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Effects of Local Inertia in the Expansion Channel Filled with Porous Media: A Finite Element Analysis

H. SHAIKH[++], S. B. SHAH, M. A. SOLANGI*, A. BALOCH**

Department of Mathematics, Shah Abdul Latif University, Khairpur Mirs, Sindh

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Abstract: A computer simulation in the 1:4 expansion ducts filled with porous material and liquid laminar flow of constant viscosity is studied. The finite element scheme is adopted to find the steady solution of flow features and the scheme is called semi–implicit predictor–corrector. The vortex reduction in the expansion duct due to increase Reynolds Number is investigated through expansion duct due to the presence of the porous material. The excellent agreements of numerical results are achieved with comparison of other numerical results existed in open literature. The effects of Reynolds number are examined though the vortex intensity, Loss of pressure differences with coutte–correction.

Keywords: Expansion flows, Finite Element Method, Porous Media.

1. INTRODUCTION

The fluid flow though porous media play a vital role many fields such as Natural science, engineering and industrial applications for example heat exchangers, nuclear reactors, composite materials, groundwater contamination, packed-bed reactors, and better oil recovery (Balhoff (2000), Hassanipour et al.(2007) and Mitsoulis et al (2009)). In these processes the modeling of fluid flow such as stokes, laminar and turbulent through porous material are presented continuum macro scale using Navier stokes equations such as continuity equation coupled with momentum equation. Whilst, Darcy's law are replaced by the velocity vector field in the constitute equations. Historically, the concept of porosity was given the Henry Darcy, examining the resistance of a fluid to flow within a solid matrix. The Darcy's law expressed as the ratio between permeability of the porous material and constant viscosity due to Newtonian fluid with pressure differences. Various concentrations of flow features though complex geometries filled with porous medium are studied and comprise Newtonian, Non-Newtonian and viscoelastic fluids. (Sochi (2009), Kumar *et al.,* (2012))

The effect of a porous medium on the flow of a watery in expansion tank was investigated through numerical simulation by Hassanipour *et al.,* (2007). The Brinkman–Hazen–Dupuit–Darcy equation was used for homogeneous, isotropic flow through porous media and applied finite volume method with successive over–relaxation technique to study the behavior of vortex development in the presence of porous materials. Through numerical simulations, the influence of permeable media properties and vortex intensity has been investigated by means of vortex flow in the presence of porous material and validated the predictions. The Reynolds number changes from 3000 to 9000 and porous media were used with different permeability (0.2 to 0.8). Two types of vortex rings (lower and upper vortex) occurs in a channel filled without porous material and when filled porous material, at low or high Reynolds number, vortex flows bounce back and cannot pass through a low–permeability porous medium but with maximum permeability porous medium, the vortices breaks into smaller vortexes in front of the porous object and does not appear in porous medium.

A numerical model for expansion flow using an integral constitutive equation for Newtonian and viscoelastic monolayer suitably modified to account for strain–thickening in the planar extensional viscosity was developed in Mitsoulis *et al.,* (2009). The finite element simulations on integral–type constitutive equations implemented and have been undertaken for the flow of Langmuir monolayers in the benchmark problems of exit flow through 1:4 planar expansions. Through experimentally and numerically, the effects of inertia (Re) on Newtonian monolayers appear in and creating strong vortex activity in the expansion with increasing Reynolds number vortex activity (size and intensity) grows and the opposite is true on viscoelastic monolayer, the vortex activity is substantially reduced in expansion flow compared with the Newtonian one.

[++]Corresponding author: hisam.shaikh@salu.edu.pk, Cell. No. +923363586010
*Department of Basic Sciences and Related Studies, Mehran University of Engineering and Technology Jamshoro
**ISRA University, Hyderabad

The simulations for the Newtonian fluid are shown at apparent shear rates of 43.1 s^{-1}, 86.2 s^{-1}, and 107.8 s^{-1}, respectively also the results are compared with the experimental flows as observed in Olson *et al*.(2000).

Historically, various researchers have used different numerical techniques to simulate the flow phenomena in expansion channel, such as, excess pressure drop, vortex development and demonstrated different limitations of various numerical algorithms. The expansion flow filled with porous material has not been focus of many researchers.

