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Application of Viscoelastic Fluid Flow through Porous Channels coupled with Oldroyd–B Constitutive Model: Analysis of Viscoelastic Stresses

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Abstract: This paper associated with Applications of Viscoelastic Flow through Porous Channels for statics related with viscoelastic stresses. The related problem is organized by means of Darcy-Brinkman model coupled with Oldroyd–B Constitutive Model.The solution is found by using the symmetries for the system through symmetry method. Numerical predictions are creating adopting solver ND–Solve function in Mathematica.

Keywords: Viscoelastic, Darcy-Brinkman model, Oldroyd– B constitutive model, Symmetry method.

1 INTRODUCTION

In the field of rheology Modeling for flow of viscoelastic fluids is imperative for foreseeing and understanding the manners of procedures and therefore for designing most favorable flow configurations and used for choosing operating situations. As difficult nature of these fluids there is not only one constitutive equation accessible in the literature which describes the properties of flow related with all non-Newtonian fluids. For this reason a range of models have been recommended and among those models, power-law and differential type fluids have acquired a great deal of attention. Some relevant contributions dealing with this type of fluids related with some essential investigation are prepared by way of Fetecau and Fetecau, (2005, 2007), Tan and Masuoka (2005), and Taha Sochi (2009, 2010) and others.

Those fluids which generally describe a linear relationship between the stress and the strain rate are said to be Newtonian fluids or giving rise to a constant viscosity, is always characterized to be a Newtonian fluid. In laminar flow, stress τ is directly proportional to strain rate $\dot{\gamma}$ that is

$$\tau = \mu \dot{\gamma} \qquad (1)$$

Here for a given fluid system, μ is the viscosity as it has an effect on other physical parameters, such as pressure and temperature and even as it is independent of the strain rate. Hence water, air and honey are example of these fluids. Figure used for a Newtonian flow related with a stress in opposition to strain rate will exist a straight line through the starting point. See Taha Sochi (2009, 2010), Dorodnitsyn [2004] and others.

The governing equation to model the flow of viscoelastic fluids through porous media, recently developed by Tan and Masuoka (2005), adapts to Darcy–Brinkman model.

There are several constitutive equations which predict qualitatively the behaviour of some of the material functions, requiring only a small number of free constants. Some of these constitutive equations, however, are complicated. The rheological properties of materials are of greatest interest. The rheological equations of state and material derivatives are described for viscoelastic, generalized inelastic non-Newtonian fluids and fibre suspensions. Here viscoelastic behaviour will be modelled by the Oldroyd-B (Oldroyd (1958) and Phan-thien Tanner (PTT) (1977) differential constitutive models.

The result of problems concerned with realistic fluid flow solved with Lie Group techniques has obtained mounting concentration during current years. Lie-Group theory of ordinary and partial differential equations as a scientific branch created from efforts of the exceptional mathematician Lie of the nineteenth century (1842–1899) Bluman and Kumei (1989),Olver,1986), Ibragimov, (19990, Basov, 2004) and others.

The symmetries for solving the differential equations are imperative and greater identified than the equations since basic physical laws have been reflected in these symmetries. Thus, when discrediting, or otherwise modifying dynamical equations, it is of interest to preserve the original symmetries. Partial differential equations, such as convection Newtonian, non- Newtonian problems are known and then we hope that using the symmetries method, we can solve the equation along with other cases.

Symmetry method has most helpful and impressive property that its symmetries map solutions to solutions and also reduce the PDE's (partial differential equations) into ODE's (ordinary differential equations) The analytical solution is found under the symmetries for the system through symmetry method. The governing system of partial differential equations

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(PDEs) is reduced in to a system of ordinary differential equations (ODEs)) applying appropriate boundary conditions corresponding to the symmetry group, so that exact invariant solutions can be achieved. Numerical predictions of the system are found adopting solver ND–Solve function in Mathematica.

