Sindh Univ. Res. Jour. (Sci. Ser.) Vol. 45 (2) 421-428 (2013)

SINDH UNIVERSITY RESEARCH JOURNAL (SCIENCE SERIES)

Vibration characteristics of submerged functionally graded cylindrical shell on Winkler and Pasternak foundations using wave propagation approach

A. G. SHAH[++], T. MAHMOOD*, M. N. NAEEM**, S. H. ARSHAD***

Govt. Post Graduate College Jhang, Jhang 35200. Punjab, Pakistan

Received 17th Feb. 2013 and Revised 21st June 2013

Abstract: This research paper deals with the influence of Winkler and Pasternak elastic foundations and functionally graded material on the free vibration characteristics of a cylindrical shells submerged in fluid. Love's thin shell theory has been employed for strain- and curvature displacement relationship. Wave propagation approach is used to solve shell dynamics equations. Influence of fluid, functionally graded material and Winkler as well as Pasternak elastic foundations are studied on the natural frequencies of submerged cylindrical shells. Results obtained are confirmed by the comparisons from the existing ones in literature.

Keywords: Vibration, Submerged functionally graded cylindrical shell, Love's theory, Wave propagation method, Winkler and Pasternak foundations.

1. INTRODUCTION

Circular cylindrical shells are widely used in many structural applications such as airplanes, marine crafts, pressure vessels, submarine hulls, offshore drilling rings and construction buildings. Goncalves and Batista (1987) studied the free vibration of simply supported shells partially filled with or submerged in a fluid. The coupling between the shell deformations and the acoustic medium of fluid is affixed by shell dynamical equations and the acoustic wave equation. Amabili (1996, 1999) examined free vibrations of circular cylindrical shells and tubes completely and partially filled with a dense and partially immersed in a different fluid respectively. Loy *et al.* (1999) investigated the vibration of functionally graded cylindrical shell. They studied the influence of the constituent volume fractions and the effects of configurations of the constituent materials on the shell frequencies (Hz). Zhang *et al.* (2001) applied wave propagation technique to investigate coupled vibration of fluid-filled cylindrical shells. They showed that the fluid effect on the shell frequencies is significant. He (2002) presented the coupled structural-acoustic analysis of finite cylindrical shells submerged in fluid by using this technique. For coupled analysis the comparisons of the natural frequencies were taken by the presented method and numerical finite element method (FEM) and boundary element method (BEM). Shah *et al.* (2011) have already studied the vibrational behaviour of FGM cylindrical shells filled with fluid resting on Winkler and Pasternak elastic foundations. But there is no evidence of the vibrational investigation of functionally graded cylindrical shells submerged in fluid and resting on elastic foundations point out an interaction between out side surface of the cylindrical shells and the fluid in which it is dip. Moreover the Winkler and Pasternak elastic foundations, in which the cylindrical shell is kept, also influence the natural frequencies of the shells. Shell dynamical equations are combined with the acoustic wave equation to implicate fluid term expressions. The fluid loaded terms are expressed by Hankel's function. Wave propagation approach is employed to analyze the vibration characteristics of cylindrical shells submerged in fluid.

2 MATERIAL AND METHODS

A cylindrical shell of the thin-wall is considered here as shown in **(Fig.1),** having geometrical parameters: length L, thickness h and mean radius R. The orthogonal coordinate system (x, θ, z) is taken to be at the surface of the shell where x, θ and z represent the axial, circumferential and radial coordinates respectively. Young's modulus E, the Poisson ratio v and the mass density ρ are the shell material parameters. The axial, circumferential and radial displacement deformations are denoted by $u(x,\theta,t)$, $v(x,\theta,t)$ and $w(x,\theta,t)$ respectively with regard to the shell middle surface.

