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Simulation of Transient Groundwater Flow in a Well Penetrating Confined Aquifer in Southern Rohri Command

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Abstract: A transient groundwater flow model is developed in the framework of FEM with standard Galerkin approach. The simulation model is applied to a real field problem comprising an area containing homogeneous, anisotropic type confined aquifer located over 75.69 km^2 horizontal impervious layer in Southern Rohri command, at Tando Allahyar district of Sindh province Pakistan. The area is discritized into triangular elements of non-uniform sizes. The confined aquifer contains a sink with radial flow to the well that fully penetrates the aquifer. The model is run for 10 time steps to simulate the transient conditions. The radius of influence of the well after continuous pumping of 88 hours maximally goes up to 900 m. Performance of the model is tested at 4 nodes. The mean residual (i.e. the difference between the predicted and the observed values of piezometric heads) at these nodes on all the 10 time steps varies between 1.44 $\times 10^{-4}$ and 6.61 $\times 10^{-4}$ m. Performance of the model is measured by statistical parameters; ME varies between 0.0060819 and 0.0000014; RMSE varies between 0.009098 and 0.0000258; and Efficiency of the model is evaluated as 99.9 %.

Keywords: Simulation, Finite element method, Groundwater modeling, hydraulic heads

1 INTRODUCTION

The modeling of groundwater is carried out with varying degrees of success to help ensure the effective management of groundwater resources. According to Ashraf and Ahmed (2008), the ability to predict the flow of groundwater is of paramount importance in planning and implementing development projects in the groundwater under the growing demand for freshwater resources. Among the many numerical methods for modeling different types of problems of groundwater flow, the FEM allows an accurate simulation of boundary and flow directions, and changes in the head inside an element (Karlheiz and Moreno 1996). For many of the problems of groundwater flow, the FEM was found to be superior to other numerical methods (Willie and Yeh 1987). Furthermore, the heterogeneity and irregular boundary conditions are easily managed by the FEM. Also, the size of elements can be easily modified to reflect the swiftly changing values of field variables in the FEM.

(Chen and Jiao 1999) used FEM with standard Galerkin scheme to simulate non-darcian one-dimensional vertical flows in well bores and two-dimensional flow in multilayer confining aquifer system. (Robinson and Gallagher 1999) developed a field scale groundwater flow model to simulate groundwater discharge within an unconfined coastal aquifer. (Dogrul and Kadir 2006) used Galerkin based FEM technique to develop a post-processing method to compute local and global mass conserving flow rates at element faces. (Ramesh and Mahesha 2008) simulated Varada basin aquifer system using FEM. KIM (2001) used FEM for solution of two-dimensional parabolic equation for modeling of heat problems. (Zhang, 2007) used a quasi-three-dimensional model for temporal and spatial discretization to avoid spurious oscillation while employing FEM to model groundwater flows.

Thus, in context of realization of FEM as an efficient and widely used numerical tool for solution of partial differential equations (PDEs), almost in all fields of engineering including groundwater hydrology, present study is aimed up to develop a computer model in the framework of FEM to simulate transient state groundwater flow. For this purpose, a two-dimensional mass conservation equation governing transient flow in porous media is selected for its resolution; and consequently the model is implemented to simulate flow field, piezometric heads creating cone of depression in a well fully penetrating confined aquifer and running under transient conditions at Southern Rohri command district Tando Allah Yar, Sindh, Pakistan.

2. Governing Equation and Problem Formulation

The transient state groundwater flow equation for a two-dimensional confined aquifer on a horizontal plane bed, obtained through mass conservation, is selected as governing equation, viz.

$$-\operatorname{div}(\mathbf{q}) \pm Q = S_s \frac{\partial h}{\partial t} \quad \text{on } \Omega \times [0, T] \quad (1)$$

where div- is the divergence operator; S_s- is the storage coefficient for a confined aquifer (dimensionless); h = h(x,y,t)- is the piezometric head (L); $\mathbf{q} = [q_x \;\; q_y]^T$- is depth-integrated Darcy flux, or simply flux, in vector form; q_x- and q_y- are components of Darcy flux in x- and y- lateral directions, respectively ($L^2.T^{-1}$); Ω- is a two-dimensional arbitrary spatial domain; Q = Q (h,x,y) is a general source/sink term that may be combination

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of point sources/sinks such as pumping and injection wells or distributed sources such as recharge from an overlaying vadoze zone; and or head dependent sources such as drains, stream groundwater interaction ($L.T^{-1}$); S is the storativity or specific yield and t- is the time (T).