Section 2 deals with the formulation of the problem. Section 3 related with invariant solutions of viscoelastic flow in channel filled with porous media, section 3.1 linked with Study State Solution for non-homogeneous equation, section 3.2 includes Lie for the partial dalgebra ifferentials equations 10(*i*) and 10(*iii*). Lie-point symmetries are presented in sections 3.3, section 3.4 connected with results concerned with invariant solution related to the operator X_1 - βX_3. Section 4 related with analysis of viscoelastic stresses, section 4.1 concerned with analytical solution of normal and shear stresses, section 4.2 associated with steady state solution of normal and shear stresses, 4.3 contains numerical solution of normal and shear stresses and section 5 concerned with conclusion of the problem.

2 PROBLEM FORMULATIONS

Consider the incompressible laminar flow of viscoelastic fluid through a porous channel. The system of equations of flow includes of the conservation of momentum and conservation of mass transport paired with the Oldroyd–B constitutive model. The flow of viscoelastic fluids through porous media is supposed to be isotropic and homogeneous. The momentum equation can be modelled by using Darcy-Brinkman model and in the absence of body force; equations of continuity and momentum may be defined in the following form:

$$\nabla.\bar{u} = 0 \qquad (2)$$

$$\frac{\rho}{\varepsilon}\frac{\partial \bar{u}}{\partial t} = \frac{1}{\varepsilon}\nabla([2\mu_2 \underline{\underline{d}}] + \tau) - \nabla p - \rho\,\bar{u}.\nabla\bar{u} - \frac{\mu}{K}\bar{u} \quad (3)$$

The Oldroyd–B constitutive equation describes the viscoelastic stresses in the flow can be stated as below:

$$\lambda\frac{\partial \tau}{\partial t} = [2\mu_1 \underline{\underline{d}}] - \tau - \lambda\{\bar{u}.\nabla\tau - \nabla\bar{u}.\tau - (\nabla\bar{u})^T.\tau\} \quad (4)$$

In the above equations, u is the velocity vector field of flow, τ is the extra stress tensor, $\underline{\underline{d}}$ is the rate of strain tensor, ∇ is the spatial differential operator, p is the isotropic fluid pressure (per unit density) and t is the time. Fluid density is denoted by ρ, whereas λ is the relaxation time of the viscoelastic fluid and K is the intrinsic permeability of the porous medium. The μ_1 and μ_2 are respectively the viscoelastic solute and Newtonian solvent viscosities. Total viscosity μ of the viscoelastic flow is $\mu = \mu_1 + \mu_2$ and is taken constant. In the above equation (2), the acceleration co–efficient tensor is assumed to be $1/\varepsilon$ and ε is porosity of porous media.

The equations are derived which govern the unsteady unidirectional flow of viscoelastic fluid through porous media adopting Oldroyd–B constitutive model. For unidirectional flow the velocity field is $\bar{u} = (0,\, v(t,y), 0)$; here the description of velocity automatically gives pleasure to the incompressibility state. The derivation of such equations by using the momentum transport equation of viscoelastic fluid and Oldroyd–B constitutive equations assuming constant pressure gradient and may be stated in the absence of body force and the governing system of equations is defined in the following dimensionless form.

$$\mathrm{Re}\,\frac{\partial v}{\partial t} = 1 + \mu_2\frac{\partial^2 v}{\partial y^2} + \frac{\partial \tau_{12}}{\partial y} - \frac{1}{Da}v \qquad \text{(i)}$$

$$We\,\frac{\partial \tau_{11}}{\partial t} = 2We\;\tau_{12}\frac{\partial v}{\partial y} - \tau_{11} \qquad \text{(ii)}$$

$$We\,\frac{\partial \tau_{12}}{\partial t} = \mu_1\,\frac{\partial v}{\partial y} - \tau_{12} \quad \text{(iii) Where } v(y,t), \quad (5)$$

$\tau(y,t)$ and y are dimensionless velocity, stress tensor and transversal coordinates and t is the non-dimensional time. The dimensionless Reynolds number (R*e*), Weissenberg number (*We*) and Darcy's number (*Da*) are defined as

Re = $\rho\, L\, Vc / \mu$, We = $\lambda^* = \lambda\, Vc$ /L and $Da = K^* / \varepsilon\, L^2$ respectively.

