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Numerical Simulation of Flow through a Periodically Constricted Tube: Effects of Inertia and Undulation on Flow Structure

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Abstract: Present research work focus on a flow of Newtonian fluid through periodically constricted tube using a time–marching finite element method. A semi–implicit algorithm, so called, Taylor–Galerken/pressure–correction scheme is employed to seek the steady–state numerical predictions. The effects of inertia and impact of undulation on flow structure and friction factor is investigated. Numerical solutions for Newtonian fluid are compared and close agreement is realised against experimental as well as other numerical results.

Keywords: Finite Element Method, Newtonian Fluids, Periodically Constricted Tube.!

1. INTRODUCTION

Flow through a periodically constricted tube has an industrial importance and present challenge to numerical researchers, due to its variety of applications and unavailable close form of the problem. This problem is simulated extensively adopting various numerical techniques. Flow through a periodically constricted tube was originally investigated by (Lahbabi, and Chang, 1986.) and subsequently by (Baloch, *et al.,* 1995, 2007), using the Finite Difference methods for testing inertial flow of Newtonian fluid and the effect on flow resistance. Later on, (Pilitsis, *et al.,* 1991) presented Finite Element predictions for both Newtonian and Non–Newtonian fluids using Power–Law shear viscosity and extensional viscosity, and demonstrated influence of the inertia and extensional viscosity on flow resistance.

The numerical method used by (Crochet, *et al.,* 1990) employed a low–order finite difference approximation of the derivatives which was criticised by (Pilitsis, *et al.,* 1991,) and presented enough evidence on invalidity and inaccuracy of their results. Recently, several finite element methods have used to solve this problem by different researchers (Baloch, *et al.,* 2007,). Present study investigates the Newtonian flow through a periodically constricted tube. This problem is particularly chosen as suitable test case as it presence both converging and diverging sections and reduce shear and elongational motion at a same time and relatively have simple domain. Two different undulation levels, i.e., thirty percent (30%) and fifty percent (50 %) are selected to investigate the effects of undulation and influence of inertia (Re) on flow structure, fraction factor "*f*", and flow resistance *f*Re is investigated. The growth of recirculation flow rate (Q_v) at different Reynolds number from zero to one thousand, are particularly focused. At both undulation levels, start of recirculation at critical points are realised in present work. The numerical simulations utilise a finite element time–stepping technique based on a semi implicit Taylor–Galerken/pressure–correction scheme by (Pilitsis, *et al.,* 1991) and solutions are obtained for inertial flows of Newtonian fluids to compare against by (Lahbabi, *et al.* 1986.), (Pilitsis, *et al.,* 1991) and (Baloch, *et al.,* 2007,).

2. MATERIAL AND METHODS

Newtonian flow through periodically constricted tube is considered as a domain of the interest. As on the assumption that flow is axi-symmetric in circular tube, therefore, the flow is only considered for thin film in upper–half of undulating tube, the schematic diagram of domain is shown in **(Fig.1).** The radius of the undulating tube wall along the flow axis is given by the following formula:

$$r_w = R\left[1 + \lambda Cos\left(\frac{2\pi z}{L}\right)\right]$$

Where R is the average radius of the equivalent straight tube, both α and L are the dimensionless amplitude of the undulation and wave length respectively. Let (r, z) be coordinates where z-axis is taken as axial direction along the axis in the flow of fibre suspension, while r is along radial direction respectively.