Eq. (1) is evaluated in the framework of standard Galerkin finite element method for the unknown function, h, then as a characteristic feature of the FEM an approximate solution is obtained for the function and owing to this a residual occurs at each point of the problem domain. The residual will vary from point to point within the domain. The domain is discretized into a mesh of simple triangular elements, whereas for time discretization, a scheme of lumped element type formulation is made in which the Crank-Nicholson scheme is adopted.

3. NUMERICAL METHODOLOGY AND MATERIAL Properties

3.1 Model Development

The model is developed in the framework of Finite Element Method with standard Galerkin approach. The geometry of the domain is discritized into simple triangular elements of non-uniform sizes with nodal points at their vertices. The finite element formulation presented in previous section is implemented in the form of a computer code resulting into a computer model. The computer model, coded in the Fortran-95, is versatile in nature such that it can be implemented for a choice of several types of elements.

3.2 Material Properties

Material properties i.e., the components of saturated hydraulic conductivity can be different for each element in the mesh. Dirichlet boundary conditions consisting of specified piezometric heads and Neumann boundary conditions comprising of specified fluxes can be prescribed. The unknown function, that is piezometric heads are computed at each node and the components of apparent groundwater velocity are computed at the center of each element in the mesh. The computer model reads the program description including node numbers and coordinates, element numbers, material properties, boundary conditions, time step, etc, from a single input file and this information is written to an output file followed by computed values of piezometric heads and apparent groundwater velocity at each time step. The validity and performance of the model is demonstrated by its application to real field situation and comparison of results with the prototype data is carried out; and efficiency of the model is checked through statistical analysis.

5 RESULTS AND DISCUSSION

5.1 Model Application

To test the validity of the simulation model, it is applied to a real field problem. An area containing homogeneous, anisotropic type confined aquifer located over a horizontal impervious layer is considered. The study area lies in Southern Rohri command, 10 miles south of Tando Allahyar city of Sindh province and accessible by Tando Allahyar-Chambar road. The site has an altitude of 17.37 m above the mean sea level. Upper surface of the area is covered by a 15 m thick clay layer characterized by low hydraulic conductivity, thus forming top confining layer of the aquifer, and below this lies very fine to medium sand with median (D_{50}) grain size of 0.20 to 0.35 mm forming a productive aquifer. The sand is normally well sorted with uniformity coefficient (CU) and coefficient of sorting (S_0) varying from 1.42 to 4.30 and 1.35 to 2.41, respectively. Another clay layer exits between 42.5 and 52 m from the ground surface, thus making the aquifer predominantly a confined one with an average thickness of 27.5 m. From the ground surface, average depth to groundwater level is about 4.57 m.

For input to the model, aquifer parameters have been evaluated by using the data of constant-rate pumping test at a tube well carried out by WAPDA (1978). The representative values of storage coefficient, S_s- and transmissivities, T_x- and T_y- are 0.00125, 0.006596 m^2/s and 0.006596 m^2/s, respectively. For the given aquifer the saturated hydraulic conductivities k_x- and k_y- are worked out as 2.399×10^{-4} m/s and 2.399×10^{-4} m/s, respectively.

In order to demonstrate the applicability of the model, the model area is considered as a two-dimensional horizontal domain comprising of the aforementioned confined aquifer, which also contains a sink with radial flow to the well that fully penetrates the aquifer. The area considered for demonstration of the model spreads over 75.69 km^2 with 8700 × 8700 m lateral dimensions; and the sink lies at its centre, which