In which K is the non-dimensional modified permeability of the porous medium. Whilst, L is the characteristic length taken as half width of the channel and Vc is the characteristic velocity assumed as reference axial velocity $Vc = \varepsilon\, L^2\,(-\partial p/\partial x)/\mu$, It is necessary to prescribe initial and boundary conditions for to complete the well posed problem specification,. So initial conditions are given as:

$v(0, y) = 0,\ \tau_{11}(0,y) = 0,.\ \tau_{12}(0,y) = 0,.$ when $t > 0$ (6)

and boundary conditions are taken as:

$v(t, -1) = 0$ and $v(t, 1) = 0$, when $-1 \le y \le 1$ (7)

3 Invariant solutions of viscoelastic flow in channels filled with porous media

3.1 Steady State Solution for non-homogeneous equation

For non–homogeneous equation (4(*i*)), first we find the study state solution equation, suppose that

$$v(t, y) = u_1(t, y) + \psi_1(y)$$
$$\tau_{11}(t, y) = u_2(t, y) + \psi_2(y)$$
$$\tau_{12}(t, y) = u_3(t, y) + \psi_3(y) \qquad (8)$$

Putting these values in Equation (8), and separating the like terms of two independent variables which give the two systems of equations. First system subject to boundary conditions $\psi_1(-1)=0,\ \psi_1(1)=0$ and after integration system result and applying the boundary conditions admit the steady–state solutions

$$\psi_1(y)=Da\ (1-\frac{Cosh\ \frac{y}{\sqrt{Da}}}{Cosh\ \frac{1}{\sqrt{Da}}}),\quad \psi_2(y)=2We\,\mu_1\,Da\ \frac{Sinh^2\ \frac{y}{\sqrt{Da}}}{Cosh^2\ \frac{1}{\sqrt{Da}}}$$

$$\psi_3(y)=-\mu_1\ \sqrt{Da}\ \frac{Sinh\ \frac{y}{\sqrt{Da}}}{Cosh\ \frac{1}{\sqrt{Da}}} \tag{9}$$

and second system which is PDE's is given as

$$\left.\begin{aligned}
&\mathrm{Re}\ \frac{\partial u_1}{\partial t}=\mu_2\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial u_3}{\partial y}-\frac{1}{Da}u_1 && (i),\\
&We\ \frac{\partial u_2}{\partial t}+u_2=2We\ \{u_3+\varphi(y)\}\frac{\partial u_1}{\partial y}+2We\ \phi'(y)u_3 && (ii),\\
&We\ \frac{\partial u_3}{\partial t}+u_3=\mu_1\frac{\partial u_1}{\partial y} && (iii)
\end{aligned}\right\} \tag{10}$$

Subject to initial and boundary conditions is given as

$u_1(t,-1)=0,$ (i) and $u_1(t,1)=0,.$ (ii) when t > 0

$$u_1(0,y)=-\ Da\ (1-\frac{Cosh\frac{y}{\sqrt{Da}}}{Cosh\frac{1}{\sqrt{Da}}}),. \quad \text{(iii)} \qquad u_2(0,y)=-2We\,\mu_1\,Da\ \frac{Sinh^2\frac{y}{\sqrt{Da}}}{Cosh^2\frac{1}{\sqrt{Da}}},\quad \text{(iv)} \tag{11}$$

$$u_3(0,y)=\mu_1\ \sqrt{Da}\ \frac{Sinh\frac{y}{\sqrt{Da}}}{Cosh\frac{1}{\sqrt{Da}}}$$ (v) when $-1\le y\le 1$ and $\mu_1+\mu_2=1$

