*Sindh Univ. Res. Jour. (Sci. Ser.) Vol.46 (4):547-552 (2014)*

**SINDH UNIVERSITY RESEARCH JOURNAL (SCIENCE SERIES)**

# Left Transitive AG-Groupoids

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Received 18th May 2014 and Revised 9th November 2014

**Abstract.** An AG-groupoid is an algebraic structure that satisfies the left invertive law: $ab\cdot c=cb\cdot a$. We prove that the class of left transitive AG-groupoids (AG-groupoids satisfying the identity, $ab\cdot ac=bc$ ) coincides with the class of $T^2$-AG-groupoids. We also developed a simple procedure to test whether an arbitrary groupoid is left transitive AG-groupoid or not. Further, we prove that (i) every left transitive AG-groupoid is transitively commutative AG-groupoid. (ii) For left transitive AG-groupoid the properties of flexibility, right alternatively, AG*-groupoid right nuclear square, middle nuclear square and commutative semigroup are equivalent.

**Keywords**- AG-groupoid, commutative semi-group, nuclear square AG-groupoid, unipotent, $T^2$-AG- groupoid, alternative AG-groupoid.

## 1. INTRODUCTION

An AG-groupoid is an algebraic structure that satisfies the left invertive law; $ab\cdot c=cb\cdot a$ (Kazim *et al.,* 1977). This structure is a generalization of commutative semi-groups. Every AG-groupoid satisfies the medial law; $ab\cdot cd=ac\cdot bd$. An AG-groupoid $S$ with the left identity element $e$ is called an AG-monoid and every AG-monoid is paramedical $ab\cdot cd=db\cdot ca$. Recently, many researchers have done a significant work in this field and a considerable work has been done on various aspects. Many classes have been discovered and enumerated up to order 6 (Gap: 2012, Distler *et al.,* 2011, Shah *et al.,* 2012, Shah *et al.,* 2013). The field is very important and AG-groupoids have applications in the theory of flocks (Kazim 1977) and geometry (Shah, 2012).

A groupoid $G$ satisfying the identity $ac\cdot bc=ab\ \forall a,b,c\in G$ is called right transitive groupoid (Polnijo 1993), and $G$ is called left transitive groupoid if it satisfies the identity $ab\cdot ac=bc\ \forall a,b,c\in G$ (Denes *et al.,* 1974). In Section 2 we prove that non-associative right transitive AG-groupoids do not exist. In Section 3 we prove the coincidence of the left transitive AG-groupoids with the class of $T^2$-AG-groupoids. We also prove that every left transitive AG-groupoid is transitively commutative AG-groupoid. We also investigated that on what conditions some left transitive AG-groupoids become commutative semigroups.

## 2. MATERIALS AND METHODS

The following table contains various AG-groupoids with their defining identities that will be used in the rest of this article.

| AG-groupoid | Defining Identities |
|---|---|
| Left nuclear square | $a^2\cdot bc=a^2b\cdot c$ |
| Middle nuclear square | $ab^2\cdot c=a\cdot b^2c$ |
| Right nuclear square | $ab\cdot c^2=a\cdot bc^2$ |
| $T^1$ | $ab=cd\Rightarrow ba=dc$ |
| $T^2$ | $ab=cd\Rightarrow ac=bd$ |
| $T_l^3$ | $ab=ac\Rightarrow ba=ca$ |
| $T_r^3$ | $ba=ca\Rightarrow ab=ac$ |
| $T^3$ | Both $T_l^3$ & $T_r^3$ |
| AG**-groupoid | $a\cdot bc=b\cdot ac$ |
| Transitively commutative | $ab=cd$ & $bc=cb\Rightarrow ac=ca$ |
| AG-monoid | AG-groupoid with identity |
| Quasi-cancellative | $x^2=xy$ & $y^2=yx\Rightarrow x=y$<br>$x^2=yx$ & $y^2=xy\Rightarrow x=y$ |
| Flexible | $ab\cdot a=a\cdot ba$ |
| Right alternative | $b\cdot aa=ba\cdot a$ |
| AG*-groupoid | $ab\cdot c=b\cdot ac$ |

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## 3. RESULTS AND DISCUSSION

We discuss both the right and left transitive AG-groupoid via in turn as in the following. To reduce the chances of mistakes we give various examples of these structures by computer using GAP software.

### A. RIGHT TRANSITIVE AG-GROUPOIDS

We begin this subsection with the following definition of right transitive AG-groupoids and prove that non-associative right transitive AG-groupoid does not exist.