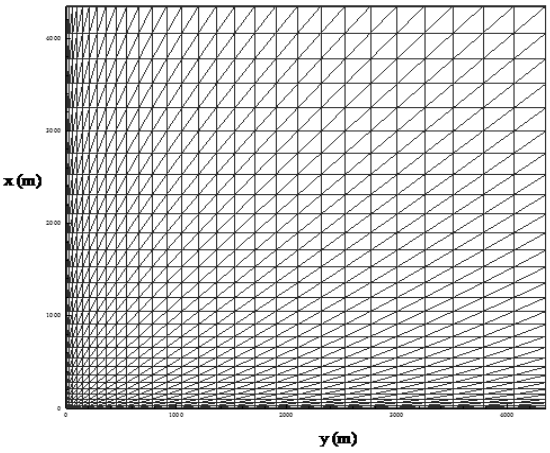


Fig. 2: A finite element mesh of non-uniform triangular sizes.

is allowed to run at a constant pumping rate of 0.043 m^3/sec. While simulating the problem, x-y plane of the model is considered to coincide with horizontal surface formed by piezometric heads of the aquifer and

positioning the origin of the coordinate system at centre of the well, whilst extending positive z-axis vertically downward along the well. Thus, the problem considered is symmetric in nature, hence, a single quadrant of the domain area is considered for simulation. The area is discritized into 1682 triangular elements of non-uniform sizes by 900 nodal points at the vertices of the elements **(Fig. 1).**

In the present case since the bottom layer of the aquifer is impervious (aquiclude), therefore Neumann type boundary with zero flux condition is imposed on it. Moreover, since piezometric head data is available for node numbers 1 through 10, therefore at initial time (t = 0), these nodes are assigned Dirichlet boundary condition, while rest of the boundary nodes are treated as Neumann nodes with zero flux condition.

A sink of constant value (0.043 m^3/sec) is assigned to node 1 during transient simulation. The computation scheme starts with an initial estimate for distribution of the field variable. For this purpose, the model is executed first to perform steady state simulation at initial time, t = 0, to give an initial estimate of unknown field of piezometric heads for the transient simulation to proceed further. The model is run for 10 time steps to simulate the transient conditions. The elapse times are selected as 1, 2, 5, 10, 20, 30, 60, 120, 240 and 360 minutes. In the present study, an implicit scheme i.e. Crank–Nicholson scheme is adopted, which is unconditionally stable. A tolerance limit of 0.001 is adopted to check the convergence during the solution process. Initially, the aquifer parameters such as T and S_s are assigned on the basis of field test results, however, these results are substantiated on the basis of minimization of residuals between simulated and observed heads. For a given value of the storage coefficient, it is also confirmed that the aquifer is predominantly confined. From the T value, it can be concluded that the aquifer has good potential for groundwater flow.

A cross-section of simulated head distribution in radial direction signifying gradient of flow toward the well at different time steps is shown in Fig. 2. Furthermore, from the simulated results it is also inferred that the radius of influence of the well after continuous pumping of 88 hours maximally goes up to 900 m.

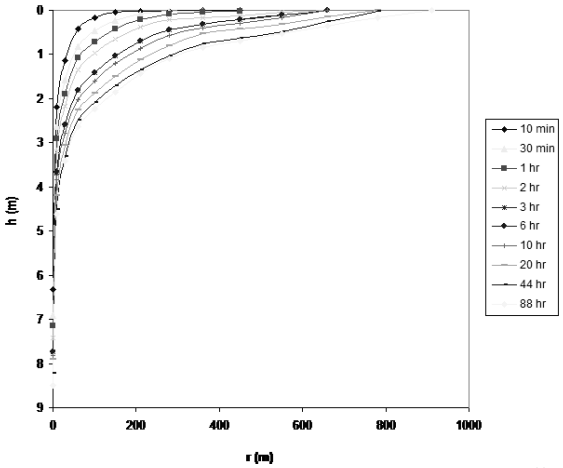


Fig. 2: Simulated head distribution in radial direction at different time steps.

5.2 Model Performance and Validity

Performance of the model can be tested by quantifying model error (residual) i.e. the difference between the predicted and the observed values of piezometric heads at the measuring locations as shown in Table 1.The mean residual varies between 1.44×10^{-4} and 6.61×10^{-4} m for the nodes under consideration. These results show that simulated heads match the observed heads fairly well for all the cases; thus indicating no significant difference between observed and simulated values.