[++] Corresponding Author's Email: aghafarshah@yahoo.com Cell. +92-3346339914
* Department of mathematics, the Islamia University Bahawalpur, Bahawalpur 63100, Pakistan
**G. C. University Faisalabad, Faisalabad 38000, Punjab, Pakistan
***Department of Civil Engineering, Iqra National University Peshawar, Peshawar 25000, Pakistan

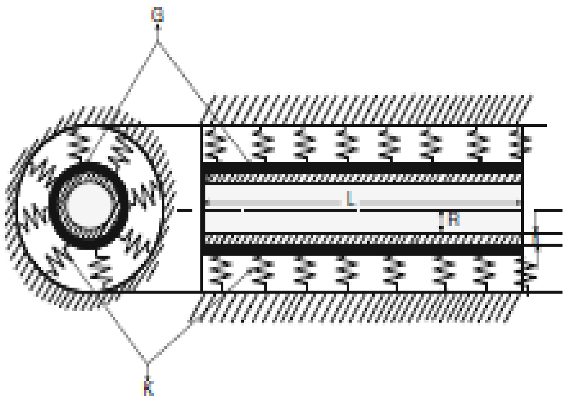

Fig. 1 Cylindrical shell on elastic foundation

The equations of motion for cylindrical shell from the Love shell theory are given in the form as:

$$\frac{\partial N_x}{\partial x} + \frac{1}{R}\frac{\partial N_{x\theta}}{\partial \theta} = \rho_t \frac{\partial^2 u}{\partial t^2} \quad (1)$$

$$\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial N_\theta}{\partial \theta} + \frac{2}{R}\frac{\partial M_{x\theta}}{\partial x} + \frac{1}{R^2}\frac{\partial M_\theta}{\partial \theta} = \rho_t \frac{\partial^2 v}{\partial t^2} \quad (2)$$

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{2}{R}\frac{\partial^2 M_{x\theta}}{\partial x \partial \theta} + \frac{1}{R^2}\frac{\partial^2 M_\theta}{\partial \theta^2} - \frac{N_\theta}{R} = \rho_t \frac{\partial^2 w}{\partial t^2} \quad (3)$$

where N_x , N_θ , $N_{x\theta}$ are force resultant and M_x , M_θ , $M_{x\theta}$ are moment resultants and given as:

$$\begin{bmatrix} N_x \\ N_\theta \\ N_{x\theta} \\ M_x \\ M_\theta \\ M_{x\theta} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \gamma \\ k_1 \\ k_2 \\ 2\tau \end{bmatrix} \quad (4)$$

Where e_1, e_2 and γ are the reference surface strains and k_1, k_2 and τ are the surface curvature and A_{ij} , B_{ij} and $D_{ij}\,(i,j = 1{,}2 \text{ and } 6)$ stand for extensional, coupling and bending stiffness respectively and defined as:

$$\left(A_{ij},\ B_{ij},\ C_{ij}\right) = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \left(1,\ z,\ z^2\right) Q_{ij}\, dz \quad (5)$$

ρ_t denotes the mass density per unit length and is defined as: $\rho_t = \int_{\frac{-h}{2}}^{\frac{h}{2}} \rho\, dz$

For the isotropic materials, the reduced stiffness $Q_{ij}\,(i, j = 1,2 \text{ and } 6)$ are given as

$$Q_{11} = Q_{22} = \frac{E}{1-\nu^2}$$

$$Q_{12} = \frac{\nu E}{1-\nu^2} \qquad Q_{66} = \frac{E}{2(1+\nu)} \quad (6)$$

Analysis is carried out using Love's first order thin shell theory. Relationships for strain-displacement and curvature-displacement are provided from this theory and given for a cylindrical shell as:

$$\{e_1, e_2, \gamma\} = \left\{\frac{\partial u}{\partial x}, \frac{1}{R}\left(\frac{\partial v}{\partial \theta} + w\right), \left(\frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta}\right\}\right. \quad (7)$$

$$\{k_1, k_2, \tau\} = \left\{-\frac{\partial^2 w}{\partial x^2}, -\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right), -\frac{1}{R}\left(\frac{\partial^2 w}{\partial x \partial \theta} - \frac{\partial v}{\partial x}\right)\right\} \quad 8$$