In order to specify the well–posed problem the essential initial and mixed Dirichlet and Neumann boundary conditions are taken as follows:

Initial and Boundary conditions for Momentum Transport Equation	
Initial Conditions	Quiescent flow condition $v_r(z,0) = v_z(r, 0) = 0$
Boundary Conditions	1. At solid wall: no-slip $v_r = v_z = 0$. 2. At exit: $v_r = p = 0$, v_z =? 3. At axis of symmetry: $v_r = 0$ and traction free condition $\frac{\partial v_z}{\partial r} = 0.$ 4. At Inlet: $v_z = v_{\max}\left\{1-\left(\frac{r}{R}\right)^2\right\}$

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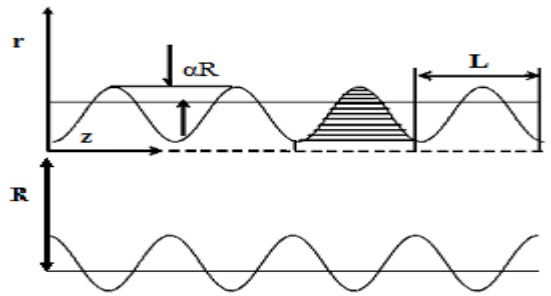
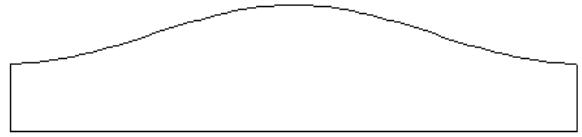
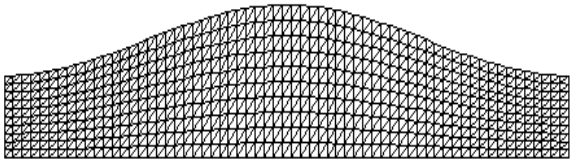


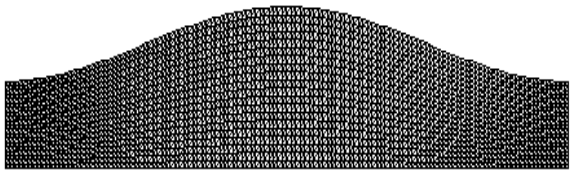
Fig.1. Domain for flow through PCT



Domain of interest



Coars Mesh



Refined Mesh

Fig.2. Domain of Interest and Finite Element Meshes for PCT.

3. Governing system of equations

Consider the flow of fibre suspension for Newtonian solvent through PCT governed by the conservation of mass and axisymmetric momentum transport equations in cylindrical polar co-ordinates in the absence of body forces and non–dimensional form equations can be presented as:

Continuity equation: $\nabla.\boldsymbol{v}=0$

Momentum Equation:

$$\frac{\partial \boldsymbol{v}}{\partial t}=\frac{1}{\mathrm{Re}}\nabla^2\boldsymbol{v}-(\boldsymbol{v}.\nabla)\boldsymbol{v}-\nabla p$$

Where $\boldsymbol{v} \equiv (v_r, v_z)$ is the velocity vector field in r and z directions, **p** is isotropic fluid pressure and t is time. Whilst, Re denotes the Reynolds number defined as: $\mathrm{Re}=\frac{\rho VR}{\mu_o}$.

4. Numerical Scheme

For the Newtonian flow of fluid the numerical simulations utilise a finite element time–stepping technique based on a semi–implicit Taylor–Galerken/pressure–correction scheme (Baloch, et al. 2007,**).** To describe this method, summary is given here for completeness. This technique contains time discretization in Taylor series combined with predicator-corrector scheme. To find the second order time derivatives, two steps Lax–Wendroff approach have been employed. These schemes show an extensive development in accurateness and steadiness with respect to both finite deference and Euler–Galerken finite element Schemes. Pressure–correction method assures second order accuracy and stability through linearised energy analysis, and when it combined with predicator- corrector Taylor–Galerken technique, the base for present numerical scheme adopted in this research work is provided to obtain the time independent results.

The numerical simulations are conducted in multi–stage form. Initially, at stage one solution is obtained at half time step. In second part of stage one, an intermediate non-solenoidal velocity field has been calculated. At stage two Pressure difference is calculated by solving the Poisson equation. The Crank-Nicolson scheme is adopted to get the second order level of accuracy, while solenoidal velocity has been obtained in third and final stage. Both stages one and three are governed by the augmented mass matrices, and are iteratively solved very efficiently in a handful of iteration by Jacoby Method which avoids the system matrix assembly that is helpful component for solution process. At stage two we adopted direct Choleski solution method, because the pressure matrix is symmetric and is positive definite with bounded structure. Solving the problem, equations are discritised through standard Galerken weighted residual method and mixed velocity pressure formulation is used for finite element approximation and triangular element chosen here for domain. Both piecewise linear and quadratic functions for velocity and pressure are employed.