**Definition-1:** An AG-groupoid $S$ is called right transitive AG-groupoid if,

$$ab \cdot cb = ac \quad \forall\ a,b,c \in S$$

**Example-1:** Associative right transitive AG-groupoid of order 4.

| · | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 |
| 2 | 2 | 1 | 4 | 3 |
| 3 | 3 | 4 | 1 | 2 |
| 4 | 4 | 3 | 2 | 1 |

**Theorem-1:** Every right transitive AG-groupoid is commutative semigroup.

**Proof:** Let $S$ be a right transitive AG-groupoid, and let $a,b \in S$, then

$$ab = ab \cdot bb = (ab \cdot bb)bb$$
$$= (bb \cdot bb)ab = bb \cdot ab = ba$$
$$\Rightarrow ab = ba.$$

Thus $S$ is commutative AG-groupoid and hence is associative.

Due to their associative nature, we avoid discussing right transitive AG-groupoids in detail.

### 3.2 LEFT TRANSITIVE AG-GROUPOIDS

**Definition-2:** An AG-groupoid $S$ is called left transitive AG-groupoid if

$$ab \cdot ac = bc \quad \forall\ a,b,c \in S \qquad (3.1)$$

Peter, V. and Stevanovic N. (Peter, V. *et al.*, 1995), provides a procedure to test any groupoid for AG-groupoid, AG*-groupoid and AG**-groupoid. Here we introduce a test to check an arbitrary AG-groupoid $(S, \cdot)$ for left transitive AG-groupoid. For this we have performed the following binary operation for some fixed $x \in S$ and $a,b \in S$

$$a * b = xa \cdot xb \qquad (3.2)$$

(3.1) holds if,

$$a * b = a \cdot b \qquad (3.3)$$

To construct table of operation $"*"$ for any fixed $x \in S$, we rewrite $x$-row of the $"\cdot"$ table as an index row of the new table and multiplying it by the elements of that row that is $x$-row of the $"\cdot"$ table from the left. If the tables for the operation $"*"$ for each $x \in S$ coincides with the $"\cdot"$ table then (3.3) holds and hence the AG-groupoid is left transitive AG-groupoid. To illustrate the procedure, in the extended table for the following example, the tables on the right of the $"\cdot"$ table is obtained for the operation, $"*"$.

**Example-2:** Check the following AG-groupoids for left transitive AG-groupoid.

| · | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 1 | 2 | 3 |
| 2 | 3 | 1 | 2 |
| 3 | 2 | 3 | 1 |

(i)

| · | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 3 |
| 3 | 1 | 2 | 1 |

(ii)

(a). We extend Table (i) in the way as described above to get the following:

| · | 1 | 2 | 3 | 1 | 2 | 3 | 3 | 1 | 2 | 2 | 3 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
| 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 |
| 3 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 | 1 |

It is clear from the extended table that tables on the right coincide with the $"*"$ table. Hence $G$ is left transitive AG-groupoid.

(b) To check Table (ii) for the desired property, we extend the table as described below.

| · | 1 | 2 | 3 | 1 | 1 | 1 | 1 | 1 | 3 | 1 | 2 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 3 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

The tables on the right do not coincide with the $"\cdot"$ table. Hence $G$ is not left transitive AG-groupoid.

## 4. CHARACTERIZATION OF LEFT TRANSITIVE AG-GROUPOIDS

In this section, we will discuss various relations of left transitive AG-groupoids with already known classes of AG-groupoids. We start with the following simple lemma which proves that the class of $T^2$-AG-groupoids (left transitive AG-groupoid) is contained in the class of uni-potent AG-groupoids. This fact further assists to prove that every left transitive AG-groupoid is

$T^2$-AG-groupoid and vice versa. For detailed study of $T^2$-AG-groupoids we suggest (Ahmad *et al.*, 2013, Shah *et al.*, 2013).

**Lemma-1:** Every $T^2$-AG-groupoid is uni-potent AG-groupoid.

**Proof:** Let $S$ be a $T^2$-AG-groupoid and let $b$ be any element of $S$, then using the definition of $T^2$-AG-groupoid, we have

$$\forall a,b,c \in S, ab = ab \Rightarrow a^2 = b^2.$$

Hence $S$ is uni-potent AG-groupoid.

**Corollary-1:** Every unipotent AG-groupoid has a unique idempotent element.

Using the above fact, we are ready to prove that the class of $T^2$-AG-groupoids is the same as our class of left transitive AG-groupoids.

