



Optimal Identification of Well Parameters Using Genetic Algorithm

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Abstract: An effort has been made to optimally identify well parameters, viz. formation loss coefficient, entry loss coefficient, and turbulence coefficient in the framework of Genetic Algorithm (GA), which consequently resulted in the development of a computer model. To check validity of the model, results of the model are compared with the ones obtained through Jacob's graphical approach. Moreover, since minimization of the objective function 'sum of square of errors (sse)' achieved through the GA approach have edge over the pertinent results obtained through Jacob's traditional method; further endorse validity of the model results. The model minimizes sse to 7.45×10^{-5} over 4.40×10^{-4} as computed by Jacob's method, signifying a percentage difference to the tune of 490. The model attains convergence smoothly except for minor level perturbations. Jacob's assumption for value of the turbulence loss coefficient is discarded, instead Rorabaugh and Todd's opinion is ascertained.

keywords: Well parameters, Optimization, Genetic Algorithms.

1. INTRODUCTION

Proper estimation of groundwater depletion to groundwater withdrawal can be estimated knowing the actual value of drawdown within the aquifer at the well site, which in turn depends upon characteristics of the aquifer and the well under consideration. These characteristics consists of formation loss coefficient ' B_f ', entry loss coefficient ' C_f ' and turbulence loss coefficient ' n '. traditionally, these parameters have been identified by conducting a step-drawdown test and analyzing the test data by a graphical method suggested by Jacob (1947). However, this method is subjective in nature, time-consuming, prone to error and possesses intrinsic limitations.

With the availability of high speed computers and evolution of different numerical methods, graphical solution can be efficiently replaced with some suitable numerical solution in the framework of a computer model. A number of conventional optimization techniques such as Dynamic programming, Gauss-Newton method and Gradient projection are available in literature, which by virtue of their inherited characteristic yield local optima. To overcome this various unconventional optimization techniques are applied by different investigators, which merit in achieving global optima, such as Genetic Algorithm (Ciaurri et al, 2011), Shuffled Complex Evolution method developed at the University of Arizona (Duan et al, 1994) and Artificial Neural Network (Rogers and Dowla, 1994)

In past few years Genetic Algorithm (GA) has been widely used by hydrologists for solution of groundwater problems (Gwo, 2001; Parsad and Rastogi,

2001; Memon et al, 2007). In this article GA technique is employed to optimally identify the well parameters, which ultimately resulted in development of a computer model coded in C++ language.

2. PROBLEM FORMULATION

A well penetrating throughout the depth of an aquifer is subjected to formation and well losses. Formation loss is domino effect of the aquifer properties, whereas well loss is on account of friction met due to radially inward motion of water to the well and of its uptake towards the pumping device (**Fig. 1**). Jacob (1947) expressed the formation loss as ' $B_f Q$ ' and the well loss as ' $C_f Q^n$ '. Thus, the total drawdown for a given discharge ' Q ' running under steady state condition is given by

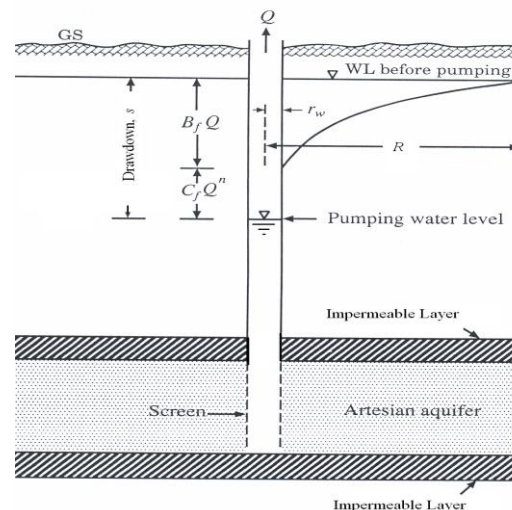


Fig. 1. Drawdown in an Artesian (Confined) Aquifer.

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$$s = B_f Q + C_f Q^n \quad (1)$$

From Eq. (1) it can be seen that for a given discharge, drawdown 's' is the function of B_f , C_f and n parameters. Thus proper estimation of s requires prior knowledge of these parameters, and in pursuit of achieving this objective; the present research is aiming at.

In order to achieve above mentioned objective, formulation of the problem is made in such a way that the aim of identification of well parameters is achieved through an optimal process of minimization of errors by applying GA approach. Formulation of the problem is made with the help of prototype data obtained through step-drawdown test.