Stage–1(a): Given $\left(v^n, p^n\right)$, find $v^{n+\frac{1}{2}}$ such that

$$\left\{\left(\frac{2}{\Delta t}\left(v^{n+\frac{1}{2}}-v^n\right),v\right)+\frac{1}{2\,\mathrm{Re}}\left(\nabla\left(\left(v^{n+\frac{1}{2}}-v^n\right),\nabla v\right)\right)\right\}=\left(p^n-\frac{1}{\mathrm{Re}}\nabla v^n,\nabla v\right)-\left(\left((v.\nabla)v\right)^n,v\right)$$

Stage–1(b): Given $\left(v^n, v^{n+\frac{1}{2}}, p^n\right)$, find v^* such that:

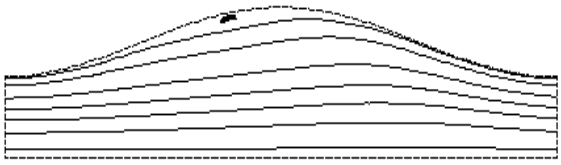
$$\left\{\left(\frac{1}{\Delta t}\left(v^{*}-v^{n}\right),v\right)+\frac{1}{2\,\mathrm{Re}}\left(\nabla\left(v^{*}-v^{n}\right),\nabla v\right)\right\}=\left(p^{n}-\frac{1}{\mathrm{Re}}\nabla v^{n},\nabla v\right)-\left(\left((v.\nabla)v\right)^{n+\frac{1}{2}},v\right)$$

Stage–2: Given v^{*} and p^{n}, find $p^{n+1}-p^{n}$ such that: $\left(\theta\nabla\left(p^{n+1}-p^{n}\right),\nabla q\right)=\frac{-1}{\Delta t}\left(\nabla v^{*},q\right)$

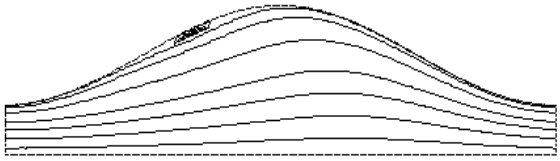
Stage–3: Given v^{*} and $p^{n+1}-p^{n}$, find v^{n+1} such that: $\left(\frac{1}{\Delta t}\left(v^{n+1}-v^{*}\right),v\right)=\left(\theta\left(p^{n+1}-p^{n}\right),\nabla v\right)$

5. DISCUSSIONS

Numerical computations are carried out for Newtonian flow through two different undulation levels of the domain, to investigate the effect of amplitude of undulation, influence of inertia on flow structure in the form of streamlines and pressure isobars. Start of recirculation has been identified at different critical limit points of Reynolds number values. Fraction factor, flow resistance and vortex enhancement at different Reynolds number are also investigated. Numerical solutions have been obtained for inertial flows of Newtonian fluids to compare against experimental data, investigated in the literature (Lahbabi and Chang, 1986) and other numerical results produced by (Crochet, *et al.,* 1990) and (Pilitsis, *et al.,* 1991,). For each undulation level, streamlines projections are plotted at constant incremental values in two main parts, core flow and recirculation area. Seven contours are chosen in the main core flow region, for thirty percent undulation, that are, 0.001, 0.01, 0.05 0.1.0.15, 0.2 and 0.24 (Q/2) on the line of symmetry of non dimensional values of stream functions while in the fifty percent undulation, again seven contours are chosen to describe the core flow, which are, 0.001, 0.02, 0.04, 0.06, 0.08, 0.1 and 0.12. In all the undulation levels, in the region where the vortex development occurs, $\psi_{min:}$ is employed to represent the slower regions. All contours in a recirculation region represented from the centre of the vortex i.e., innermost contour to the periphery $\psi_{per:}$ of the vortex i.e., outermost contour. Two finite element meshes are adopted for computation to validate the algorithm, which has been used in this investigation.