**Theorem-2:** An AG-groupoid S is left transitive AG-groupoid if and only if it is $T^2$-AG-groupoid.

**Proof:** Let $S$ be a left transitive AG-groupoid and $a,b,c,d \in S.$ Assume that $ab = cd$ since $S$ is left transitive we have

$$ac = ba \cdot bc = (ab \cdot aa)(bc).$$

Which implies that by left invertive law; $ac = (bc \cdot aa)(ab).$ Repeatedly using the medial and left invertive laws and the assumption $ab = cd$ we get,

$$ac = (bc \cdot c)(aa \cdot d) = (cc \cdot b)(aa \cdot d)$$
$$= (cc \cdot aa)(bd) = (ca \cdot ca)(bd).$$

Thus the left transitivity in $S$ and the medial law further implies that

$$ac = aa \cdot bd = ab \cdot ad = bd \Rightarrow ac = bd.$$

Hence $S$ is $T^2$-AG-groupoid.

Conversely, let $S$ be a $T^2$-AG-groupoid and let $a,b,c \in S.$ Then by Lemma-1 there exist $c$ in $S$ such that $a^2 = c^2 = c \ \forall a,b,c \in S.$ Now by medial law $ab \cdot ac = aa \cdot bc,$ this implies by Lemma-1 and Corollary-1 that $ab \cdot ac = c \cdot bc \Rightarrow aa \cdot bc = c \cdot bc$ (by medial law).

Again Corollary-1 implies that $bb \cdot bc = c \cdot bc,$ which by left invertive law becomes $(bc \cdot b)b = c \cdot b.$ This; further implies that the use of the $T^2$-AG-groupoid and the left invertive law alternatively; we have

$$(bc \cdot b)c = b \cdot bc \Rightarrow cb \cdot bc = b \cdot bc$$
$$\Rightarrow cb \cdot b = bc \cdot bc \Rightarrow bb \cdot c = bc \cdot bc.$$

Thus by Lemma-1 and Corollary-1 we get $cc = bc \cdot bc.$ Hence $S$ is left transitive AG-groupoid.

Therefore, the left transitive AG-groupoids and the previous $T^2$-AG-groupoids coincide. Since we know the facts that every $T^2$-AG-groupoid is $T^1$-AG-groupoid, each $T^1$-AG-groupoid is $T^3$-AG-groupoid and $T^1$-AG-groupoid is paramedial and AG** -groupoid (Ahmad *et al.*, 2013, Shah *et al.*, 2013, Rashad *et al.*, 2013) thus we have the following corollary in terms of transitive AG-groupoids.

**Corollary-2:** Every left transitive AG-groupoid is:

(i) $T^1$-AG-groupoid
(ii) AG** -groupoid
(iii) $T^3$-AG-groupoid
(iv) Paramedial AG-groupoid
(v) Left nuclear square AG-groupoid
(vi) Unipotent AG-groupoid

**Theorem-3:** Every left transitive AG-groupoid is transitively commutative AG-groupoid.

**Proof:** Let $S$ be a left transitive AG-groupoid and let $a,b,c \in S.$ Assume that $ab = ba$ & $bc = cb.$ Since $S$ is left transitive we have, $ac = ba \cdot bc.$ Thus, by use of left invertive law and assumption implies that, $ac = (bc \cdot a)b = (cb \cdot a)b$ and once again using the left invertive law and the assumption we get $ac = (ab \cdot c)b = (ba \cdot c)b = bc \cdot ba.$ But since $S$ is left transitive we have $ac = ca.$ Hence $S$ is left transitively commutative AG-groupoid.

The converse of the previous theorem is not true (see example-3).

**Example-3:** Transitively commutative AG-groupoid of order 4 which is not a left transitive AG-groupoid.

| * | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 2 | 3 | 1 | 4 |
| 2 | 4 | 1 | 3 | 2 |
| 3 | 3 | 2 | 4 | 1 |
| 4 | 1 | 4 | 2 | 3 |

The following examples show that neither left transitive AG-groupoid nor AG-monoid is quasi-cancellative AG-groupoid. However, we can prove that the intersection of these two classes indeed become quasi-cancellative AG-groupoid.

