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Numerical Study of Newtonian Blood Flow through a Plaque Deposited Artery

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Abstract: Numerical simulations of Newtonian flow of blood along a plaque deposited arteries have been performed to investigate the flow structure of blood in the form of stream line patterns. A mathematical model has been established, for rigid and axially symmetric deposited levels of plaque by employing Taylor-Galerkin/Pressure-correction finite element scheme. The results show that stream line patterns are formed in the downstream of the artery and their intensity is associated with the level of deposition and Reynolds number and linear trend is observed increasing in fashion. Predicted response of blood is compared against analytical and experimental study of blood flow and observed in good agreement.

Keywords: Blood Flow, Arteries, Constant Viscosity Fluid and Stream Line Patterns.

1. INTRODUCTION

The deposition of plaque in the flow field of blood is considered as a major cause of several diseases, including heart attack. Hence, the role of hemodynamics cannot be ignored. Generally, it is accepted that hemodynamic factors, i.e., blood fluid mechanics are very closely related with the state of deposition level, which causes disturbances in the flow field and as a result, wall and deposition degenerative processes generates the arterial diseases (Ahmed, 1998; Andersson, *et al.,* 2000). The variation in the flow field has been studied for many years. Some studies have been performed under experimental side, considering rigid deposition of plaque (Baloch, *et al.,* 1998; Buchanan *et al.,* 2003). Whereas, numerical computations along axially symmetric deposition levels have been reported by many researchers, considering the blood as a Newtonian fluid (Carew, *et al.,* 1994; Mandal, *et al.,* 2010). Whilst, under some investigations, irregular shape of blockage is considered (Cuvelier, *et al.,* 1986). Furthermore, some discussions have been made for the non-Newtonian flow of blood (Gnasso, 1997; Hawken, *et al.,* 1990), for the prediction of impact of blockage in the flow field of blood.

A time steeping finite element study for incompressible laminar flow of blood has been presented to capture the Newtonian behaviour of blood flowing in a axi symmetric plaque deposited artery segment. To simulate the Navier-Stokes equations, along with the incompressibility constraints, a semi–implicit, Taylor-Galerkin/ pressure-correction scheme has been employed. This technique is referred as highly accurate time-stepping scheme for computing transient as well as steady flow problems (Ishikawa, *et al.,* 1998; Long, *et al*., 2001).

This paper covers a numerical technique for simulating blood flow in arteries with applications in the field of medical science along with treatment planning of cardiovascular diseases. The flow field at downstream in a partially plaque filled artery is investigated and is demonstrated by stream line patterns at various inertial levels.

2. MATERIALS AND METHODS

The axially symmetric plaque deposited artery segment is mathematically modelled with solid walls on assumption side having a circular section. The blockage is considered in parabolic form increasing in length as well as in depth and is modelled as:

$$r_n = r_0\left[1+\frac{k}{100}\frac{(a-Z_c)(b-Z_c)}{h^2}\right] \qquad (1)$$

where a and b are taken as two points of blockage, whilst h and k are referred as depth and percentage of deposition respectively and r_0 is taken as radial coordinates in the non blockage region of the artery.

After specifying the problem, initial and boundary conditions are to be described necessarily as: $v(z,0) = v_0(z)$ subject to $\nabla . v_0 = 0$.

At the inlet of the artery; the boundary conditions are fixed maximum for velocity defined by the function $v_z = v_{\max}(1-r^2)$. Whereas, in reality, artery walls are elastic, but for this initial study, artery walls are assumed solid, therefore, treated as free from slip boundary conditions. The simulation of blood flow is made in the upper-half of the artery segment, as by

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assumption it is treated as axi-symmetric. Whereas, pressure is considered zero at out let of the artery.

3. Governing Equations

Navier-Stokes and continuity equations have been employed to model Newtonian flow structure of blood, defined in equations (2-3) respectively.

$$\nabla.\boldsymbol{v} = 0 \tag{2}$$

$$\rho\frac{\partial \boldsymbol{v}}{\partial t} = \mu\nabla^2\boldsymbol{v} - \rho\boldsymbol{v}.\nabla\boldsymbol{v} - \nabla p \tag{3}$$

4. Numerical Scheme

This investigation adopts a time marching finite element algorithm for computing numerical solution through a semi implicit Taylor-Galerkin/Pressure-Correction scheme (Siouffi, *et al.*, 1998; Tu and Deville, 1996; Zienkiewicz, 1983). The scheme is based on fractional step formulation as under:

Stage-1(a):

$$\left[\frac{2}{\Delta t}M + \frac{1}{2\,\mathrm{Re}}S_{rr,j}\right]\left(V_{r,j}{}^{n+\frac{1}{2}} - V_{r,j}{}^{n}\right) = \left[-\frac{1}{\mathrm{Re}}S_{rr}\,V_{r,j} - L_1^t\,P_k\right]^n - N(V)\,V_{r,j} \tag{4}$$

$$\left[\frac{2}{\Delta t}M + \frac{1}{2\,\mathrm{Re}}S_{z\,z,j}\right]\left(V_{z,j}{}^{n+\frac{1}{2}} - V_{z,j}{}^{n}\right) = \left[-\frac{1}{\mathrm{Re}}S_{zz}\,V_{z,j} - L_2^t\,P_k\right]^n - N(V)\,V_{z,j} \tag{5}$$

Stage-1(b):

$$\left[\frac{1}{\Delta t}M + \frac{1}{2\,\mathrm{Re}}S_{rr}\right]\left(V_{r,j}{}^{*} - V_{r,j}{}^{n}\right) = \left[-\frac{1}{\mathrm{Re}}S_{rr}\,V_{r,j} - L_1^t\,P_k^n\right]^n - N(V)V_{r,j}^{n+\frac{1}{2}} \tag{6}$$

$$\left[\frac{1}{\Delta t}M + \frac{1}{2\,\mathrm{Re}}S_{zz}\right]\left(V_{z,j}{}^{*} - V_{z,j}{}^{n}\right) = \left[-\frac{1}{\mathrm{Re}}S_{zz}\,V_{z,j} - L_2^t\,P_k^n\right]^n - N(V)\,V_{z,j}^{n+\frac{1}{2}} \tag{7}$$

Stage- 2:

$$K\left(Q^{n+1}\right) = -\frac{2}{\Delta t}\left(L_1\,V_{r,j} + L_2\,V_{z,j}\right) \tag{8}$$

Stage 3:

$$M\left(V_{r,j}{}^{n+1} - V_{r,j}{}^{*}\right) = \frac{\Delta t}{2}L_1{}^{t}\left(p^{n+1} - p^{n}\right) \tag{9}$$

$$M\left(V_{z,j}{}^{n+1} - V_{z,j}{}^{*}\right) = \frac{\Delta t}{2}L_2{}^{t}\left(p^{n+1} - p^{n}\right) \tag{10}$$

Where M, S, N(V), L; K are matrices, defined in references (Solangi, *et al.,* 2012; Memom, *et al.,* 2011).
The system matrices are defined as:

$$M = \int_{\Omega}\Phi_i\Phi_j\,r\,d\Omega \tag{11}$$

$$L_1 = \int_{\Omega}\psi_i\,\frac{\partial\Phi_j}{\partial r}\,r\,d\,\Omega + \int_{\Omega}\psi_i\,\Phi_j\,d\,\Omega \tag{12}$$

$$L_2 = \int_{\Omega}\psi_i\,\frac{\partial\Phi_j}{\partial z}\,r\,d\,\Omega \tag{13}$$

$$L = \int_{\Omega}\psi_i\,\frac{\partial\Phi_j}{\partial z}\,r\,d\,\Omega \tag{14}$$

$$N(V) = \int_{\Omega} \Phi_i \left(\Phi_l \, V_l \, \frac{\partial \Phi_j}{\partial r} + \Phi_l \, V_l \, \frac{\partial \Phi_j}{\partial z} \right) r \, d\Omega \tag{15}$$

$$K_{ij} = \int_{\Omega} \frac{\partial \psi_i}{\partial r} \; \frac{\partial \psi_k}{\partial r} \, r \, d\Omega \tag{16}$$

$$S = \begin{bmatrix} S_{rr} & 0 \\ 0 & S_{zz} \end{bmatrix} \tag{17}$$

Where

$$S_{rr} = \int_{\Omega} \left(\frac{\partial \Phi_i}{\partial r} \frac{\partial \Phi_j}{\partial r} + \frac{\partial \Phi_i}{\partial z} \frac{\partial \Phi_j}{\partial z} + \frac{\Phi_i \Phi_j}{r^2} \right) r \, d\Omega \tag{18}$$

$$S_{zz} = \int_{\Omega} \left(\frac{\partial \Phi_i}{\partial r} \frac{\partial \Phi_j}{\partial r} + \frac{\partial \Phi_i}{\partial z} \frac{\partial \Phi_j}{\partial z} \right) r \, d\Omega \tag{19}$$

