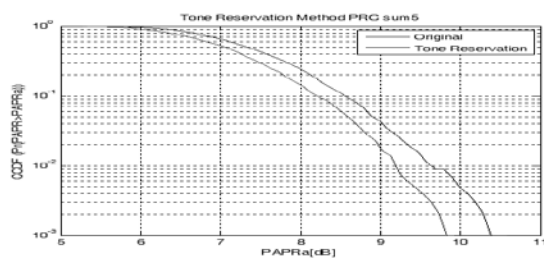


Fig. 7 TR-OFDM signal for -r1-r2-r3+r4 combination

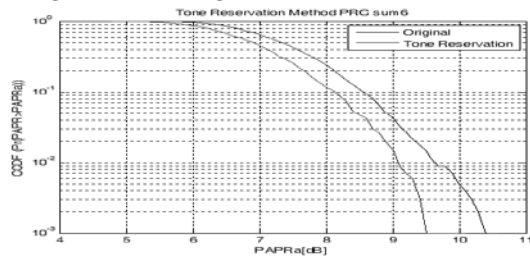


Fig. 8 TR-OFDM signal for -r1+r2+r3+r4 combination

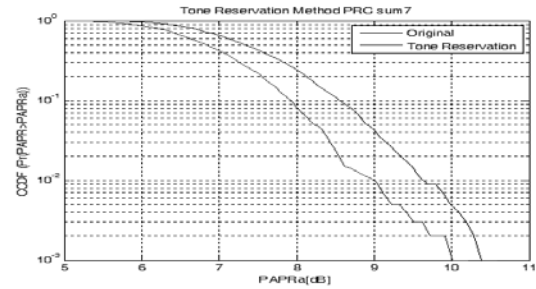


Fig. 9 TR-OFDM signal for +r1-r2-r3-r4 combination

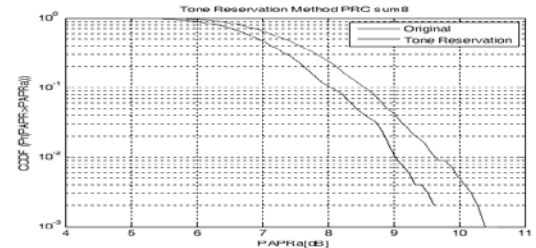


Fig. 10 TR-OFDM signal for +r1-r2-r3+r4 combination

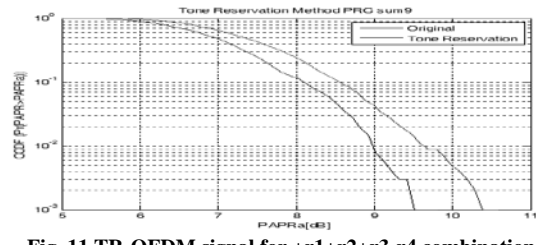


Fig. 11 TR-OFDM signal for +r1+r2+r3-r4 combination

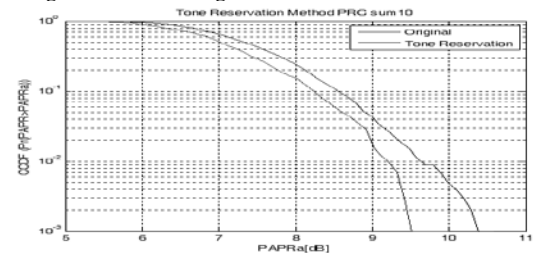


Fig. 12 TR-OFDM signal for +r1+r2-r3+r4 combination

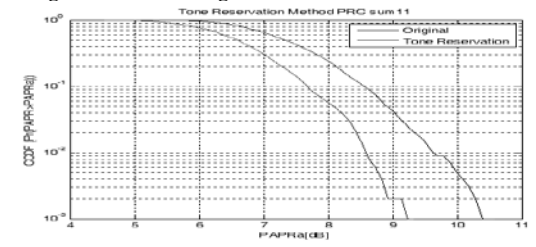


Fig. 13 TR-OFDM signal for +r1+r2+r3-r4 combination

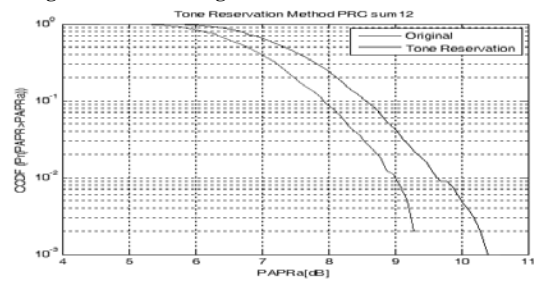


Fig. 14 TR-OFDM signal for +r1-r2+r3+r4 combination

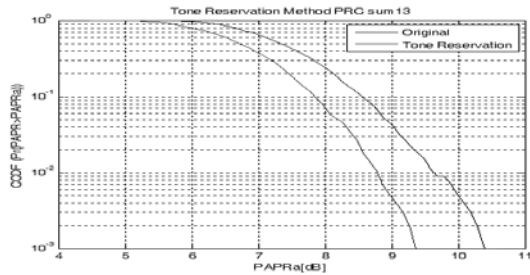


Fig. 15 TR-OFDM signal for +r1+r2-r3+r4 combination

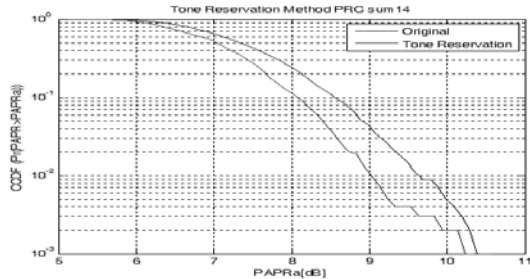


Fig. 16 TR-OFDM signal for -r1+r2-r3-r4 combination

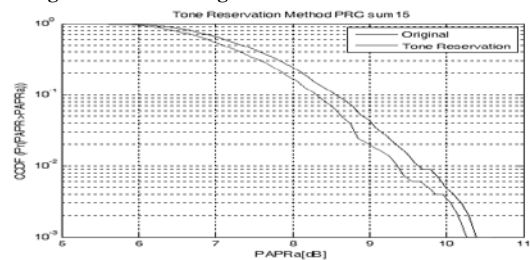


Fig. 17 TR-OFDM signal for -r1-r2+r3-r4 combination

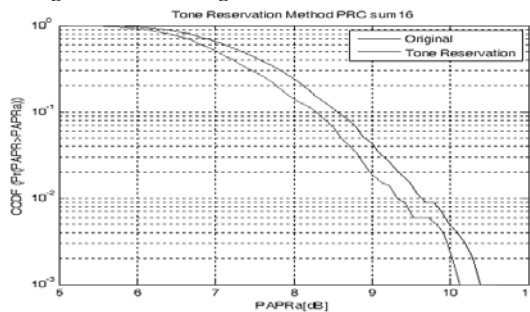


Fig. 18 TR-OFDM signal for -r1+r2+r3-r4 combination

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