**Example-4:** (i) Left transitive AG-groupoids of order 4 which is not quasi-cancellative AG-groupoid. (ii) AG-monoid of order 4 which is not quasi-cancellative AG-groupoid.

| · | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 1 | 2 | 2 | 4 |
| 2 | 4 | 1 | 1 | 2 |
| 3 | 4 | 1 | 1 | 2 |
| 4 | 2 | 4 | 4 | 1 |

(i)

| · | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 1 | 3 |
| 3 | 1 | 1 | 1 | 2 |
| 4 | 1 | 2 | 3 | 4 |

(ii)

**Theorem-4:** Every left transitive AG-monoid is quasi-cancellative AG-groupoid.

**Proof:** Let $S$ be a left transitive AG-monoid and $a,b,c \in S$. For quasi-cancellative AG-groupoid, we have to prove that $S$ is both left quasi-cancellative AG-groupoid and right quasi-cancellative AG-groupoid. For left quasi-cancellative, let $aa = ab$ & $bb = ba$, then

$$a = ea = be \cdot ba = be \cdot bb = eb = b$$
$$\Rightarrow a = b.$$

Thus $S$ is left quasi-cancellative AG-groupoid. Similarly it can be proved that $S$ is right quasi-cancellative AG-groupoid. Hence $S$ is quasi-cancellative and the theorem is proved.

The following theorem explains the conditions for which a left transitive AG-groupoid becomes a commutative semigroup.

**Theorem-5:** For every left transitive AG-groupoid $S$ the following are equivalent:

(i) $S$ is flexible AG-groupoid.
(ii) $S$ is right alternative AG-groupoid.
(iii) $S$ is AG*-groupoid.
(iv) $S$ is middle nuclear square AG-groupoid.
(v) $S$ is right nuclear square AG-groupoid.
(vi) $S$ is commutative semigroup.

**Proof:** Let $S$ be a left transitive AG-groupoid and $a,b,c \in S$. (i) $\Rightarrow$ (ii) Assume that (i) holds. Now by left invertive law $ab \cdot b = bb \cdot a.$ since $S$ is left transitive, this gives $ab \cdot b = (ab \cdot ab)a.$ Which by left invertive law implies that $ab \cdot b = (a \cdot ab)(ab).$ The left transitivity and medial law further imply that;

$$ab \cdot b = (a \cdot ab)(ba \cdot bb)$$
$$= (a \cdot ba)(ab \cdot bb).$$

By assumption we get; $ab \cdot b = (ab \cdot a)(ab \cdot bb).$ Since $S$ is left transitive, we have $ab \cdot b = a \cdot bb.$ Thus $S$ is right alternative and hence (ii) holds.

(ii) $\Rightarrow$ (iii) Now assume that (ii) holds. Since by left invertive law $ab \cdot c = cb \cdot a.$ As $S$ is left transitive, so by this and by left invertive law we have $ab \cdot c = (bc \cdot bb)a = (a \cdot bb)(bc).$ Thus using the assumption we have $ab \cdot c = (ab \cdot b)(bc).$ This further implies by the left transitivity in $S$ and by medial law that

$$ab \cdot c = (ab \cdot b)(ab \cdot ac)$$
$$= (ab \cdot ab)(b \cdot ac) = (bb)(b \cdot ac)$$
$$b \cdot ac \Rightarrow ab \cdot c = b \cdot ac.$$

Thus $S$ is AG* -groupoid and hence (iii) holds.

(iii) $\Rightarrow$ (iv) Assume that (iii) holds, i.e. $S$ is AG$^*$-groupoid, then $ab^2 \cdot c = b^2 \cdot ac.$ Since $S$ is left transitive this implies that $ab^2 \cdot c = b^2(ba \cdot bc).$ and by assumption again this implies that $ab^2 \cdot c = (ba \cdot bb)bc.$ Again since $S$ is left transitive and by assumption we have

$$ab^2 \cdot c = ab \cdot bc = (b \cdot ab)c$$
$$= (ab \cdot b)c.$$

Thus by left invertive law and assumption we have $ab^2 \cdot c = (bb \cdot a)c = a \cdot b^2 c.$

Hence $ab^2 \cdot c = a \cdot b^2 c$ and thus $S$ is middle nuclear square, i.e. (iv) holds.

(iv) $\Rightarrow$ (v) Next assume that $S$ is middle nuclear square AG-groupoid. Since $S$ is left transitive, we have $a \cdot bc^2 = a(ab \cdot ac^2)$ this implies by medial law that $a \cdot bc^2 = a(aa \cdot bc^2),$ which by assumption implies that $a \cdot bc^2 = aa^2 \cdot bc^2.$ This by the twice use of medial law gives $a \cdot bc^2 = ab \cdot a^2 c^2 = ab(ac \cdot ac).$ Since $S$ is left transitive we have $a \cdot bc^2 = ab \cdot c^2.$ Hence $S$ is right nuclear square and thus (v) holds.