In Eq. (1), if an observed value of Q is inserted; and if values of B_f , C_f and n are also incorporated by selecting them randomly within the range of their given boundary conditions, then a drawdown (si_i) for an i th data-set could be identified and may be computed as

$$si_i = B_f Q_i + C_f Q_i^n \quad (2)$$

where $i = 1, 2, 3, \dots, M$; and M is total number of data-sets. Definitely, the difference of identified drawdown (si_i) and observed drawdown (so_i) will result into an error ($si_i - so_i$). Now, integration of the square of errors (e_i) over the whole domain of step-drawdown test data may be formulated as

$$sse_j = \sum_{i=1}^M (si_i - so_i)^2 \quad (j = 1, 2, 3, \dots, N) \quad (3)$$

where sse_j is the objective function required to be minimized to have an optimal solution for B_f , C_f and n ; and N is the total number of iterations, amongst which a minimal value of the objective function is chosen.

The objective function, formulated as above, is minimized in the framework of GA. The GA works in accordance with biological processes of survival and adaptation (Goldberg, 1989). Computationally it is simple, yet a powerful search algorithm that seeks to reproduce mechanisms of natural selection and population genetics. Main functional mechanisms of GA include; genetic representation, reproduction, crossover and mutation.

Jacob's Graphical Approach

Jacob uses Eq. (1) in his graphical approach. He assumes a constant value of 2 for n , while the other two parameters B_f and C_f are estimated by him. A little

modification of Eq. (1) and incorporation of the n value, changes the equation to

$$s/Q = B_f + C_f Q \quad (4)$$

Jacob, then uses the data-sets of s vs. Q obtained through step-drawdown test. Of this data, a plot of s/Q vs. Q is made on an arithmetic scale paper. The resultant graph is a straight line. Hence, it may be concluded that Eq. (4) is a straight line equation with B_f and C_f as its intercept and gradient respectively.

For comparison and demonstration of results, a published data-set (Sayed, 1988) of the step-drawdown test is used. This data-set is presented in (Table 1).

Table 1. Discharge-Drawdown Data of Step-Drawdown Test

Discharge Q (m^3/min)	Drawdown S (m)	Cumulative Drawdown Σs (m)	Specific Drawdown $\Sigma s/Q$ (min/m^2)
3.40	6.89	6.89	2.026471
4.73	3.29	10.18	2.152220
6.05	3.66	13.84	2.287603

Using the above data-set, a graph (Fig. 2) in accordance with Jacob's approach is prepared; results deduced are as shown in Table 2.

Source: Sayed, S. A. S., "Handbook of Aquifer Test Analysis", IIDE, MUET, Jamshoro, Pakistan.

The Genetic Algorithm Model

A computer model coded in C++ language is developed in the framework of GA. The model prerequisites data set of step-drawdown test. This data-set (Table 1) is used to demonstrate the results identified

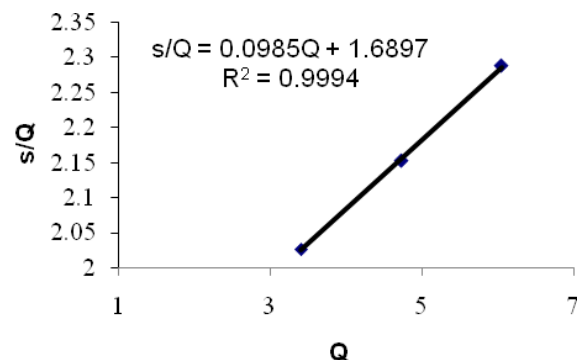


Fig. 2. Estimation of Well Parameters by Jacob's Method

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by the model. well parameters are optimally identified by the model as shown in Table 2.

Table 2. Well Parameters

Parameters	Jacob's Graphical Method	Genetic Algorithms Method	% Difference
B_f	1.69	1.814	7.3
C_f	0.099	0.039	-60.6
n	2	2.385	19.3
sse	4.40E-04	7.45E-05	-83.1

In the computer model, boundary conditions for B_f are set in accordance with the governing Eq. (1). Since B_f is sort of a head loss, it may only have a positive value. Thus it can be construed from Eq. (1) that when ' $C_f Q$ ' tends to ' s/Q ' then B_f reaches to 0, which characterizes its lower boundary, and when ' $C_f Q$ ' attains to 0 then B_f tends towards ' s/Q ', which marks its upper boundary condition. Therefore, the boundary conditions for B_f are set as 0 and 3.0. Boundary conditions for C_f are set in accordance with suggestions made by Walton (1962). According to him C_f depends upon well condition; Table 3 contains such information. Thus in line with his suggestions the boundary conditions for C_f are set as 0 and 5.0. Jacob (1947) in his approach assumes a constant value of 2 for n , whereas Rorabaugh (1953) argues that n may considerably be different than 2.0. Todd (1995) opines that exact value of n cannot be fixed since it varies from well to well depending upon in and outside flow conditions of the well. Thus the boundary conditions for n are set at 1.0 and 3.0.

