



SER and Outage Probability Analysis of Network Coding over Identical Terrestrial and Non-identical Satellite-terrestrial Fading Channels

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Received 28th December 2013 and Revised 18th February 2014

Abstract: Network Coding (NC) is a new paradigm that gave height to cooperative networks in wireless communication by increasing reliability, throughput and capacity of the systems. Analog Network Coding (ANC) and the Physical-layer Network Coding (PNC) are two most studied variants of NC. In this paper, we present the simulation analysis for both of these techniques for terrestrial identical fading channels (Rician) and hybrid satellite-terrestrial non-identical fading channels (Rician+Nakagami-m). For former case, relay optimization is considered as an important facet in our simulation results during evaluating SER and Outage probability while for the later case, we evaluated both SER and Outage Probability over non-identical fading channels. Our results show the comparison of both ANC and PNC which indicates the reliability of PNC over ANC while processing advantage of ANC over PNC.

Keywords: Network Coding; Fading Channels; Terrestrial Networks; Nakagami-m.

1.

INTRODUCTION

Network Coding (NC), proposed in (Ahlswede et al., 2000), has provided elevations to wireless cooperative networks by increasing the overall efficiency, capacity and throughput of the system over traditional routing or relaying techniques (Ahlswede et al., 2000), (Hoand and Lun, 2008). Intermediate nodes perform some algebraic operations on incoming signals in NC and then broadcast the composite signal to the users. The strength of NC is recognized in information theory as it can achieve the capacity up to two times the traditional networks. Different surveys have been performed by network information analysts to show the valuable aspects of NC and its challenging variants (Hoand and Lun, 2008), (Yeung, 2011). To understand the benefits of NC and its basics, consider the famous Alice Bob topology with intermediate node for the explanations. The traditional routing need four time slots for the successful communication between Alice and Bob and allows them to exchange pair of packets. Conventional NC, also called digital NC, requires three time slots. In this case, Alice will send the packet xA to relay in the first time slot and Bob will send xB in second time slot. Intermediate node will perform the XOR operation on both incoming packets and will broadcast the combined signal, xA ⊕ xB to both Alice and Bob in third the time slot. Conventional network coding is further enhanced to Physical-layer Network Coding (PNC) (Zhang et al., 2006) and Analog Network Coding (Katti et al., 2007) , which disclosed further improvement of networks in terms of throughput and capacity while reducing the number of time slots to 2.

Alice and Bob send their information simultaneously to relay node in case of both PNC and ANC. The PNC adds reliability to the networks by carrying out the Modulo 2 Addition (XOR) on the incoming signals at relay nodes with proper modulation and demodulation kept the same as in the case of NC. While its counterpart ANC has further decreased the processing burden via performing the simple addition on electromagnetic signals (Katti et al., 2007). Instead of avoiding the interference, we actually exploit it in case of ANC which is greatly advantageous for wireless broadcast networks In this draft, we are considering only these two techniques of NC for our results evaluation. We are considering terrestrial fading channel and also satellite-terrestrial non-identical fading channels to evaluate our results. Relay optimization has always been an important consideration to enhance the performance of cooperative networks (Thakur et al., 2011). Analysis has revealed that these performances can further be enhanced with different NC techniques. Optimum relay location has been determined for both ANC and PNC to achieve the better performance of the system in this paper. We evaluate the outage probability and SER results over Rician fading channels with relay optimization. It is to be noted that the main objective of this paper is not to revisit path-selection algorithms but to focus on giving an exact analysis of optimal relay position that is valid for all SNR regions. The concept is given to justify the different network coding techniques in relay optimization and how these techniques can affect the performance of network in this scenario. Satellite-earth stations' propagation channel models define the Quality of Service (QoS). For

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systems` efficiency, the work of (Loo, 1985), (Lutz *et al.*, 1991), (Yi *et al.*, 2011) have devised the Land Mobile Satellite (LMS) where the cooperative relaying satellite-terrestrial network has been proven to be more efficient under the given set conditions. According to (Iqbal and Ahmad, 2011), various probability combinations are required as terrestrial radio propagation models are not feasible for satellite networks. The study of various network coding schemes is important for consideration due to their better performance in the next generation satellite networks. Propagation model proposed in (Loo, 1985) having Rice Distribution and LMS channel with large scale path-loss has been used our proposed work. As in (Iqbal and Ahmad, 2011), various NC schemes for non-identical fading channels have been introduced for the integrated satellite-terrestrial network's feasibility report. Furthermore, due to the significant advantages in cooperative networks, ANC and PNC have used and cross-compared.