3.2 Lie algebra for the partial differentials equations

Once symmetry Lie algebra of the differential equation is recognized, it can be used in the study of transformations that will reduce the equation to simpler form and it is powerful method in finding analytical solutions of differential equations. In this section, symmetry conditions and method for finding the Lie point symmetries of the equations (10(*i*)) and (10(*iii*)). The operator

$$X=\varphi(t,y,u_1,u_3)\frac{\partial}{\partial t}+\xi(t,y,u_1,u_3)\frac{\partial}{\partial y}+\eta^1(t,y,u_1,u_3)\frac{\partial}{\partial u_1}+\eta^2(t,y,u_1,u_3)\frac{\partial}{\partial u_3} \tag{12}$$

is the Lie point symmetry generator for partial differentials equations (9(*i*)) and (9(*iii*)) iff,

$$X^{[2]}(\mu_2\ u_{1yy}+u_{3y}-\frac{u_1}{Da}-\mathrm{Re}\ u_{1t})\Big|_{(10-i\ \&\ iii)}=0\quad X^{[1]}(\mu_1 u_{1y}-We\ u_{3t}-u_3)\Big|_{(10-i\&iii)}=0$$

Where first and second extended infinitesimal generator of *X* are

$$X^{[1]}=X+\eta_t^{1[1]}\frac{\partial}{\partial u_{1_t}}+\eta_y^{1[1]}\frac{\partial}{\partial u_{1_y}}+\eta_t^{2[1]}\frac{\partial}{\partial u_{3\,t}}+\eta_y^{2[1]}\frac{\partial}{\partial u_{3y}},\quad X^{[2]}=X^{[1]}+\eta_{yy}^{1[2]}\frac{\partial}{\partial u_{1yy}} \tag{13}$$

In which

$$\eta_t^{1[1]}=D_t\eta^1-u_{1_t}D_t\phi-u_{1_y}D_t\xi\ ;\qquad \eta_y^{1[1]}=D_y\eta^1-u_{1_t}D_y\phi-u_{1_y}D_y\xi\ ;$$

$$\eta_t^{2[1]}=D_t\eta^2-u_{3t}D_t\phi-u_{3y}D_t\xi\ ;\qquad \eta_y^{2[1]}=D_y\eta^2-u_{3t}D_y\phi-u_{3y}D_y\xi\ ;$$

$$\eta_{yy}^{1[2]} = D_y \eta_y^{[1]} - u_{1ty} D_y \phi - u_{1yy} D_y \xi \tag{14}$$

Where Dx_i is the total derivative operator, so we get

$$\frac{-1}{Da}\eta^1 - \mathrm{Re}\,\eta_t^{1[1]} + \eta_y^{2\,[1]} + \mu_2 \eta_{yy}^{1[2]}\Big|_{(10-i\ \&\ iii)} = 0 \;\; -\eta^2 + \mu_1 \eta_y^{1[1]} - We\ \eta_t^{2\,[1]}\Big|_{(10-i\&iii)} = 0 \tag{15}$$

3.3 Lie-point symmetries of the PDE's (10(*i*)) and (10(*iii*))

The development of (14) could be set in the form of symmetry (15). Subsequent to equating the partial differentials of u_1, u_3 and powers of differentials of these equations, the operators can be establish after making simpler the over resolved linear partial differentials equations systems, as over determined systems of linear partial differentials is explained and after solving these two over determined systems of linear PDE's, result of linear equating systems presents values of the functions ϕ, ξ, η^1 and η^2 and re written as under:

$$\phi = c_1,\ \xi = c_2,\ \ \eta^1 = c_3 u_1 + g(t,y) \ \text{ and } \eta^2 = c_3 u_3 + h(t,y) \tag{16}$$