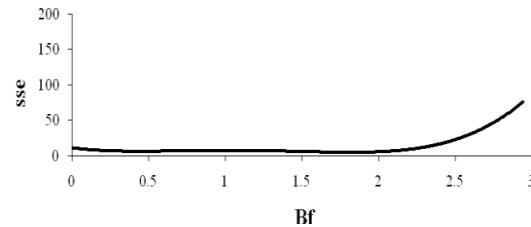
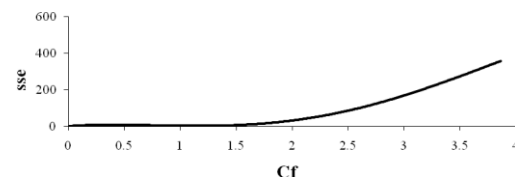
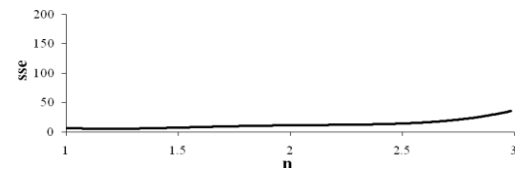
Table 3. Entry Loss Coefficient for Various Conditions of Well After Rorabaugh

Condition of Well	C_f
Properly designed and well developed	< 0.5
Mildly deteriorated or clogged	0.5 – 1.0
Severely deteriorated or clogged	1.0 – 4.0
Difficult to restore to its original condition	> 4.0

Source: Walton, W. C., "Selected analytical methods for well and aquifer evaluation", Illinois state water survey bulletin no 49, 1962.

In line with the boundary conditions for above mentioned parameters a precision of 2, 2 and 3 digits after decimal point is fixed for B_f and C_f and n , respectively. This precision level requires chromosome lengths of 9, 9 and 11 binary digits for B_f , C_f and n , respectively; making a chromosome of 29 binary digits. In the computer model the parameters population size 'POPSIZE', probability of crossover 'Pc' and probability of mutation 'Pm' are fixed by trial and error as 110, 0.85 and 0.015, respectively.

Sensitivity analysis of these parameters is carried out as shown in **(Fig. 3 (a)-(c);)** which delineate that there exhibit non-linear phenomena as B_f , C_f and n vary from 0 to 2.941, 0 to 3.865 and 1 to 2.984 respectively; with sse varying from 7.45×10^{-5} to 662.148.

**Fig. 3(a) Sensitivity Analysis of the Formation Loss Coefficient****Fig. 3(b) Sensitivity Analysis of the Entry Loss Coefficient****Fig. 3(c) Sensitivity Analysis of the Turbulence Coefficient**

3.

RESULTS

In the present study, effort has been made to develop a computer model in the framework of GA for optimal identification of well parameters (B_f , C_f and n). The model results are presented in Table 2.

The objective function 'sse' by the model minimizes to 7.45×10^{-5} , whereas it through Jacob's method reduces to 4.40×10^{-4} .

As far as turbulence loss coefficient ' n ' is concerned, Jacob's (1947) assumption of a constant value is discarded, but Rorabaugh (1953) and Todd's (1995) opinion is ascertained.

The model results illustrated in Table 2 delineate that with 19.3% change in value of n , there is successive change in the B_f and C_f values too (B_f increases by 7.3% whereas C_f decreases by 60.6%). Furthermore, value of objective function 'sse' achieved by GA model is just 16.9% of that by Jacob's graphical approach.

4.

CONCLUSION

In the light of above facts, conclusions are made as outlined below.

1. A computer model is developed in the framework of GA that identifies well parameters optimally.
2. Assumption of a constant value of turbulence loss coefficient is discarded.
3. Results obtained by the model are more dependable and resolute as compared to Jacob's graphical approach; for the reason that minimization of the objective function achieved through GA approach has edge over the results obtained through Jacob's method.
4. GA based computer model developed through the process of present research is valid enough to use, it is easy, iterative in nature, decisive and substantive enough to be used in future by groundwater planners, managers and engineers.
5. GA technique, which in fact is a non-conventional optimization technique, could be used for solution of groundwater flow problems or of any other relevant field.

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