BER analysis with relay optimization has been performed in (Jadoon *et al.*, 2012) for both ANC and PNC. Outage probability closed form expression for PNC has been derived in (Liu and Kim, 2010). Outage probability and SER analysis is evaluated in (Tanooli *et al.*, 2010) for ANC over Rician and Nakagami- m fading channels using space time coding. In (Rossetto, 2010), comparison of different NC techniques is performed in satellite environment.

In this paper, we have evaluated the:

Case1:- Performance of ANC and PNC for optimal relay locations and compared the performance of both techniques in terms of outage probability and SER. We have derived results by taking path loss and fading degradation over Rician fading channels into considerations.

Case2:- The simulation results for ANC and PNC over hybrid satellite-terrestrial non-identical fading channels for their comparative studies. This analysis is performed in terms of SER and outage probability. All results are investigated for BPSK modulation.

The rest of paper is organized as follows: Section 2 discusses the materials and methods. Section 3 presents the results and discussion. Finally, the paper is concluded in Section 4.

2. MATERIALS AND METHODS

System Model

(Fig. 1) illustrates the arrangement of the proposed system model where four nodes have been considered in the system, *i.e.*, User A (source 1), User B (source 2) and two relay nodes R₁ and R₂. Sources A and B send their information signals, x_{1j} and x_{2j} to j^{th} relay in first time slot, here $j = \{1, 2\}$. Both relays then broadcast the information to nodes A and B in second

time slot. All nodes are equipped with single antenna and relay nodes have Decode and Forward (DF) mode in case of PNC and amplify and forward (AF) mode in case of ANC.

A. Fading Model

" h_{ij} " is the fading magnitude between i^{th} terminal (sources) and j^{th} relay ($i = \{1, 2\}$ A and B and $j = \{1, 2\}$, R₁ and R₂). $\gamma_{ij} = |h_{ij}|^2$ is gamma-distributed random variable.

In the first case, we analyze the relay locations as an important parameter where " h_{ij} " has been considered as a Rician Fading. The probability density function (PDF) of Rician distribution can be expressed as (Simon and Alouini, 2005):

$$f(\alpha_{ij}) = (1 + K_{ij}) \exp(-(1 + K_{ij})\alpha - K_{ij}) \times I_0 \left(2\sqrt{K_{ij}(1 + K_{ij})} \gamma_{ij} \right) \quad (1)$$

The term $I_0(\cdot)$ in the Equation (1) is the modified version of Bessel function of the zero while K represents the Rician fading factor. The Rician fading factor becomes the Rayleigh distribution at $K=0$.

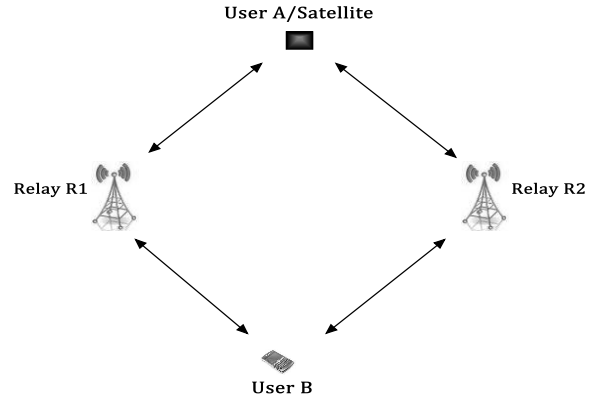


Fig. 1: System Model

In the second case, we analyze the performance of ANC and PNC for hybrid satellite-terrestrial network for the non-identical fading channels. Channels between satellite and Relays will be considered as Rician Fading channel while channel between Relays and User B will be taken as Nakagami- m fading channel. The PDF of Nakagami- m fading channels is written mathematically as (Simon and Alouini, 2005):

$$f(\alpha) = \frac{m^m \alpha_j^{m-1}}{\Gamma(m)} \exp(-m\alpha_j) \quad (2)$$

Where $\Gamma(\cdot)$ in the Equation (1) is a gamma function. Here, we consider $E(|h_{ij}|^2) = 1$ and $E(\cdot)$ is the expectation operator. For $m = 1$ the Nakagami- m

distribution becomes the exponential distribution. It is assumed that Channel state information CSI is only at sender and both sources are perfectly synchronized.