These expressions for the surface strains e_1, e_2 and γ and the curvatures k_1, k_2 and τ from the relations (7) and (8) respectively, are substituted into Eq. (4) and the resulting expressions for N_x , N_θ , $N_{x\theta}$, M_x , M_θ , $M_{x\theta}$ into Eqs. (1)-(3), and introducing the submerged cylindrical shell, which satisfies the acoustic wave equation and the terms describing the Winkler and Pasternak foundations ($Kw - G\nabla^2 w$) in the z-direction, the equations of motion for a cylindrical shell can be written in a differential operator form as:

$$L_{11}u + L_{12}v + L_{13}w = \rho_t \frac{\partial^2 u}{\partial t^2} \quad (9)$$

$$L_{21}u + L_{22}v + L_{23}w = \rho_t \frac{\partial^2 v}{\partial t^2} \quad (10)$$

$$L_{31}u + L_{32}v + L_{33}w = \rho_t \frac{\partial^2 w}{\partial t^2} + P + Kw - G\nabla^2 w \quad (11)$$

where $L_{ij}\,(i, j = 1{,}2{,}3)$ state the differential operators with regard to x and θ and are given as

$$L_{11} = A_{11}\frac{\partial^2}{\partial x^2} + \frac{A_{66}}{R^2}\frac{\partial^2}{\partial \theta^2},$$
$$L_{12} = \frac{(A_{12}+A_{66})}{R}\frac{\partial^2}{\partial x \partial \theta} + \frac{(B_{12}+2B_{66})}{R^2}\frac{\partial^2}{\partial x \partial \theta},$$
$$L_{13} = \frac{A_{12}}{R}\frac{\partial}{\partial x} - B_{11}\frac{\partial^3}{\partial x^3} - \frac{(B_{12}+2B_{66})}{R^2}\frac{\partial^3}{\partial x \partial \theta^2},$$
$$L_{21} = \frac{(A_{12}+A_{66})}{R}\frac{\partial^2}{\partial x \partial \theta} + \frac{(B_{12}+B_{66})}{R^2}\frac{\partial^2}{\partial x \partial \theta},$$
$$L_{22} = \left(A_{66} + \frac{3B_{66}}{R} + \frac{2D_{66}}{R^2}\right)\frac{\partial^2}{\partial x^2} + \left(\frac{A_{22}}{R^2} + \frac{2B_{22}}{R^3} + \frac{D_{22}}{R^4}\right)\frac{\partial^2}{\partial \theta^2},$$
$$L_{23} = \left(\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3}\right)\frac{\partial}{\partial \theta} - \left(\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4}\right)\frac{\partial^3}{\partial \theta^3} - \left(\frac{B_{12}+2B_{66}}{R} + \frac{D_{12}+2D_{66}}{R^2}\right)\frac{\partial^3}{\partial x^2 \partial \theta},$$
$$L_{31} = -\frac{A_{12}}{R}\frac{\partial}{\partial x} + B_{11}\frac{\partial^3}{\partial x^3} + \frac{(B_{12}+2B_{66})}{R^2}\frac{\partial^3}{\partial x \partial \theta^2},$$
$$L_{32} = -\left(\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3}\right)\frac{\partial}{\partial \theta} + \left(\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4}\right)\frac{\partial^3}{\partial \theta^3} + \left(\frac{B_{12}+2B_{66}}{R} + \frac{D_{12}+4D_{66}}{R^2}\right)\frac{\partial^3}{\partial x^2 \partial \theta},$$
$$L_{33} = -\frac{A_{22}}{R^2} + \frac{2B_{12}}{R}\frac{\partial^2}{\partial x^2} + \frac{2B_{22}}{R^3}\frac{\partial^2}{\partial \theta^2} - D_{11}\frac{\partial^4}{\partial x^4} - 2\frac{D_{12}+2D_{66}}{R^2}\frac{\partial^4}{\partial x^2 \partial \theta^2} - \frac{D_{22}}{R^4}\frac{\partial^4}{\partial \theta^4}, \quad (12)$$

where G stands for Pasternak elastic foundation and K for the Winkler foundation modulus. The expression for the differential operator ∇^2 is:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2}{\partial \theta^2} \quad (13)$$

The fluid exterior of cylindrical shell satisfies the acoustic wave equation which is given in cylindrical co-ordinate system (x, θ, t) as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2}+\frac{\partial^2 p}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \qquad (14)$$

Where P is the acoustic pressure and c is the sound speed of the fluid and t is the time.