Table 1: Observed and simulated piezometric heads under transient conditions

Time (hrs)	Piezometric heads (m)											
	Node1 $(D.W)_1$			Node 3 $(O.W)_3$			Node 4 $(O.W)_4$			Node 9 $(O.W)_9$		
	Obs. (ho_i)	Simulated (hs_i)	Residual (ho_i - hs_i)	Obs. (ho_i)	Simulated (hs_i)	Residual (ho_i - hs_i)	Obs. (ho_i)	Simulated (hs_i)	Residual (ho_i - hs_i)	Obs. (ho_i)	Simulated (hs_i)	Residual (ho_i - hs_i)
0.1	6.16	6.157000	-0.000040	0.91	0.910300	0.001052	0.25	0.252364	0.000620	0.02	0.000000	0.021336
0.5	6.91	6.909800	0.000016	1.57	1.569400	0.000320	0.80	0.795716	-0.000188	0.02	0.027840	-0.003456
1	7.15	7.147600	-0.000040	1.89	1.889580	0.000180	1.05	1.048884	-0.000372	0.03	0.037150	-0.003622
5	7.68	7.677900	0.000012	2.50	2.495620	0.000692	1.68	1.679292	0.000156	0.26	0.253012	0.006068
10	7.82	7.818100	0.000020	2.76	2.756660	0.001780	2.00	2.001252	0.001284	0.41	0.398160	0.007224
18	7.88	7.879100	-0.000020	2.97	2.967640	0.001112	2.19	2.184818	0.000598	0.49	0.484094	0.009682
28	7.99	7.988800	0.000008	3.22	3.217800	0.000888	2.34	2.343598	0.000314	0.60	0.587994	0.012462
40	8.12	8.122900	0.000020	3.28	3.278220	0.001428	2.43	2.431432	0.000872	0.73	0.724586	0.003886
56	8.41	8.412500	-0.000020	3.32	3.323060	-0.000740	2.53	2.528054	-0.001262	0.80	0.790748	0.007828
74	8.47	8.473400	0.000040	3.35	3.353080	-0.000280	2.61	2.612902	-0.000766	0.82	0.820770	0.002190
88	8.50	8.503900	0.000020	3.40	3.397680	0.000840	2.61	2.611812	0.000324	0.83	0.825753	0.003303

D.W = Discharging well; O.W = Observation well

Furthermore, performance of the model is quantified by computing statistical parameters, viz. mean error (ME), root mean square error (RMSE) and model efficiency (EF). The parameters indicate the goodness-of-fit between measured and simulated values. The error parameters used for evaluating the validation quality of the model are tabulated in Table 2. Efficiency of the model evaluated on the basis of simulated and observed data at the well site is 99.9 %, which is good omen to rank the model efficiency (EF) very satisfactory.

Table 2: Goodness-of-fit statistics between observed and simulated piezometric heads

Measures of error	Well location (Node number)			
	Node 1 $(D.W)_1$	Node 3 $(O.W)_3$	Node 4 $(O.W)_4$	Node 9 $(O.W)_9$
M.E (m)	0.0000014	0.0006611	0.000144	0.0060819
RMSE	0.0000258	0.0009097	0.0007219	0.009098
R^2	1.0	1.0	1.0	0.9995

D.W = Discharging well; O.W = Observation well

6 CONCLUSIONS

A computer model is developed in the framework of finite element method with standard Galerkin approach. The geometry of the domain is discritized into simple triangular elements of non-uniform sizes with nodal points at their vertices. The model simulates groundwater flow under transient conditions. The finite element formulation presented in section 3 is implemented in the form of a computer code resulting into a computer model. The model is coded in Fortran-95 and is applied to a real field problem of two-dimensional horizontal domain comprising of confined aquifer, which also contains a sink with radial flow to the well. The results, simulated from the model pertain to identification of flow field, the radius of influence of the well due to sink and equipotential head distribution signifying a cone of depression at the well site. The results are substantiated on the basis of minimization of residuals between observed and simulated piezometric heads. Performance of the model is tested statistically by comparing simulated values against the observed ones. Efficiency of the model, evaluated is on the basis of simulated and observed data at the well site, is 99.9 %., which is good sign to grade the model very acceptable. The model is adaptable in nature such that it can be implemented for a choice of several types of elements and solves other problems of similar nature with slight alterations.

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