B. Input Output Equations

As we know that signal x_{11} and x_{11} are transmitted by source nodes A and B to relays R_1 and R_2 , respectively. It is also assumed that transmissions of both the signals are simultaneous. We have to take the distances into consideration and for this purpose d_{ij} is the distance between i^{th} node and j^{th} relay. The received signals at the relays R_1 and R_2 can be expressed individually as:

$$\begin{aligned} y_{R1} &= (h_{11}x_{11})(d_{11})^{-\alpha} + h_{21}x_{22}(d_{21})^{-\alpha} + n_{R1} \\ y_{R2} &= (h_{22}x_{22})(d_{22})^{-\alpha} + (h_{12}x_{11})(d_{12})^{-\alpha} + n_{R2} \end{aligned} \quad (3)$$

Where $i \in \{1, 2\}$ and $j \in \{1, 2\}$.

Relay, thereby, normalizes the received signal and broadcasts the analog network coded signal to the terminals A and B. Both users extract the required information by subtracting their own signal. The general form of the received signal from j^{th} relay is represented as:

$$R_{ij} = h_{ij}\beta_j y_{Rj} (d_{ij})^{-\alpha} + n_{Rj} \quad (4)$$

Where β_j in the Equation (4) represents amplification factor at j^{th} relay. It is assumed that the value of β_j is known to both the users. Further, as user A already knows the value of x_{1j} , h_{1j} , therefore, it can reconstruct the signal as:

$$y_{ij} = R_{ij} - |h_{R1}|^2 \beta_j x_{ik} ((d_{ij})^{-\alpha})^2 \quad (5)$$

Hence, the recovered signals from both the relays can be written as:

$$\begin{aligned} y_{11} &= \beta_1 h_{11} h_{21} x_{21} (d_{11} d_{21})^{-\alpha} + \acute{n}_{11}, \\ y_{12} &= \beta_2 h_{12} h_{22} x_{22} (d_{12} d_{22})^{-\alpha} + \acute{n}_{12}, \\ y_{21} &= \beta_1 h_{11} h_{21} x_{11} (d_{11} d_{21})^{-\alpha} + \acute{n}_{21}, \\ y_{22} &= \beta_2 h_{12} h_{22} x_{12} (d_{12} d_{22})^{-\alpha} + \acute{n}_{22} \end{aligned} \quad (6)$$

Where $\acute{n}_{ij} = \beta_j h_{ij} n_{rj} ((d_{ij})^{-\alpha})^2 + n_j$ in the Equation (6) having variance $(\beta_j^2 |h_{ij}|^2 + 1)N_0$. These parameters will remain the same for both cases, but their values will change according to fading channels as discussed in the next section.

3. RESULTS AND DISCUSSION

A. Simulation Model

Outage Probability:

Mathematically, the End-to-End outage probability for non-regenerative systems is defined as:

$$\begin{aligned} P_{out} &= P[\gamma_{eq} \leq \gamma_{th}] \\ P_{out} &= \int_0^{\gamma_{th}} p_{\gamma_{eq}}(\gamma) d\gamma = p_{\gamma}(\gamma_{th}) \end{aligned} \quad (7)$$

Where γ_{eq} in the Equation (7) represents the value of SNR required whereas γ_{th} shows predetermined threshold SNR. A value above γ_{th} shows satisfactory

system's quality which is dependent on the modulation schemes and type of service. Outage probability is one of the important parameter to evaluate the performance of any wireless system. We have simulated outage probability separately for both PNC and ANC over Rician Fading channels and analyzed the optimum relay location for both techniques which will be discussed in next section in detail.