Governing System of Equations

For the liquid flow the governing equations continuity and momentum equations are in Non–Dimensional form are expressed as follows:

$$\nabla . \underset{\sim}{v} = 0 \qquad (1)$$

$$\frac{\rho}{\varepsilon}\frac{\partial v}{\partial t} = \frac{\mu}{\varepsilon}\nabla^2 v - \frac{\rho}{\varepsilon} v.\nabla v - \frac{\mu}{k} v - \nabla p \qquad (2)$$

Where $\boldsymbol{v} = \left(v_x, v_y, v_z\right)$ is the velocity vector field of the fluid, whilst, v_x, v_y and v_z are velocity components in x, y and z directions respectively. Whereas ρ is the mass density of the fluid (per unit liquid volume for liquid fluid assumed to be constant), μ is the absolute viscosity of fluid, p is the isotropic pressure per unit density, k and ε are the intrinsic permeability and porosity of the porous medium respectively and t is the time.

The component of velocity vector v_z is turned to be zero in case of two-dimensional flow and convective acceleration vanishes also, the pressure gradient is assumed to be constant. The above equation in one-dimension reduces as follows:

The governing system of equations is cast in non-dimensional form, using dimensionless variables

$$v^* = \frac{v}{V_c},\ x^* = \frac{x}{L},\ y^* = \frac{y}{L},$$

$$K^* = K \text{ and } t^* = \frac{\mu . t}{\rho L^2}, \text{ where } v^* \text{ and } y^* \text{ are}$$

dimensionless velocity and transversal coordinates respectively K^* and t^* are the non-dimensional modified permeability of the porous medium and the non-dimensional time respectively. While, L is the characteristic length taken as half width of the channel and V_c is the characteristic velocity assumed as reference axial velocity $V_C = \frac{\varepsilon L^2}{\mu}\left(-\frac{\partial p}{\partial x}\right)$, where dimensionless Darcy's number Da defined as:

$$\mathrm{Da} = \frac{K^*}{\varepsilon L^2},$$

The dimensionless continuity and momentum equation becomes

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} = 0 \qquad (3)$$

$$\frac{\partial u}{\partial t} = \frac{1}{\mathrm{Re}}\left(\frac{\partial u^2}{\partial x^2} + \frac{\partial u^2}{\partial y^2}\right) - \frac{1}{Da}u - \frac{\partial p}{\partial x}$$

$$\frac{\partial v}{\partial t} = \frac{1}{\mathrm{Re}}\left(\frac{\partial v^2}{\partial x^2} + \frac{\partial v^2}{\partial y^2}\right) - \frac{1}{Da}v - \frac{\partial p}{\partial y} \qquad (4)$$

To provide a well-posed specification for flow problem, it is necessary to prescribe appropriate initial and boundary conditions. Simulations commence from a quiescent initial state $v(0, y) = 0$. Boundary conditions are taken as v (t, 0) = V_{max} and v (t, 1) = 0.

Numerical Scheme

In the literature, many numerical techniques have been applied to obtain the solution of the above constitute equations. The best suitable method that used here, the method is called finite element Taylor Galerkin pressure correction scheme. Firstly, these equations, together with the associated boundary conditions, are based on suggestions of Van Kan(1986) , this was acknowledged by Townsend and Webster (1987) for viscoelastic flows and developed a finite element Taylor-Galerkin/Pressure-Correction (TGPC) scheme to attain second-order accuracy. After that, the many researchers adopted the scheme in complex flows by using different type of geometries such as Al-Bayati (2010) and Rong (2007). The scheme is splitting into three distinct fractional stages.

At stage one (a and b) a velocity field **u** at half steps (n + ½) is computed from level n and non–divergence free velocity field $\mathbf{u}^*$ is computed utilizing the information gathered from level n and (n+1/2) respectively, with the step–1b confirming the second order level of accuracy. The Poisson equation is resolved for pressure variance, adopting Crank–Nicolson approach of θ = 0.5 at stage two, then successively occupied at a 3rd and last stage to conclude a solenoid–free velocity field.

To develop semi–discrete system over a time step $[t_n, t_{n+1}]$, for equations (1 and 2), Taylor series expansions and pressure–correction splitting is employed. Given solutions ($\mathbf{u}^n$, p^n) at $t = t_n$ to find $u^{n+1/2}$, $\mathbf{u}^*$, $\mathbf{u}^{n+1}$ and p^{n+1} over the various fractional stages. The detail study of the Taylor Galerkin finite element scheme is already explained in Shaikh (2012).