Where $f(t,y)$ & $g(t,y)$ are arbitrary functions of the following equations,.

$$\mathrm{Re}\ g_t = \mu_2\ g_{yy} + h_y - \frac{1}{Da} g \ \text{ and } We\ h_t = \mu_1 g_y - h$$

In (16), c_i are arbitrary constants and symmetry Lie algebra of PDE's system (10(*i*)) and (10(*iii*)) is three-dimensional and spanned by the following generators

$$X_1 = \frac{\partial}{\partial t}, \quad X_2 = \frac{\partial}{\partial y}, \quad X_3 = u_1 \frac{\partial}{\partial u_1} + u_3 \frac{\partial}{\partial u_3} \ \text{ and } \ X_m = f(t,\ y)\frac{\partial}{\partial u_1} + g(t,\ y)\frac{\partial}{\partial u_3} \tag{17}$$

Where *m* are non-negative numbers

3.4 Results concerned with invariant solutions related to $X_1 - \beta X_3$

In this section only those operators are used that related to solution of physical problem (10(*i*)) and (10(*iii*)) subject to same initial and boundary conditions. Method of solutions depends on the applications of Lie group of transformation related with one-parameter to the PDE's system. As $X = X_1 - \beta X_3 = \frac{\partial}{\partial t} - \beta \frac{\partial}{\partial u_1} - \beta \frac{\partial}{\partial u_3}$

Hence applying the operator *X*, and applying the super position principle he final result of the system (5) to (7) show the following solutions

$$v(t,\ y) = \sum_{n=1}^{\infty} \frac{2\,Da\,(-1)^{n-1}}{\lambda_n (Da\ \lambda_n^{\ 2} + 1)} \left(\frac{\beta_2 (1-\beta_1 We)}{(\beta_1 - \beta_2)} e^{-\beta_1 t} - \frac{\beta_1 (1-\beta_2 We)}{(\beta_1 - \beta_2)} e^{-\beta_2 t} \right) Cos\, \lambda_n y + Da \left(1 - \frac{Cosh \frac{y}{\sqrt{Da}}}{Cosh \frac{1}{\sqrt{Da}}}\right) \tag{18-i}$$

$$\tau_{11}(t,\ y) = \left(\sum_{n=1}^{\infty} \frac{Da\,(8\,We\ \mu_1)^{\frac{1}{2}}}{(Da\ \lambda_n^2 + 1)} \left(\begin{array}{l} \frac{\beta_2^2 (1-\beta_1 We)\,(e^{-2\beta_1 t} - e^{\frac{-t}{We}})}{(\beta_1 - \beta_2)^2\,(1 - 2\,\beta_1 We)} + \frac{\beta_1^2 (1-\beta_2 We)\,(e^{-2\beta_2 t} - e^{\frac{-t}{We}})}{(\beta_1 - \beta_2)^2 (1 - 2\beta_2 We)} \\ - \frac{\beta_1 \beta_2 (2 - \beta_1 We - \beta_2 We)\,(e^{-(\beta_1 + \beta_2)t} - e^{\frac{-t}{We}})}{(\beta_1 - \beta_2)^2 (1 - \beta_1 We - \beta_2 We)} + \frac{\beta_2\,(2 - \beta_1 We)\,(e^{-\beta_1 t} - e^{\frac{-t}{We}})}{(\beta_1 - \beta_2)(1 - \beta_1 We)} \\ - \frac{\beta_1 (2 - \beta_2 We)\,(e^{-\beta_2 t} - e^{\frac{-t}{We}})}{(\beta_1 - \beta_2)(1 - \beta_2 We)} \end{array} \right)^{\frac{1}{2}} Sin\ \lambda_n y \right)^2$$

$$+ 2 We\, \mu_1\, Da \left(1 - e^{\frac{-t}{We}}\right) \frac{Sinh^2 \frac{y}{\sqrt{Da}}}{Cosh^2 \frac{1}{\sqrt{Da}}} \tag{18-ii}$$