Symbol Error Rate:

For analyzing the performance of systems, another important factor is the Symbol Error Rate (SER) where the average SER equation depicts dependency on the modulation type which has been used. For the M-PSK modulation scheme, the average SER is represented as (Simon and Alouini, 2005):

$$P_{e(MPSK)} = \frac{1}{\pi} \int_0^{(M-1)\pi/M} \mathcal{M}_{\gamma_i} \left(\frac{g_{PSK}}{\sin^2 \theta} \right) d\theta \quad (8)$$

Where, $g_{PSK} = \sin^2(\pi/M)$. Here, we are using BPSK modulation for which $M = 2$.

C. SER and Outage Probability over Rician Fading Channels

Simulation model is shown in the (Fig. 2) for this case. It is assumed that users A and B cannot communicate directly. Relays are intermediary nodes that complement the transmission. The performances for both ANC and PNC are recorded sequentially. Results are evaluated for sub-urban area by taking the value of path-loss exponent $\alpha = 3$; while $(d)^{-\alpha}$ is total path loss. Total distance between users A and B is taken as d_o and is normalized to 1 for simplicity in simulations. All distances are normalized to d_o . We will keep the distances of both nodes A and B at same distance from relay nodes R1 and R2, and then we will move relays along their axes at different positions from both nodes in square distances as shown in (Fig. 2).

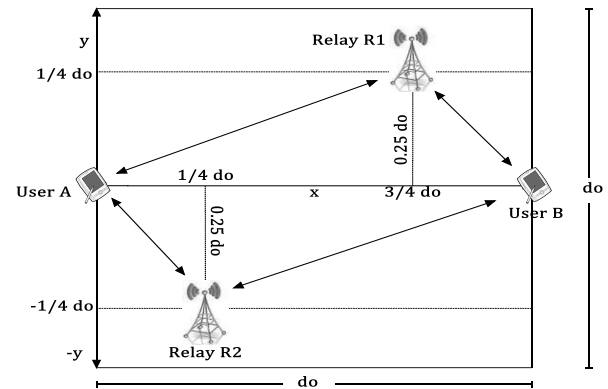


Fig. 2: Simulation Model for Case 2

For simplicity, results are only shown for node A for symmetric reasons as performances of both nodes A and B are similar. Node A is located at coordinates (0, 0)

and the node B at $(0, d_o)$ within a square of side length of d_o , while the two relays are located between node A and B as shown in **(Fig. 2)**. The distance between relays' axes is set to be $d_o/2$. Thus the y-axis of the relay 1 and 2 are set to be equal to $d_o/4$ and $-d_o/4$, respectively, as shown in simulation model of **(Fig. 2)**. We take different combinations for optimum location of both relays at their different projections on x-axis. We can find the distance of the user from relay nodes by using simple trigonometric equation:

$$d_{ij} = \sqrt{(jth \text{ relay projection})^2 + \left(\frac{d_o}{4}\right)^2} \quad (9)$$

Different combinations of projections for both relays have been simulated for both SER and outage probability.

(Fig. 3) and **(Fig. 4)** show the outage probability of various projections for R1 and R2 on *x-axis* graphs for both PNC and ANC. These 3D graphs are simulated for SNR = 15dB and value of threshold SNR is; $\gamma_{th} = 10$ dB. **(Fig. 5)** and **(Fig. 6)** show SER versus both relays' projections on *x-axis* graphs for both PNC and ANC, respectively. These graphs are drawn for SNR = 8dB. Rician K factor is also an important parameter for analysis in Rician fading channels. We have taken K = 1dB for overall simulation of results. It should be noted that for $K = 0$ dB, Rician distribution becomes Rayleigh distribution.