(v) $\Rightarrow$ (vi) Assume that (v) holds. Since $S$ is left transitive, we have

$$ab = aa \cdot ab = (ba \cdot ba)(ab)$$
$$= (ba \cdot ba)(ba \cdot bb),$$

which by assumption implies that $ab = ((ba \cdot ba)(ba))bb.$ This further implies by left invertive law that

$$ab = (bb \cdot ba)(ba \cdot ba).$$

Thus by left transitive and medial properties in $S$ we have $ab = ba(ba \cdot ba) = (ba)(bb \cdot aa),$ but since $S$ is right nuclear square by assumption we have $ab = (ba \cdot bb)aa.$ Thus left transitivity implies that $ab = ab \cdot aa \Rightarrow ab = ba.$ Hence $S$ is commutative, and since every commutative AG-groupoid is associative. Therefore, $S$ is commutative semi-group.

Finally,(vi) $\Rightarrow$ (i) is obvious. Hence the theorem is proved.

In the following we define a congruence relation $\rho$ on the left transitive AG-groupoids. We also show that the congruence is separative, i.e., if $ab\rho a^2,\ ab\rho b^2$ then $a\rho b.$

Theorem-6: Let $S$ be a left transitive AG-groupoid. Then the following are true for a relation $\rho$ on $S$ defined by $a\rho b \Leftrightarrow a^2 = b^2 \quad \forall a,b \in S.$

(i) $\rho$ is a congruence relation

(ii) $\rho$ is separative

(iii) $ab\,\rho\,ba$, for any $a,b \in S$

**Proof**: Let S be a left transitive AG-groupoid, and $a,b \in S$.

Let $\rho$ defined on $S$ as given. Then using Lemma 1 it is an easy exercise to prove that $\rho$ is equivalence and hence a congruence relation.

(i) Now to prove that $\rho$ is separative, let $ab\rho a^2,\ ab\rho b^2$.

This implies that $(ab)^2 = (a^2)^2$ and $(ab)^2 = (b^2)^2\ \forall a,b \in S$.

Now by repeated use of the left transitive property and the assumption, we have,

$$a^2 = aa = aa \cdot aa =$$
$$= (a^2)^2 = (ab)^2 = (b^2)^2 =$$
$$= b^2 \cdot b^2 = bb \cdot bb = b^2$$
$$\Rightarrow a^2 = b^2 \Leftrightarrow a\rho b.$$

Thus $\rho$ is separative.

(ii) Now by the use of left transitive AG-groupoid property, medial law, left invertive law and the assumption, we have,

$$(ab)^2 = ab \cdot ab = a^2 \cdot b^2 = ba^2 \cdot bb^2 =$$
$$= (bb^2 \cdot a^2)b = (a^2b^2 \cdot b)b =$$
$$= ((aa \cdot bb)b)b = ((ab \cdot ab)b)b =$$
$$= ((ab)^2 b)b = ((a^2a^2)b)b =$$
$$= (a^2b)b = bb \cdot a^2 =$$
$$= ba \cdot ba = (ba)^2$$
$$\Rightarrow (ab)^2 = (ba)^2 \Leftrightarrow ab\rho ba.$$

## 5. CONCLUSION

The paper extends and investigates the concept of right (left) transitive groupoid $G$ (by Polnijo 1993) satisfying the identity $ac \cdot bc = ab\ \ \forall a,b,c \in G\ (ab \cdot ac = bc\ \ \forall a,b,c \in G)$ to right (left) transitive AG-groupoid. Various results have been investigated such that the class of $T^2$-AG-groupoids (an AG-groupoid satisfying the identity $ab = cd \Rightarrow ca = bd$ and the class of left transitive AG-groupoids coincide. A simple verification test whether an arbitrary AG-groupoid is left transitive or not, is discussed in this paper. Many other results are included such as,

(i) Every left transitive AG-groupoid is transitively commutative AG-groupoid. (ii) For left transitive AG-groupoid the properties of flexibility, right alternativity, AG*, right nuclear square, middle nuclear square and commutative semigroups are equivalent. (iii). Many examples and counterexamples have been provided to support the results produced in this paper.

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