Re=41



Re=28

Fig.3. Streamlines for both thirty and fifty percent undulation at critical Reynolds numbers.

5.1 Effects of Inertia on Flow Structure

The inertial flow of Newtonian fluid through all selected levels of periodically constricted tube is analysed to understand the effects of inertia on the flow structure. Streamline projections are described here with increasing the values of Reynolds number (Re). In **(Fig. 3)**, from computed data it is clear that in thirty percent undulation no vortex is observed at low Reynolds number up to 40, and streamlines representing main core flows are almost attached with undulation wall. The development of vortex starts approximately at Re = 41 close to undulation upstream wall. As Reynolds number increases vortex centre moving from upstream to the centre of the undulation and grow larger. This process of the development of vortex grows more and strengthens, and the centre of vortex moves further to the centre and occupy whole undulation. Whilst, in fifty percent undulation, recirculation develops at low Reynolds number Re= 28 and similar phenomenon of vortex enhancement occurs. In **(Fig. 4–5),** recirculation increased as inertia increased at high Reynolds number and size of vortex is enhanced as level of undulation increased and observed at the centre of undulation. At Reynolds number 500, core flows becomes almost straight, vortex enhanced more, and centre of the vortex shifted towards the downstream, while by increasing the inertia at Re=1000, it is observed that centre of the larger recirculation region moves further to downstream and instability is observed in the shape of the vortex, the main core flows observed very much straight. In **(Table- 1),** the numerical results of present study are compared against the experimental results of (Lahbabi, and Chang, 1986.), and numerical results of (Pilitsis, *et al.,* 1991,) and (Baloch, *et al.,* 2007).



Re=100

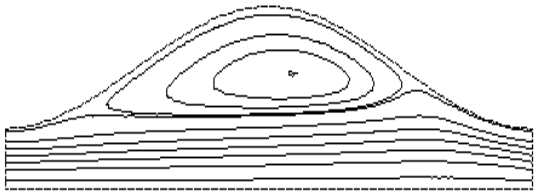


Re=500

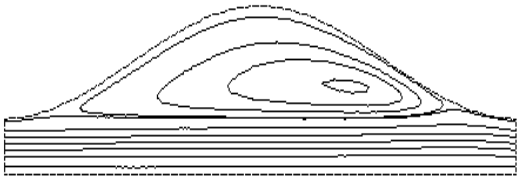


Re=1000

Fig.4. Streamlines for thirty percent undulation with increasing Reynolds Numbers.