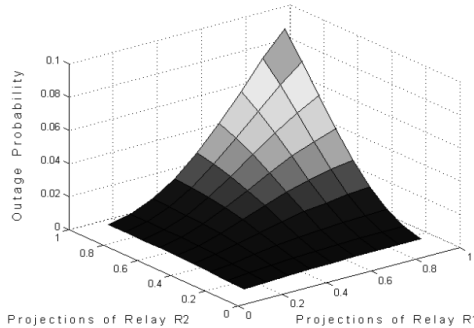


Fig. 3: Outage Probability versus projections of both relays for PNC at SNR = 15dB

B. Outage Probability and SER over Satellite-terrestrial non-identical Fading Channels

In this case, we have normalized all the distances in the Equation (9) to 1 ($d_{ij} = d_o = 1$), *i.e.*, no relay optimization is performed in this case. We have taken Satellite and User B as shown in the **(Fig. 1)**. Non-identical fading channels are used between satellite, relays and User B as we have discussed above. Again, it should be noted that the use of terrestrial channels for satellite environment is not feasible and we will use

non-identical channels for this purpose. The importance of usage of these non-identical channels is not our goal in this paper; it has been shown by authors as discussed above.

From graphical standpoint of outage probability, as shown in the **(Fig. 3)** and **(Fig. 4)**, we can examine the better performance of PNC and ANC respectively when both relays are near user A, but we also need to encounter the better performance of user B. So, there must be some tradeoff in locating both the relays for better performance of all users. It is clear that we are getting better outage probabilities when at least one relay is nearer to the user. So, if one relay is kept near the user A and one near user B, then, we will attain less outage probabilities. So, $0.1d_o$ projection for one relay and $0.9d_o$ for other relay will be an optimum location for both NC techniques. We can also keep them in the range of $0.1d_o - 0.3d_o$ or $0.7d_o - 0.9d_o$ to get better performance.

We have analyzed the SER and Outage Probability by using these channels for different NC techniques. **(Fig. 7)** in the subsequent sub-section shows the respective outage probability and SER versus the discrete values of SNR over non-identical fading channels in satellite-terrestrial network. In this case, we have taken Rician and Nakagami-*m* fading channels for our simulations.

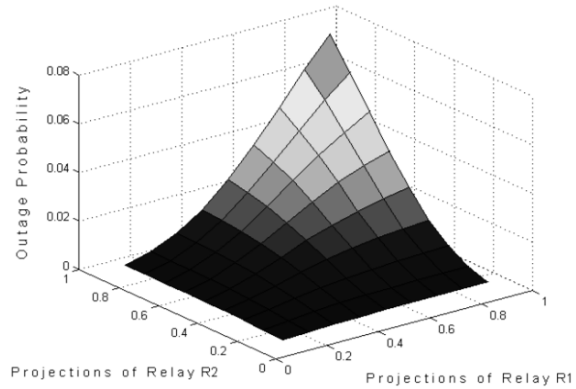


Fig.4: Outage Probability versus projections of both relays for ANC at SNR = 15dB

C. Evaluation and Results Discussion

This sub-section formally discusses the graphical results attained during the performance evaluation. The results are obtained from Monte Carlo simulations in MATLAB. The channel co-efficient is assumed to be constant during the span of transmission. Furthermore, receiver having perfect channel estimation and transmitter with no Channel State Information (CSI) has been considered.

Proceeding with the ensuing work, the projection of various distances for both the relays on *x-*

axis are encountered as illustrated in (Fig. 2), whose graphical aspect was analyzed along their outage probability values. For this premium location, a judgment was set that figured out the comparison along most important network topologies discussed above that is ANC and its variant counterpart PNC.

Similarly, in case of SER and PNC, minimum outage probabilities are obtained when either one or both relays is placed in the center, *i.e.*, at $0.5d_0$. A better performance can be seen at $0.4d_0$ and $0.6d_0$, respectively. However, by moving away from these positions towards the users, there is a tremendous increase in SER as shown in (Fig. 5). PNC outperforms SER when only one relay is at the center position. (Fig. 6) shows the SER and ANC performance with respect to relays projections. Minimum SER is obtained at $0.3d_0$ projections. However, by moving towards user A from this location, degradation in SER performance can be seen. This means that better SER is obtained by placing one relay at $0.3d_0$ or $0.4d_0$.