$$\tau_{12}(t,y) == \sum_{n=1}^{\infty} \frac{2\mu_1 Da\,(-1)^n}{(Da\,\lambda_n^{\,2}+1)} \left(\frac{\beta_2}{(\beta_1-\beta_2)} e^{-\beta_1 t} + \frac{\beta_1}{(\beta_1-\beta_2)} e^{-\beta_2 t}\right) Sin\,\lambda_n y - \mu_1 \sqrt{Da}\ \frac{Sinh\,\frac{y}{\sqrt{Da}}}{Cosh\,\frac{1}{\sqrt{Da}}} \qquad \text{(18-iii)}$$

Where $\lambda_n = (\frac{2n-1}{2})\pi$ and $\mu_1 + \mu_2 = 1$ and

$$\beta_1 = \frac{1}{2}\left(\frac{1}{We} + \frac{\mu_2 \lambda_n^2}{\mathrm{Re}} + \frac{1}{Da\,\mathrm{Re}}\right) + \frac{1}{2}\sqrt{\left(\frac{1}{We} + \frac{\mu_2 \lambda_n^2}{\mathrm{Re}} + \frac{1}{Da\,\mathrm{Re}}\right)^2 - \frac{4(Da\,\lambda_n^2+1)}{Da\,\mathrm{Re}\,We}}$$

$$\beta_2 = \frac{1}{2}\left(\frac{1}{We} + \frac{\mu_2 \lambda_n^2}{\mathrm{Re}} + \frac{1}{Da\,\mathrm{Re}}\right) - \frac{1}{2}\sqrt{\left(\frac{1}{We} + \frac{\mu_2 \lambda_n^2}{\mathrm{Re}} + \frac{1}{Da\,\mathrm{Re}}\right)^2 - \frac{4(Da\,\lambda_n^2+1)}{Da\,\mathrm{Re}\,We}}$$

4 ANALYSIS OF VISCOELASTIC STRESSES

4.1 Analytical solution of normal and shear stresses

The analytical result of governing system of equation have been found by Lie group method and normal and shear stresses components are defined in the equations (18-ii) and (18-iii). The solutions of equations (18-ii) and (18-iii) are expressed in figure-1 and 2 for several of parameters and at different time *t*.

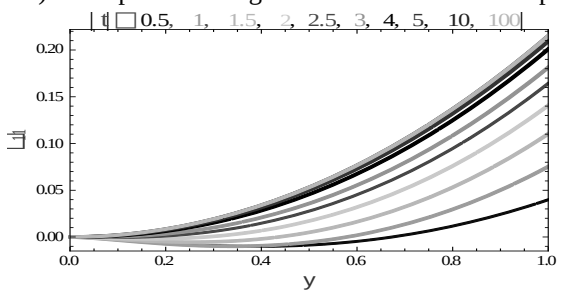


Fig.1: Normal stress τ_{11}

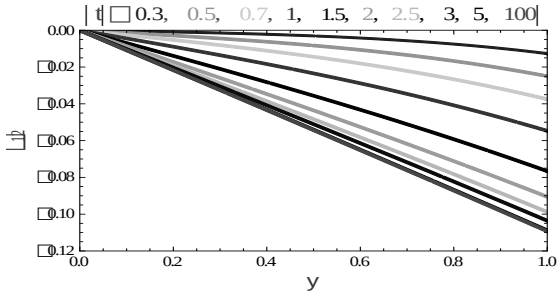


Fig. 2: Shear stress τ_{12}

Analytical solution of the normal stress component τ_{11} (18-ii) and shear stress component τ_{12} (18-iii) wit $Da = 20,\ \mathrm{Re} = 1,\ \mu_1 = 1/9,\ We = 1$ and at different time *t*

The time dependent effect on the normal stress component and shear stress are displayed in figure–1 and 2 respectively. The first normal stress component τ_{11} is shown in figure–1 which illustrates that the normal stress τ_{11} increases with increasing time and attain an upper limit at same time level in non–linear fashion as time increased from initial state and reached at steady–state whose maximum value is equal to $\tau_{11} = 0.215$ and Whilst, the shear stress component τ_{12} is displayed in figure–2 and the shear stress component τ_{12} decreases as time increases and there is a lower limit for this decrease. There is no further change in shear–stress τ_{12} if it reached from transition to steady–state whose minimum value is equal to $\tau_{12} = -0.11$.