Problem Definition and Mathematical Formulation

Expansion flow is very complex problem and the analytical solutions are not possible due to its complexity, numerical solution is preferred. In the literature various researchers Baloch *et al.,* (2003) and Baloch and Townsend (2008)) vindicates the robustness of a fractional–stage finite element semi–implicit scheme, i.e., second order accurate Taylor–Galerkin / pressure–correction schemes for the solution of Navier–stokes equation. Therefore, for this complex problem a semi–implicit Taylor–Galerkin/pressure–correction numerical scheme is employed to resolve the laminar incompressible Navier–Stokes equations.

Discuss here liquid flow of constant viscosity fluid through an expansion channel filled with porous media and this problem has significance to the biomechanics and pepper industry Liu, *et al.,* (1998) and Kumar *et al.,* (2012). Two channels are interconnected whose length is 49L and 27.5L for upstream and downstream and height of the channel is L and 4L for small channel and large channel. In the domain to calculate 2987, 6220 and 14057 number of total elements, number of total nodes and degree of freedom respectively taken as triangular elements

At inlet and outlet the Dirchlet and no–slip boundary conditions are organized on the stationary boundary walls. Whereas, at exit the Cartesian coordinates are fixed with zero pressure gradient. Initially the predictions begin with initial conditions at small value of inertia and steady solution is achieved at high values of inertia by a continuum approach.

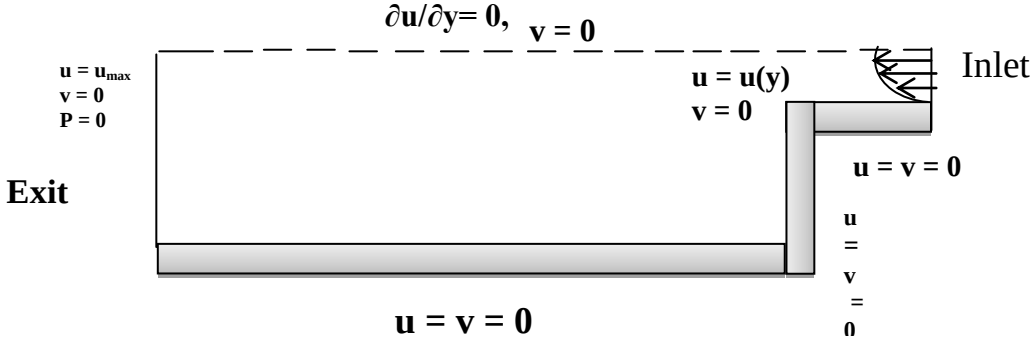
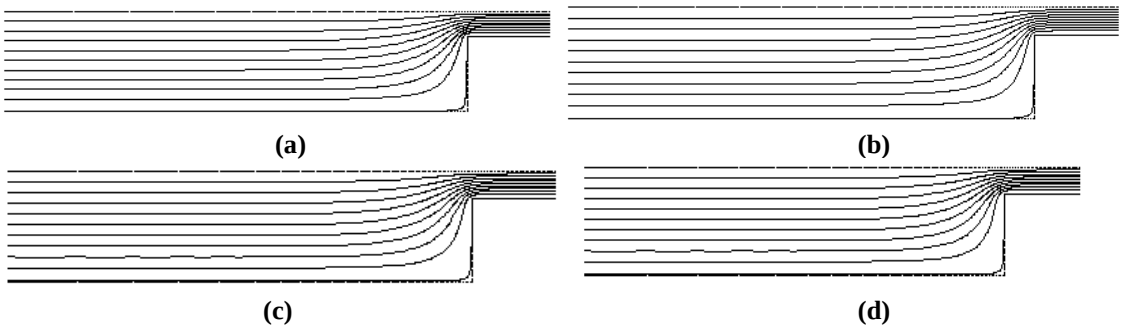


Fig.1: **Schematic diagram 1:4 planner expansion channel with boundary conditions.**

2. RESULTS AND DISCUSSION

Flow through porous media

Numerical simulations are performed to describe the transport of vortical flow with various velocities through porous media. In numerical results under investigation, Reynolds number changes from 0.001 to 50 and at different Darcy's number for Da = 0.001, 0.01 and 0.1. It is clearly demonstrated that the texture of porous media and inertia have been affected the flow structure and behavior. Flow structure does not vindicate the vortex developed, this may be due to presence of porous material in the domain and inertia does not have any influence on flow. Various flow phenomena on flow structures are predicted at different porosities and Reynolds number for both the Stokes flow and the laminar flow. In Figure–2 to 4 illustrates the streamlines contours of the flow at different Darcy's number (Da) and Reynolds numbers (Re), clearly demonstrate that vortex activity is completed vanished and no recirculation appeared in the salient corner, and flow expands right from expansion corner up to salient corner wall.