Numerical procedure

The following modal displacement shape functions are adopted to separate the time and space variables

$$u(x,\theta,t) = A\cos(n\theta)e^{i(\omega t - k_m x)}$$
$$v(x,\theta,t) = B\sin(n\theta)e^{i(\omega t - k_m x)}$$
$$w(x,\theta,t) = C\cos(n\theta)e^{i(\omega t - k_m x)} \qquad (15)$$

in the longitudinal, circumferential and transverse directions respectively. The constants A, B and C are the amplitudes of vibrations in the x, θ and z directions respectively, n is the number of circumferential waves and k_m stands for axial wave number that is associated with a boundary conditions. Wave numbers for four types of boundary conditions are given in (**Table 1**):

Table 1 Axial wave number for different boundary conditions

Boundary conditions	Wave numbers
Simply supported - simply supported	$k_m = m\pi/L$
Clamped – clamped	$k_m = (2m+1)\,\pi/2L$
Clamped - simply supported	$k_m = (4m+1)\,\pi/4L$
Clamped – free	$k_m = (2m-1)\,\pi/2L$

These axial wave numbers k_m are selected to satisfy boundary conditions at both edges of a cylindrical shell. ω denotes the natural angular frequency for the cylindrical shell.

The associated form of the acoustic pressure field exterior of the shell, which satisfies the acoustic wave Eq. (14) is given as:

$$P=P_m\cos(n\theta)\,H_n^{(2)}(k_r r)\,e^{i(\omega t - k_m x)} \qquad (16)$$

where P_m is the pressure amplitude, $H_n^{(2)}(\ )$ is the second kind Hankel function of order *n*. The radial wave number k_r and axial wave number k_m are related by a usual vector relation $k_r = (k_0 - k_m)^{\frac{1}{2}}$, where $k_0 = \omega/c$ is the fluid acoustic wave number. In order to ensure that the acoustic fields satisfy the conditions, usually $k_r = (k_0 - k_m)^{\frac{1}{2}}$ is taken when $k_0 \geq k_m$ and $k_r = -i(k_m - k_0)^{\frac{1}{2}}$ is chosen for $k_0 < k_m$.

To ensure that fluid remains in contact with shell wall, the fluid radial displacement must be equal to the shell radial displacement at the interface of the outer shell wall and the fluid. This coupling condition is:

$$-\left\{\frac{1}{i\omega\rho_f}\right\}\left(\frac{\partial P}{\partial r}\right)_{r=R} = \left(\frac{\partial w}{\partial t}\right)_{r=R} \qquad (17)$$

$$P_m = \left[\frac{\omega^2\rho_f}{k_r H_n{}^{/(2)}(k_r R)}\right] C \qquad (18)$$

where ρ_f is the fluid density and $H_n{}^{/(2)}(\ )$ denotes differentiation with respect to the argument $k_r R$. Making substitution for the displacement functions u, v and w from the expressions (15) in system of equations (9-11) and simplifying the algebraic expressions and rearranging the terms, the frequency equation is written in the following eigenvalue form:

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ -c_{12} & c_{22} & c_{23} \\ -c_{13} & c_{23} & c_{33} \end{bmatrix}\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \omega^2\begin{bmatrix} \rho h & 0 & 0 \\ 0 & \rho h & 0 \\ 0 & 0 & \rho h - \frac{\rho_f}{k_r}\frac{H_n{}^{(2)}(k_r)}{H_n{}^{/(2)}(k_r)} \end{bmatrix}\begin{bmatrix} A \\ B \\ C \end{bmatrix} \qquad (19)$$

where C_{ij} $(i, j = 1,2,3)$ are coefficients of stiffness matrix and their values are given below:

$$C_{11} = k_m^2 A_{11} + n^2\frac{A_{66}}{R^2}$$

$$C_{12} = ink_m\frac{A_{12}+A_{66}}{R} + \frac{2B_{66}+B_{12}}{R^2}$$

$$C_{13} = i\,k_m\left(\frac{A_{12}}{R} + B_{11}k_m{}^2 + n^2\frac{B_{12}+2B_{66}}{R^2}\right)$$

$$C_{22} = n^2\left(\frac{A_{22}}{R^2}+\frac{2B_{22}}{R^3}+\frac{D_{22}}{R^4}\right) + k_m{}^2\left(A_{66}+\frac{4B_{66}}{R}+\frac{4D_{66}}{R^2}\right)$$

$$C_{23} = n\left(\frac{A_{22}}{R^2}+\frac{B_{22}}{R^3}+n^2\left(\frac{B_{22}}{R^3}+\frac{D_{22}}{R^4}\right)+k_m{}^2\left(\frac{B_{12}+2B_{66}}{R}+\frac{D_{12}+4D_{66}}{R^2}\right)\right)$$

$$C_{33} = \frac{A_{22}}{R^2} + \frac{2k_m{}^2}{R}B_{12} + 2n^2\frac{B_{22}}{R^3} + D_{11}k_m{}^4 + 2n^2k_m{}^2\left(\frac{D_{12}+2D_{66}}{R^2}\right) + n^4\frac{D_{22}}{R^4} + K + G\left(k_m{}^2+\frac{n^2}{R^2}\right) \qquad (20)$$