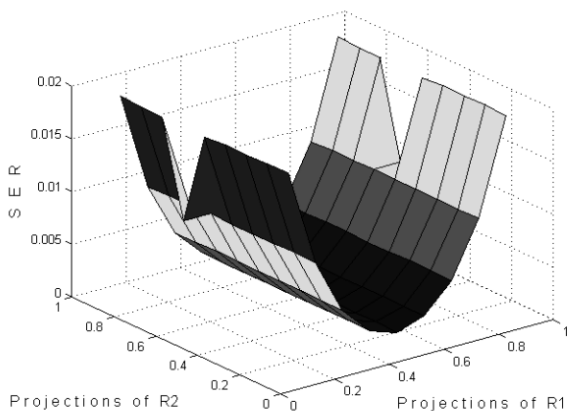


Fig.5: SER versus projections of both relays for PNC at SNR = 8dB

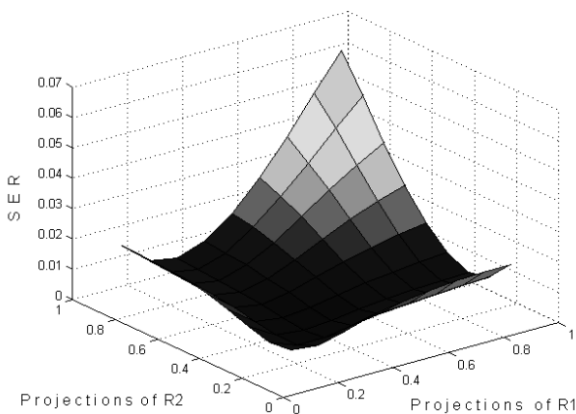


Fig.6: SER versus projections of both relays for PNC at SNR = 8dB

It can also be observed that the asymmetric behavior from the SER graphs of ANC and PNC has a progressive decremented or incremental trend of SER in case of ANC, but for PNC, when we move away from optimum positions; there is a sudden SER degradation. So, PNC strictly demands a specific location for its better performance. This demonstrates prominent system performance, which suddenly got declined with the changing position of relays towards the edges. These results are supplemented with the fact that outage probability of ANC is better than PNC for every projection. This is because of the processing burden of PNC over ANC, which decodes and then forwards the signals while ANC directly performs arithmetic operation on EMWs. Also, SER of PNC is better than ANC which shows the greater reliability of PNC over ANC.

(Fig. 7) shows the comparative plot between ANC and PNC for Symbol Error Rate (SER) at constant K where the value of K is 1 and the value of m is varied from 1 to 3. It can be seen that various values of m results in different results. For $m = 1$ (*i.e.*, Rayleigh fading), ANC outperforms PNC up to certain SNR value. When the value of $m = 1$ and the SNR value is below 12, PNC shows better performance than ANC. However, ANC outperforms PNC when the SNR value is increased above 12. The same trend can be seen when the system is simulated for higher values of m, *i.e.*, $m = 2$ and 3, PNC gives better performance below SNR 15 and 17, respectively. With an increase in the value of m both the PNC and ANC exhibit a constantly better performance. However, when both PNC and ANC are cross-compared for the same values of m, *i.e.*, either $m = 1, 2$ or 3, the former outperforms the latter at lower values of SNR while the latter performs better than the former at higher SNR.

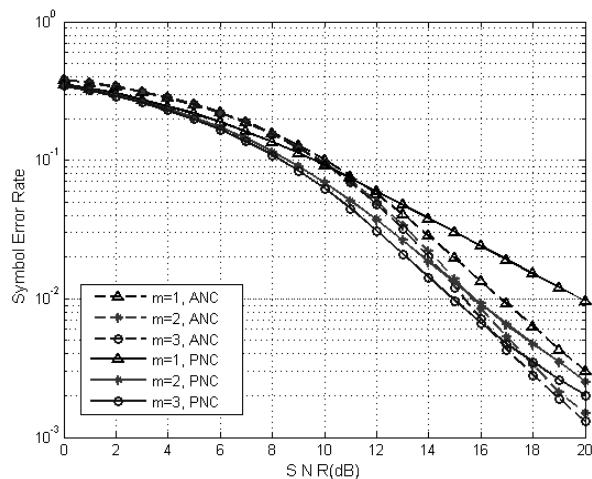


Fig.7: Outage Probability comparison for ANC and PNC in satellite-terrestrial network K=1 and m = 1, 2, and 3