Fig–2(a–d): Stream lines for 1 : 4 expansion flow; Da = 0.01, (a) Re = 01 (b) Re = 25and (c) Re = 50

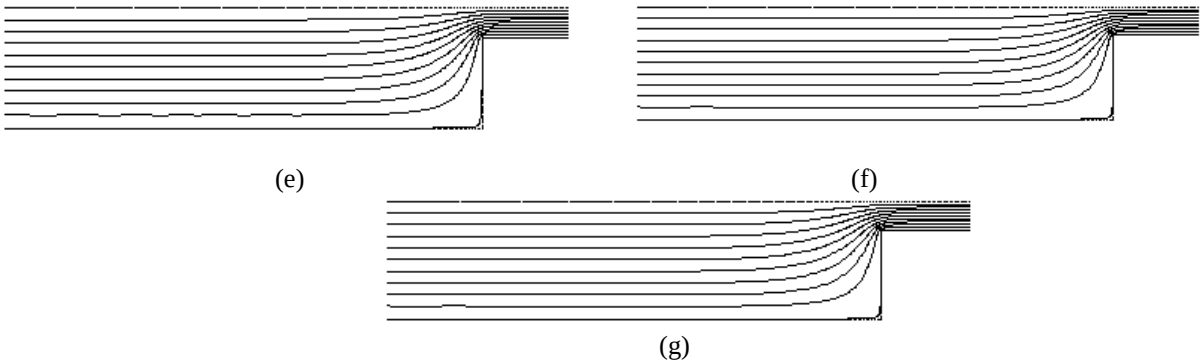


Fig. 3 (e – g): Stream lines for 1 : 4 expansion flow; Da = 0.001, (e) Re = 01 (f) Re = 25and (g) Re = 50

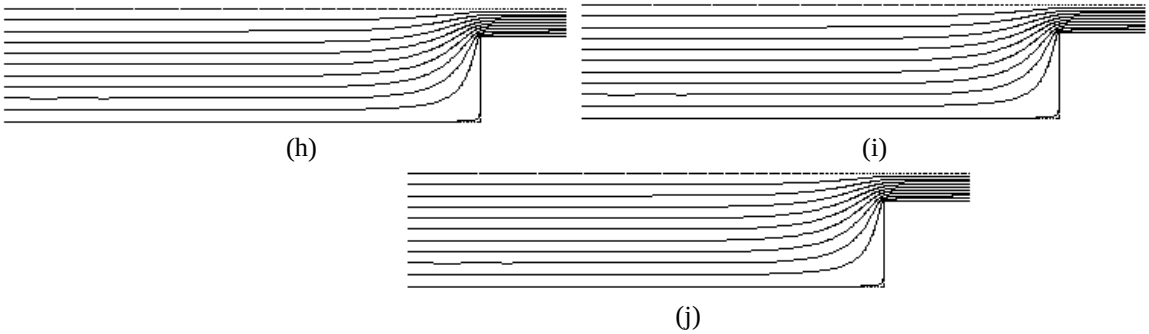


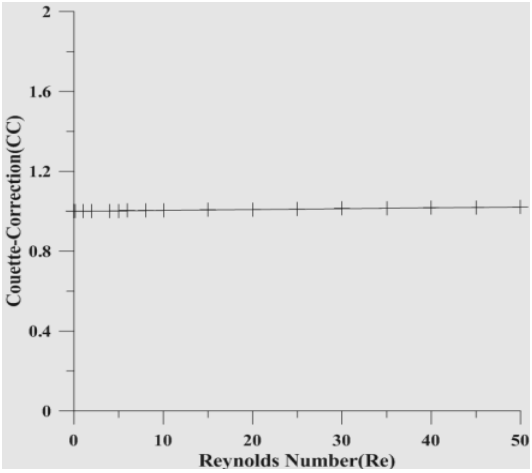
Fig. 4 (h – j): Stream lines for 1 : 4 expansion flow; Da = 0.0001, (h) Re = 01 (i) Re = 25and (j) Re = 50