Equation (20) is solved for shell frequencies and mode shapes using computer software MATLAB. The three frequencies are obtained corresponding to the axial, circumferential and radial displacements.

2. RESULTS AND DISCUSSION

To check the validity and efficiency of the current approach, comparison is made for the frequency parameter of cylindrical shells with those found in the literature. In Table 2, frequency parameters $\Omega = \omega R\sqrt{(1-\nu^2)\rho / E}$ for an isotropic cylindrical shell are compared against circumferential wave number n, with those results evaluated by Naeem and Sharma, (1999) under the influence of simply supported-simply supported boundary condition.

Table 2 Comparison of frequency parameter $\Omega = \omega R\sqrt{(1-\nu^2)\rho / E}$ for a SS-SS cylindrical shell ($m = 1, L/R = 6,\ h/R = 0.002,\ \nu = 0.3$)

n	Naeem and Sharma (1999)	Present
1	0.140641	0.140642
2	0.054323	0.054324
3	0.027074	0.027075
4	0.017776	0.017767
5	0.017088	0.017074
6	0.021303	0.021304

Table 3 Comparison of coupled natural frequencies (Hz) of a clamped-clamped cylindrical shell submerged in water by FEM/BEM, ABAQUS and Love with the natural frequencies of present method (*L=20,R=1, h=0.01*)

n	ABAQUS (1998)	Present	Modal Shape (m, n)
1	5.00	4.95	(1, 2)
2	9.62	8.93	(1, 3)
3	11.22	10.61	(2, 3)
4	11.39	11.67	(2, 2)
5	15.18	14.57	(3, 3)
6	20.58	18.25	(1, 4)
7	20.96	18.70	(2, 4)
8	22.10	19.90	(3, 4)

In **(Table 3),** a comparison of coupled natural frequencies (Hz) of clamped-clamped isotropic cylindrical shell submerged in fluid is done against circumferential wave number n, with the results of ABAQUS, (1998) for different pattern of modal shapes of circumferential and axial wave numbers n and m respectively. It is evident from Tables 2 and 3 that present results are in good agreement with that of the literatures.