Re=100



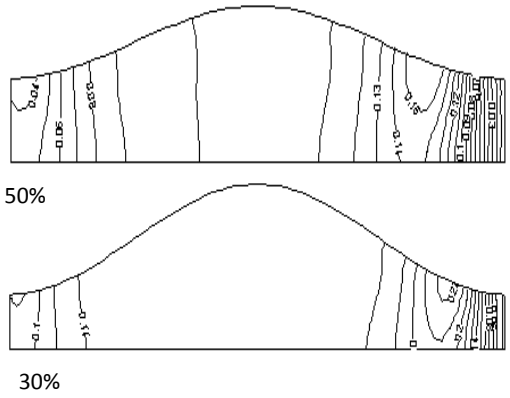
Re=500



Re=1000

Fig.5. Streamlines for fifty percent undulation with increasing Reynolds Numbers

For the flow of pure Newtonian case at Re = 500, pressure drop is demonstrated in figure-6 within periodically constricted tube at both undulation levels of 30% and 50%. Pressure contours in thirty percent undulation are plotted from 5.5126e-023 to 0.16003, whilst for fifty percent the contours are plotted from 7.1593e-023 to 0.24885. The figure clearly illustrates that maximum pressure drop moves from inlet in the vicinity of converged wall to outlet in the neighbourhood of diverging wall. Therefore, non–linear inertial influence on the friction factor (*fRe)* as function of rise in Reynolds number. However, in recirculation region non–linearity is not observed. It is extremely complex flow phenomenon, even at low inertial values flow resistance is also low where recirculation region is not developed. On the contrary, the value of friction factor increased than the corresponding Stokian value. The predicated numerical results in this study are investigated at three different maximum velocities. Due to non–availability of precise information on flow–rate solution are not in fully agreement with the experimental results of (Lahbabi, and Chang, 1986.), and numerical predictions of (Pilitsis, *et al.,* 1991) and (Baloch, *et al.,* 2007)



50%

30%

Fig. 6. Graph of Pressure in both 30% and 50% undulation at Re= 500.

Table–1: Comparison Table For Flow Risistance (Fre) Of Newtonian Fluid

	Lahbabi and Chang	Pilitsis, et al.,	Baloch and Memon	Present Study		
				(V_{max}= 2) ψ_{max} 0.245	V_{max} = 1.299224	V_{max} = 1.297959
00	26.4	26.4483	26.382	26.674	26.724	26.700
12	27.1	27.179	26.913	26.891	26.815	26.800
22	28.5	28.5536	27.889	27.346	27.238	27.100
51	31.7	31.7484	30.643	32.525	29.2026	30.600
63	32.6	32.6758	32.601	34.468	31.520	31.500
73	33.4	33.4488	33.150	35.754	33.117	33.100
132	36.7	36.5264	35.961	40.458	38.928	37.100
264	39.7	40.2446	39.964	45.519	42.401	42.400
400	40.6	42.3491	41.966	48.655	45.440	45.400
600	40.9	44.0371	43.103	52.162	48.521	48.500
783	41.8	45.5828	44.058	54.880	50.815	50.800

5.2 Impact of undulation on flow structure

The impact of amplitude of undulation on flow structure for Newtonian flow is analysed in the form of flow structure, vortex enhancement and fraction factor. Streamlines represents flow structure and recirculation flow rate demonstrate the vortex growth in the form of size and intensity. The streamline projections show the distinct effects on flow field at both the levels of undulation. The behaviour of flow analysed at various Reynolds number. It is observed that development of recirculation starts when value of inertia decreased as level of undulation increase. For both undulations the graph of the vortex intensity is plotted in **(Fig.7)** from Reynolds number 1 to 1000.

Another primary interest of the present investigation is to analyse the flow resistance (friction factor) as the function of Reynolds number and relative pressure difference, defined as:

$$f = \frac{\pi^2 \Delta p R^5}{\rho L Q^2},$$

Where R is the average radius, L is the amplitude wave length, Q is the flow rate, ρ is the fluid density and Δp is relative pressure difference and can be defined as:

$$\Delta p = \frac{\mathrm{Re}.\left\| p_{\max} - p_{\min} \right\|_{\mathrm{Re}>1}}{\left\| p_{\max} \right\|_{\mathrm{Re}=1}}$$

In **(Fig. 8)** the computed results of friction factor are graphically illustrated for both undulation levels. The values of inertia are taken from zero to Re = 1000. Presented figure clearly vindicates that as inertia increase, friction factor increase non–linearly in an exponential fashion.

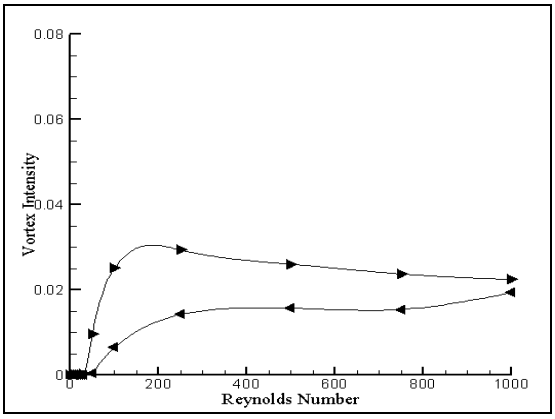


Fig.7. Vortex Intensity at both undulations with increasing Reynolds numbers.