4. CONCLUSION

In this paper, comparative analysis of analog n and physical layer network coding is performed with optimum relay allocation in terms of SER and outage probability for identical terrestrial fading channels, *i-e*. Rician. Also, the same comparison is evaluated for non-identical satellite-terrestrial fading channels *i-e*. Rician and Nakagami-*m*. Some useful results were deduced which added more to the arena of network coding. We have seen that analog network coding has decreased the system processing over PNC with better performance while PNC shows greater reliability over ANC in proposed system model. Optimum relay location is found, for both ANC and PNC and its alignment is mandatory for PNC better performance while ANC gave some radical and flexible results for different relay positions in the first case. In the second case, PNC performs better than ANC in terms of SER for lower values of SNR while the latter beat former at higher SNR. Further, an improved performance has been shown due to the use of multiple relays. A further extension of the proposed model can be the use of multiple relays with multiple users and by removing the assumption made to make the system more realistic and practical. Similarly, the system can be extended with analytical proof and mathematical modeling. Also, we can have such arrangements or algorithm to calculate these parameters where we can perform both of these techniques by switching one to other at lower and higher SNR regimes.

REFERENCES:

Ahlsweide R., N. Cai, S.Y. R. Li and R. W. Yeung (2000) Network Information Flow IEEE Trans. on Info. Theory, Vol. (46): No 4: 1204–1216.

Hoand T. and D. Lun (2008) Network Coding: An Introduction. Cambridge Univ. Press Cambridge, U.K.

Iqbal A. and K. Ahmed (2011) A Hybrid Satellite-Terrestrial. Cooperative Network over Non Identically Distributed Fading. Channels Journal of comm Vol. (6) No7:581-589.

Jadoon M.A., Z. A. Khan, A. Ahmed, M. Usman, I. Khan and S. Jan (2012) Relay Optimization for ANC and PNC over Rician Fading Channels. Int. Journal of Applied Info. Systems Vol. (1) No 9:22-27.

Katti S. , S. Gollakota and D. Katabi (2007) Embracing wireless interference: analog network coding Proceeding of ACM Conference on Computer Communications (SIGCOMM '07): 397–408. Kayoto Japan.

Liu P. and I. M. Kim (2010) Outage probability analysis of physical layer network coding in bidirectional relay network 25th Biennial. Symposium on Communications (QBSC):130-133. Canada.

Loo C. (1985) A Statistical Model for a Land Mobile Satellite Link IEEE Trans. on Vehicular Tech. Vol. (34): No 3: 123-127.

Lutz E., D. Cygan, M. Dippold, F. Dolainsky and W. Papke (1991)The land mobile satellite communication channel-recording, statistics, and channel modeling IEEE Trans. on Vehicular Tec Vol(40) No 2:375-385.

Rossetoo F. A. (2010) Comparison of different physical layer network coding techniques for the satellite environment 5th Advanced Satellite. Multimedia Systems Conf: 25-30, Germany.

Simon M. K.and M. S. Alouini (2005) Digital Communication over. Fading Channels John Wiley & Sons, New York, NY, USA.

Tanoli S. A., I. Khan, N. Rajatheva and F. Adachi (2010) Advances in Relay Networks: Performance and Capacity Analysis of Space-Time Analog Network Coding EURASIP Journal on Wireless Comm. And Networking, 1-10.

Thakur M., N. Fawaz, and M. Medard (2011) Optimal relay location and power allocation for low SNR broadcast relay channels IEEE Proceedings, INFOCOM:2822-2830. Orlando, USA.

Yeung R. W. (2011) Network Coding: A Historical Perspective IEEE.Proceedings Vol (99): No 3: 366-371.

Yi Z., M. Ju I. M. Kim (2011) Outage Probability and Optimum Power Allocation for Analog Network Coding IEEE Trans. On Wireless Comm Vol. (10) 2:407-412.

Zhang S., S. C. Liew and P. P. Lam. (2006) Hot topic: physical layer network coding Proc. of 12th ACM Annual International Conference on Mobile Computing and Networking: 358-365. Florida, USA.