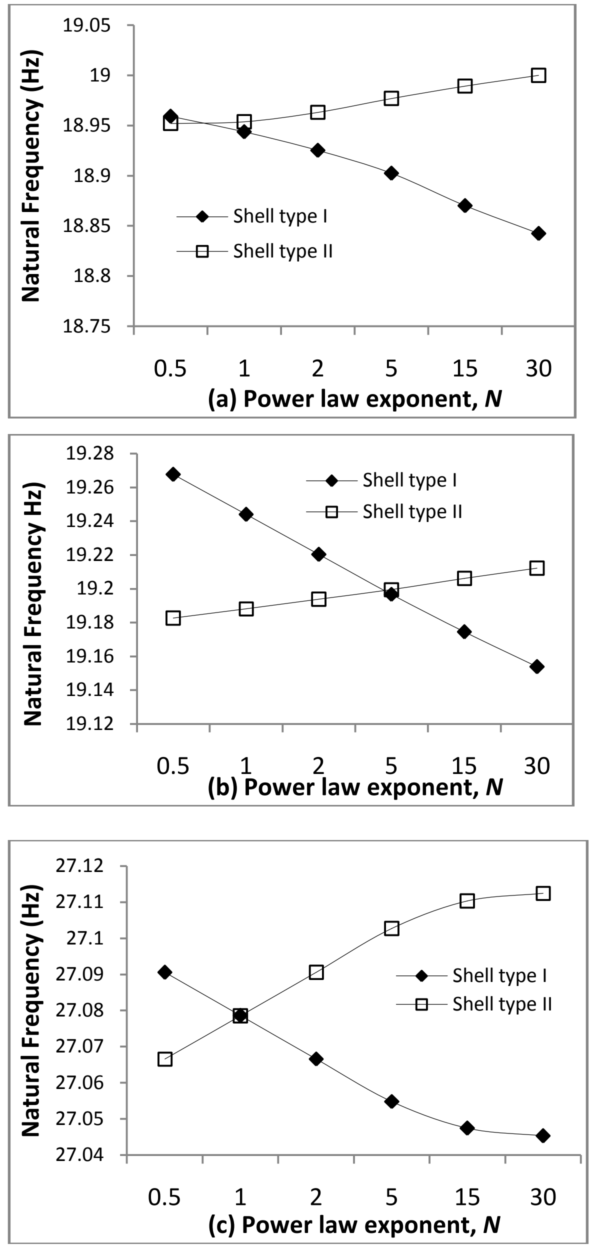

Fig. 2 Variation of natural frequencies (Hz) against power law exponent, *N* of a simply supported submerged functionally graded cylindrical shell for Type I and Type II shells having shell parameters *m*=1, n=1, *h/R*=0.002, *L/R*=20 on elastic foundations (a) G=0, *K*=1.5×10^7 (b) *K*=0, *G*=1.5×10^7 (c) *K*=1.5×10^7, *G*=1.5×10^7

In **(Fig. 2),** variation of natural frequencies (Hz) has been drawn against power law exponents, *N* for Type I and Type II cylindrical shells submerged in fluid for different set of elastics foundations G and K for shell parameters as given in the figure. It is observed that natural frequencies (Hz) decrease in shell type I whereas increase in shell type II. Moreover natural frequencies (Hz) of both types of shells intersect at different values of power law exponent for different pattern of Winkler and Pasternak foundations as shown in Fig. 2 (a)-(c) respectively.

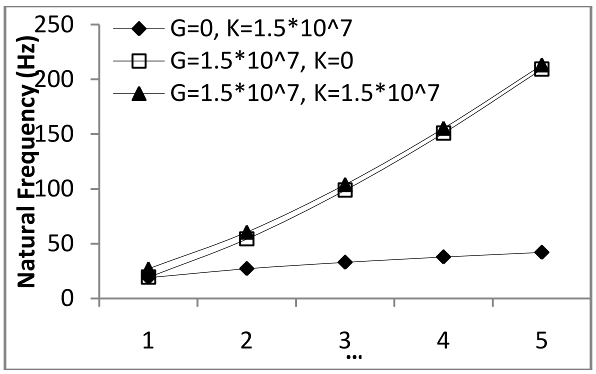

Fig. 3 Variation of natural frequencies (Hz) against circumferential wave number, *n* of a simply supported submerged functionally graded cylindrical shell on elastic foundations for different pattern of elastic foundations having shell parameters *m*=1, *h/R*=0.002, *L/R*=20 (a) G=0, *K*=1.5×10^7 (b) *K*=0, *G*=1.5×10^7 (c) *K*=1.5×10^7, *G*=1.5×10^7

In **Fig. 3,** variation of natural frequencies (Hz) of FGM submerged cylindrical shells are drawn against circumferential wave number, *n* for different patterns of elastic foundations G and K. It is observed from this figure that influence of Pasternak elastic foundation *G* on the natural frequencies (Hz) of the shells is significant whereas influence of Winkler elastic foundation *K*, on the natural frequencies is minor.

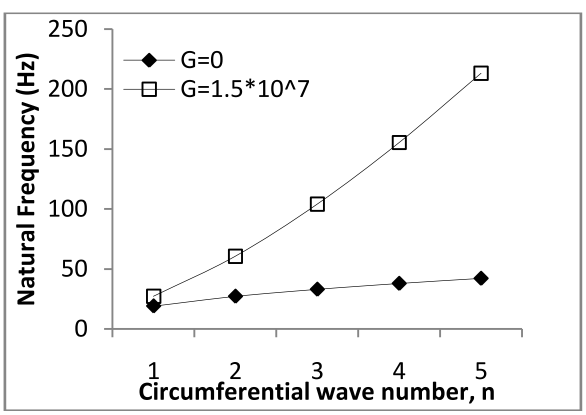

Fig. 4 Variation of natural frequencies (Hz) against circumferential wave number, *n* of a simply supported submerged functionally graded cylindrical shell on elastic foundations having shell parameters *m*=1, *h/R*=0.002, *L/R*=20, *K*=1.5×10^7 (a) G=0 (b) *G*=1.5×10^7

In **(Fig. 4),** variation of natural frequencies (Hz) against circumferential wave number *n* of submerged FGM cylindrical shells are sketched against circumferential wave number *n*, by taking Winkler elastic foundation constant and for different values of Pasternak elastic foundations. It is noted that variation of G affects significantly, the natural frequencies of the shells.