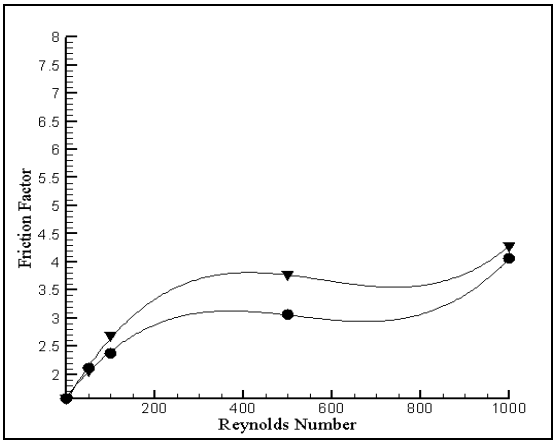


Fig. 8. Friction Factor (*fRe*) on both undulation levels with increasing Reynolds Number.

The empirical relationships for friction factor are developed based on computed solutions against the five different Reynolds number for both the levels of undulation are calculated and presented in **(Fig. 9)** and tabulated in **(Table–2)**.

Table 2: Empirical Equations of thirty and fifty percent undulation.

S. No:	Fibre Level	Empirical Equations	
		30% undulation	50% Undulation
1	00	X=(0.0056)Re + 1.51540	X= (0.0025)Re + 2.0488

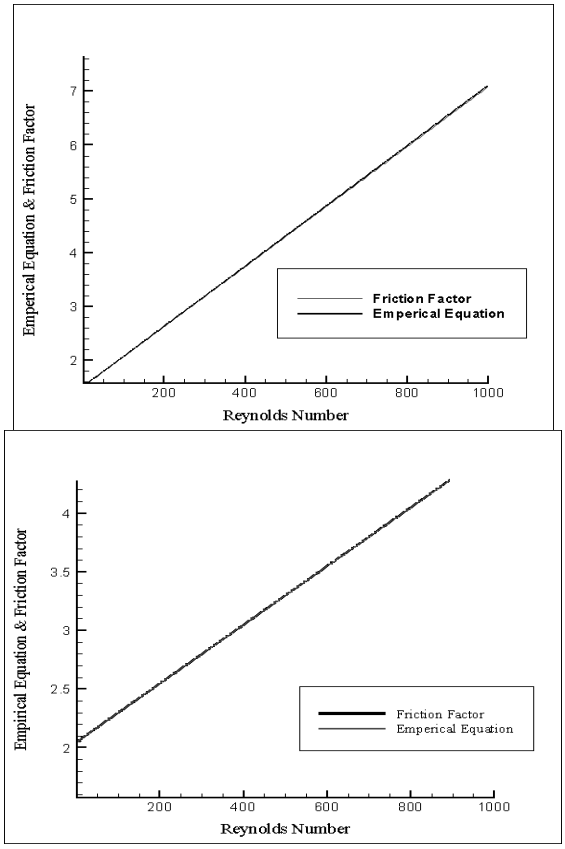


Fig. 9.Graph of the Empirical equations on both undulations

6. CONCLUSIONS

Adopting Taylor–Galerkin/Pressure–Correction Scheme the steady–state numerical solutions are obtained. The comparison is made between computed solutions against other numerical predictions and experimental results. The influence of inertia (Re) on flow structure, fraction factor "*f*" and flow resistance "*f*Re" is investigated. The vortex enhancement (Q_v) at different Reynolds number is particularly focused. At different undulation levels, start of recirculation, i.e. "Critical Limit Points of Reynolds number" is realised.

7. ACKNOWLEDGEMENT

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