5 RESULTS AND DISCUSSIONS

Stream line patterns for the flow structure of blood, defining the intensity, have been formatted from the stream function, at fixed incremental values for the two regions, labelled as main core flow and recirculation regions. The main core flow region covers five contours and recirculation region occupies four contours. As a result of imposed vanishing boundary conditions, on the walls of the arteries, the stream lines are negative in the recirculation region of blood in the downstream of the artery segment. Stream line patterns are presented for Reynolds numbers at their values (100, 200 and 300) respectively, along with various levels of deposition taken in percentages as (30, 50 and 70) respectively. From this study, it is observed that the formation of stream line patterns starts from 30% level of deposition at Reynolds number 100. The activity of stream line patterns is observed increasing as level of deposition or Reynolds number increases. Further, significant formation of stream line patterns is observed at 50% level of deposition at Reynolds number 200. Hence at this stage, the flow of blood is considered moderate, beyond this stage, the flow over shoots, being confirmed from 70% deposition of Reynolds number 200 and 300 respectively, see from the **Figs.2 - 3**. From the **Figs.1 - 3**, it is further more observed that at 30% level of deposition, the changes in the flow field are negligible, 50% moderate and 70% overshooting, from 200 and onwards values of Reynolds number. Whereas, at 100 Reynolds number, the changes in the flow structure are negligible.

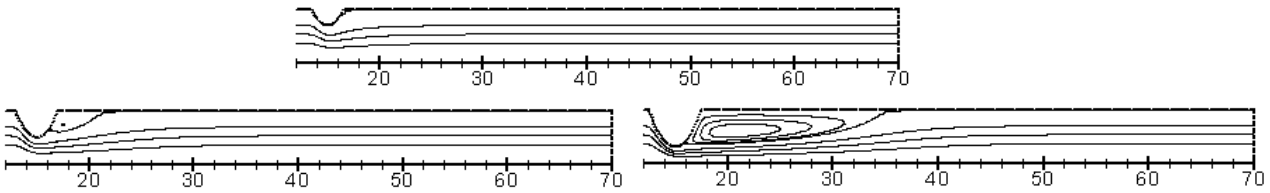


Fig.1: Streamline patterns for Reynolds number 100 at various levels of deposition

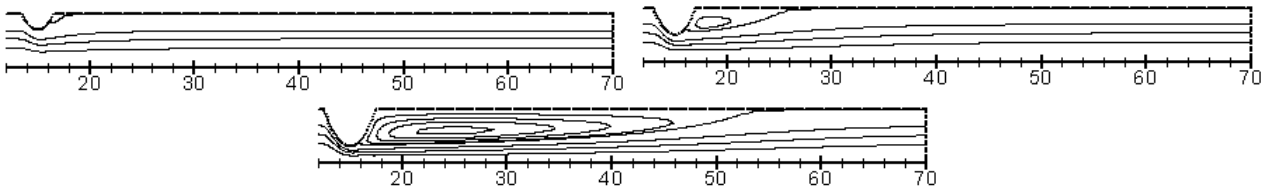


Fig.2: Streamline patterns for Reynolds number 200 at various levels of deposition

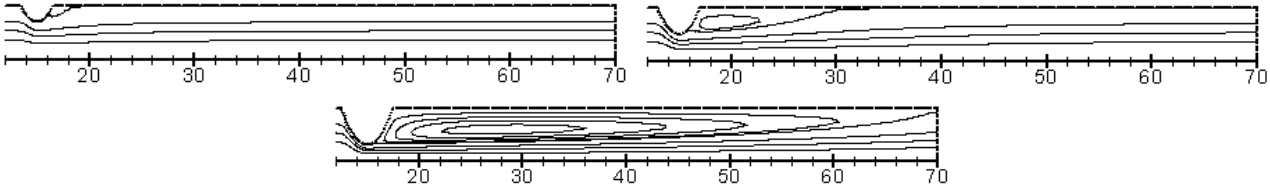


Fig.3: Streamline patterns for Reynolds number 300 at various levels of deposition

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The predicted numerical solutions for recirculation flow rate of blood are compared against the experimental results (Ahmed and Giddens, 1983) as well as analytical solution for Power law model (Neofytou and Tsangaris, 2004): at different Reynolds numbers having various levels of depositions. It is observed that the predicted results of intensity are in good agreement with analytical as well as experimental work.

6. CONCLUSION

Stream line patterns for the flow structure of blood have been studied at different levels of inertia for various deposition levels in the artery segment. It is concluded that the formation of stream line patterns starts at 30% level of deposition of Reynolds number 100 and their intensity is going to be on increasing side as level of deposition or Reynolds number is increasing. Up to 50% level of deposition of Reynolds number 200, moderate flow is observed, whilst, at 70% of 200 and 300 labelled Reynolds numbers flow overshoots.

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