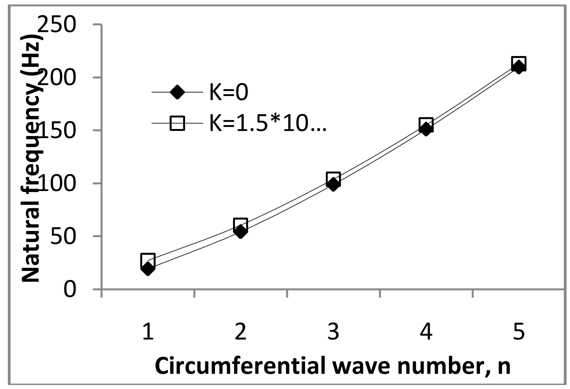

Fig. 5 Variation of natural frequencies (Hz) against circumferential wave number, *n* of a simply supported submerged functionally graded cylindrical shell on elastic foundations having shell parameters *m*=1, *h/R*=0.002, *L/R*=20, *G*=1.5×10^7 (a) K=0 (b) K=1.5×10^7

In **(Fig. 5),** variation of natural frequencies (Hz) against circumferential wave number *n* of a submerged FGM cylindrical shells are sketched against circumferential wave number *n*, by taking Pasternak elastic foundation constant and for different values of Winkler elastic foundations. It is noted that variation of G affects minutely, the natural frequencies of the shells and converge for higher values of *n*.

Tables 4 represent variation of natural frequencies (Hz) with circumferential wave number *n* of type I & type II FGM cylindrical shells respectively, submerged in fluid on Winkler and Pasternak elastic foundations for various set of power law exponents N=0.5, 1, 2, 5, 15, 30 by taking Pasternak and Winkler elastic constant to be G=0, $K=1.5 \times 10^7$. Behavior of natural frequencies (Hz) is seen to be increasing for ascending values of circumferential wave number *n*. Moreover natural frequencies decrease for rising values of power law exponent, *N* in type I FGM shell while increase for increasing values of N in type II FGM cylindrical shell. In the vertical variation of natural frequencies, rate of increment at the start is higher for lower values of circumferential wave number *n* than the higher values of *n* whereas in the horizontal variation of the natural frequencies (Hz) in shell type I and shell type II is very slow against rising values of power law exponent *N*.

Table 4 Variation of natural frequencies (Hz) against circumferential wave number *n* of a submerged functionally graded cylindrical shell on elastic foundations, G=0, *K*=1.5×10^7 for shell Type I & Type II having shell parameters *m*=1, *h/R*=0.002, *L/R*=20

	n	N=0.5	N=1	N=2	N=5	N=15	N=30
Shell type I	1	18.95938	18.94389	18.9254	18.9026	18.8503	18.8096
	2	27.07673	27.07373	27.06955	27.086146	27.05426	27.04107
	3	32.91444	32.90893	32.90127	32.89533	32.88529	32.87527
	4	37.77452	37.76651	37.75020	37.74009	37.73997	37.72992
	5	42.01161	42.00071	41.98986	41.96210	41.90178	41.84163
Shell type II	1	18.95216	18.95388	18.96321	18.97694	18.98928	18.99999
	2	27.07756	27.07968	27.08561	27.07582	27.18490	27.41499
	3	32.91433	32.92887	32.99537	33.22542	33.72546	33.92548
	4	37.77427	37.78060	37.79072	37.80081	37.86089	37.99239
	5	42.01098	42.05251	42.03303	42.08315	43.03335	43.89350