Table –1: Maximum and Minimum values of Pressure isobars

Re	Pressure isobars			
	Non-Porous		Porous	
	Min	Max	Min	Max
0.001	-3496.6	100350.0	0.82938×10^{-17}	$0.62338\times10^{+07}$
1.0	-3.6692	100.04	0.82943×10^{-20}	$0.62334\times10^{+04}$
10	-0.41601	9.7796	0.82940×10^{-21}	$0.62278\times10^{+03}$
20	-0.24856	4.8036	0.41469×10^{-21}	$0.31108\times10^{+03}$
30	-0.19996	3.1517	0.27645×10^{-21}	$0.20719\times10^{+03}$
40	-0.18296	2.3276	0.20733×10^{-21}	$0.15524\times10^{+03}$
50	-0.17107	1.8356	0.16585×10^{-21}	$0.12407\times10^{+03}$

Subsequently, pressure gradient is investigated in the porous media and a correlation between the Couette-Correction and the inertia (Re). If Eq. (2) is made non-dimensional, the Couette-Correction may be defined as ratio of pressure difference, between computed and analytical in both upstream and downstream, and twice the wall shear stress. The coutte–correction is computed as a function of Reynolds number, with change in Darcy number a relationship has been developed a model that characterize the flow through porous material.

Couttee–correction (CC) is calculated with a different Darcy number (Da = 0.001, 0.01 and 0.1). Here CC is scaled by purely stokes flow (vanishing local inertia i.e., Re = 0). In Figure–5 CC is displayed as a function of Reynolds number and presented in a common Logarithm, whilst, figure–6 describe the empirical relationship of coutte-correction depending up on the Reynolds number. At all Reynolds number, the variation in Couttee–Correction is negligible and remains constant as illustrated in **(Fig.5)**.



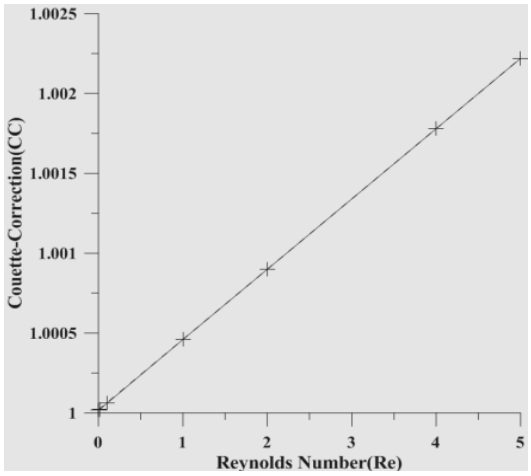


Fig. 5: Couttee-Correction is a function of Reynolds number (Re) in Porous Medium

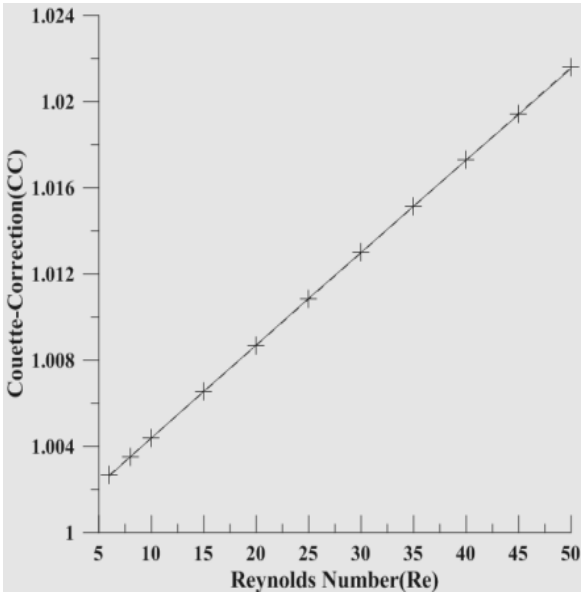


Fig. 6: Empirical relationship as Couttee-Correction is a function of Reynolds number (Re) in Porous Medium

6. CONCLUSION

The liquid flow through 1: 4 expansion channel with the presence of porous material are studied by employing a Taylor Galerkin finite element scheme to obtain the steady–state solution. The numerical predictions are achieved with excellent agreement as compared with the other numerical results. The flow features are presented by a stream lines contours with increasing inertia and various Darcy' number and examined that the vortex is completed vanished at lower inertia even at higher inertia and also varies a Darcy's number. The loss of pressure differentials are investigated through Couttee–Correction that are presented in **(Fig. 5 and 6)**.

The numerical results of the expansion flow in the presence of porous material are completed altered with the previous results of the expansion flow without porosity H. Shaikh (December 2012) and juxtapose with the other numerical results given in literature.

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