4.2 Steady State Solution of normal and shear stresses

Steady state solution have been already solved and defined in the relation (9) and normal and shear stresses components are given as

$$\tau_{11}(t,y) = \psi_2(y) = 2We\,\mu_1\,Da\ \frac{Sinh^2\,\frac{y}{\sqrt{Da}}}{Cosh^2\,\frac{1}{\sqrt{Da}}} \quad \& \quad \tau_{12}(t,y) = \psi_3(y) = -\mu_1\sqrt{Da}\ \frac{Sinh\,\frac{y}{\sqrt{Da}}}{Cosh\,\frac{1}{\sqrt{Da}}} \qquad (19)$$

Steady–state solutions of normal and shear stresses (19) are plotted in figures 3 and 4 respectively at different values of Darcy's number *Da*. The figures–3 and 4 shows that in the steady–state, if channels having small Darcy's numbers Da, then steady normal stress component τ_{11} have small values that if permeability decreases, then resistance increases and hence normal stress τ_{11} decreases in the steady–state, and figure–4 shows that in the steady–state, if channels having small Darcy's numbers *Da*, steady shear stress component τ_{12} have large values that is if permeability decreases, then shear stress τ_{12} increases in the steady state.

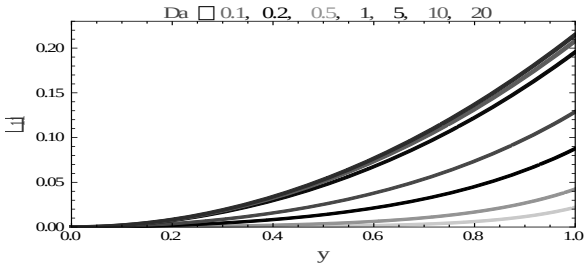


Fig. 3: Normal stress τ_{11}

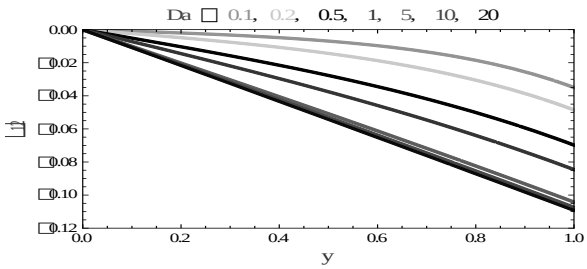


Fig. 4: Shear stress τ_{12}

Steady–state solution of the normal stress component τ_{11} and shear stress component τ_{12} (19) wit $\mu = 1$, $\mu_1 = 1/9$ and $We = 1$ at different values of *Da*

5 CONCLUSION

In this paper, the purpose of our research to solve the problem of PDE's system analytically by Lie group technique to obtain new solutions from identified ones. Lie-point symmetries have been achieved by applying symmetry conditions and adopted to arrive at the result of the problems and main purpose of research to obtain the exact solutions of normal and shear stress components. This method can provide some useful insights into comprise of solutions and may help for finding particular solutions in many cases. Also, the numerical solutions obtained through Mathematica by ND–Solver are in close agreement with analytical solutions normal and shear stress components. In both analytical and numerical techniques, comparison of both techniques was conducted. We hope that the results may be useful for other workers in the field. Our advices for the future work are developing and putting into practice other steady-state and transient viscoelastic algorithms.

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