Table 5 Variation of natural frequencies (Hz) against circumferential wave number *n* of a submerged functionally graded cylindrical shell on elastic foundation *K*=0, *G*=1.5×10^7. for shell Type I & Type II having shell parameters *m*=1, *h/R*=0.002, *L/R*=20

	n	N=0.5	N=1	N=2	N=5	N=15	N=30
Shell type I	1	19.26767	19.21396	19.15032	19.10672	19.01448	18.08384
	2	54.28369	54.25368	54.23367	54.20365	54.17364	54.12364
	3	98.893485	98.873478	98.80751	98.791029	98.780731	98.77774
	4	151.04639	151.01469	150.9830	150.95137	150.93159	150.92584
	5	209.56679	209.51373	209.4607	209.40775	209.37465	209.36503
Shell type II	1	19.18261	19.18820	19.19383	19.19954	19.20305	19.20415
	2	54.22612	54.23301	54.239912	54.24681	54.25113	54.25238
	3	98.80751	98.82399	98.84048	98.85699	99.10707	100.11234
	4	150.9830	151.01472	151.04641	151.07812	151.0979	151.103728
	5	209.46077	209.51379	209.5668	209.6199	209.6531	209.6628

In **(Tables 5),** variation of natural frequencies (Hz) with circumferential wave number *n* of submerged type I & type II FGM cylindrical shells resting on Winkler and Pasternak elastic foundations respectively, are investigated for different values of power law exponents *N*=0.5, 1, 2, 5, 15, 30 by keeping Pasternak and Winkler elastic constant to be *K*=0, $G=1.5\times10^7$. Same behavior of natural frequencies (Hz) is observed as seen in Table 4, but in this case rate of change of the natural frequencies (Hz) along circumferential wave number *n* is too much greater for higher values of *n* than that of lower values of *n*.

4. CONCLUSION

In the present study influence of functionally graded material and elastic foundations such as Winkler and Pasternak elastic foundations on the natural frequencies of FGM cylindrical shell submerged in fluid is carried out for different pattern of power law exponents and Winkler and Pasternak foundations with simply supported boundary condition at both ends. Two types of FGM cylindrical shells are fabricated by changing the configuration of the constituents in the shell. Variation of the natural frequencies for increasing values of the power law exponent is found to of reverse order in both cases that is it is increasing in one type and decreasing for the other type of the cylindrical shell. Behaviour of natural frequencies of the shells is found to be identical to the isotropic one. Influence of the fluid on the natural frequencies in which an empty FGM cylindrical shell is submerged found to be considerable whereas Winkler and Pasternak elastic foundations also influence the natural frequencies of submerged FGM cylindrical shells. Pasternak foundations influence significantly as compared to the Winkler foundations. This work can be extended to study buckling, post buckling and different mode shapes for various boundary conditions of fluid-filled cylindrical shell submerged in fluid on elastic foundations.

REFERENCES:

Amabili, M. (1996) Free vibration of partially horizontal cylindrical shells, *J. Sound and Vib;* 191, 757-780.

Amabili, M., (1999) Vibrations of circular tubes and shells filled and partially immersed in a dense fluids. *J. Sound and Vib*; 221, 567-585 .

Goncalves, P. B. and R.C. Batista, (1987) Frequency response of cylindrical shells partially submerged or filled with liquid, Journal of Sound and Vibration, 113 (1) 59-70.

Loy, C.T., K.Y. Lam and J. N. Reddy, (1999) Vibration of functionally graded cylindrical shells *International Journal of Mechanical Sciences*, 41: 309-324.

LMS International SYSNOISE User's manual, (1998), Version 5.3, Belgium; Leuven.

Naeem, M. N. and C.B. Sharma., (1999) Prediction of natural frequencies for thin circular cylindrical shells. Proc Instn Mech Engrs, 214 (C), 1313-1328.

Shah, A. G., T. Mahmood., M. N. Nawaz and S. H. Arshad, (2011) Vibrational study of fluid-filled functionally graded cylindrical shells resting on elastic foundations, ISRN Mechanical Engineering,

Article ID 892460, 1-13 , DOI: 10.5402/2011/892460

Zhang, X. M., G. R. Liu and K. Y. Lam, (2001) Coupled vibration analysis of fluid-filled cylindrical shells using the Wave propagation approach, Applied Acoustics, 62, 229-243.

Zhang, X. M., (2002) Frequency analysis of submerged cylindrical shells with Wave propagation approach, International J. of Mechanical Sc. 44, 1259-1273.