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Blood flow simulation in Carotid Artery Bifurcation using Finite Element Method

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Abstract: The Finite Element Simulation of steady state flow of blood in carotid artery bifurcation is performed using Taylor-Galerkin Finite Element Method. Simulation is performed Reynolds number 10 to observe the behavior of the blood flow. Generalized Newtonian fluid is considered for the blood flow simulation. The obtained results are compared with the Newtonian model. The results show that generalized non-Newtoian model has not influenced the general formation of the blood flow. The parameter of interest in this paper is to observe the behavior of secondary velocity, wall shear stress. In the results, small discrepancies have been observed in the parameters of interest between Newtonian and non-Newtonian blood flow simulations

1. INTRODUCRTION

The study of the flow of Blood in human arteries has gained importance because of the majority of the people of almost every country are suffering from heart attacks that subsequently cause heart failure. The studies of vascular diseases like abnormalities of carotid artery, brain hemorrhage and abdominal aortic aneurysm are becoming a challenge to the researchers. Every year millions of peoples are being affected by these diseases.

The process to simulate the blood flow experimentally, analytically and even numerically is difficult due to its complex hemodynamic nature.2The objective of simulating the blood flow is to understand the development of arteriosclerosis (cardiovascular disease) within the carotid artery and its effect on flow behavior. Atherosclerosis is a disease that deposits waxy plaque inside the arterial walls. Due this plaque, the blood vessels become narrow and prevent the smooth flow of blood in the arteries.

Blood to the neck is supplied through the carotid artery (CA). CA is a large artery on either side of the neck that supplies blood to the head. The main carotid artery is called Common Carotid Artery (CCA). It further branches into Internal Carotid Artery (ICA) and External Carotid Artery (ECA). The function of ICA is, supplying blood to the brain and ECA feeds blood to the neck and other facial parts.

In the present paper, the domain of our study is ICA. Research Scholars have been using numerous experimental and numerical techniques to investigate the formation of plaque in the carotid arteries. Some of the observations of the research scholars is given below. Glagov et al. (1988) in their work observed that the instabilities in the blood flow play a major influence in the localization of the disease. Similar study was made by Zarins et al. (1983). Bharadvaj et al. (1982) in their research work analyzed the hemodynamic flow patterns in the carotid artery. Santiago et al.(2003)performed Finite Element Simulation of blood flow in arteries particularly, the carotid artery bifurcation.

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Huang et al. (1995) in their numerical and experimental studied the fluid dynamics of a plaque developed artery for the restrictions of 44%, 56% and 75% at different Reynolds numbers ranging from 100 to 1000. Blackshear et al. (1980) investigated the velocity of flow through a normal and stenosis artery in a series of patients' images obtained by arteriography. They found that the flow velocity through a stenosis present artery depends upon the degree of constriction. Bathe and Kamm (1999) performed a finite element analysis of pulsatile blood flow through a compliant stenotic artery.

2. GOVERNING EQUATIONS

The fluid flow is considered laminar, isothermal and incompressible in carotid arteries. In this article the blood is considered Generalized Newtonian. In the absences of body forces, the Navier-Stokes equation and the equation of continuity comprise the governing equations:

$$\rho \frac{\partial u}{\partial t} = \nabla.\mathrm{T} - \rho u.\nabla u - \nabla p \qquad (1)$$

$$\nabla.u = 0 \qquad (2)$$

Where T is the stress, D is the rate-of-deformation tensor; L is the velocity gradient and μ_0 is the Newtonian viscosity. For Generalized Newtonian viscoelastic fluids an additional stress term in the momentum equation (1) is introduced as;

$$T = 2\mu_2 D + \tau \qquad (3)$$

$$D = \frac{L + L^t}{2} \text{ and } L^t = \nabla u \qquad (4)$$

Employing continuity equation (2) with constant viscosity, famous Navier-stokes equation is obtained as follow:

$$\rho \frac{\partial u}{\partial t} = \mu_0 \nabla^2 u \text{ - } \rho u.\nabla u - \nabla p \qquad (5)$$

Characterizing non-dimensional variables u = *U*u* and p = [μU/L]p*, discarding the * notation, we obtained an equivalent non-dimensional system of equations to (1) and (5) as:

$$\mathrm{Re} \frac{\partial u}{\partial t} = \nabla^2 u \text{ - } \mathrm{Re}\, u.\nabla u - \nabla p \qquad (6)$$

$$\nabla.u = 0 \qquad (7)$$

Where *Re* is the non-dimensional Reynolds number *i.e*.

$$\mathrm{Re} = \frac{\rho U L}{\mu}$$

3. NUMERICAL METHOD

Analytical methods are not applicable for the simulation of complex flow problems; alternatively numerical techniques are the ways to simulate the complex flows. In this article finite element technique is used. The present study is based on a Taylor-Galerkin/Pressure Correction (TGPC) algorithm, first purposed by Townsend and Webster (1987). Zienkiewicz and Morgan (1983), Chung (2002) and Crochet *et al*. (1984) described briefly on Galerkin finite element methods, which are the accurate, stable and good capable techniques. Chandio *et al.*,(2002,2004) employed this algorithm for free surface filament stretching and reverse roller coating problem. By using the Cuvelier *et al*. (1986) notations U(x, t) = U_j (t) Φ_j(x) and pressure field P(x, t) = P_k(t) Ψ_k(x), where Φ_j is piecewise quadratic basis function and Ψ_k is linear basis function. The fully-discrete semi-implicit Taylor-Galarkin/Pressure-Correction system of equations may be expressed in matrix form as Chandio and Webster (2002).

Stage1a: $$\left(\frac{2\,\mathrm{Re}}{\Delta t} M + \frac{1}{2} S\right)\left(U^{n+\frac{1}{2}} + U^n\right) =$$

$$\left\{-\left[S + \mathrm{Re}\, N(U)\right] U + L^T P\right\}^n$$

Stage1b: $\left(\frac{\mathrm{Re}}{\Delta t}M+\frac{1}{2}S\right)\left(U^{*}-U^{n}\right)=$

$$-\left[SU+L^{T}P)\right]^{n}+\left[\mathrm{Re}\,N(U)U\right]^{n+\frac{1}{2}}$$

Stage 2: $K(P^{n+1}-P^{n})=-\frac{2}{\Delta t}LU^{*}$

Stage3: $\frac{1}{\Delta t}M(U^{n+1}-U^{*})=\frac{1}{2}L^{t}\left(P^{n+1}-P^{n}\right)$

where M represents mass, S diffusive, N(U) convection, L pressure gradient, and K pressure stiffness matrices. The detailed summary about the indicial notations of above matrices are given in Chandio and Webster (2002).

5. BOUNDARY CONDITIONS

The geometry in the current study is based on the dimensions and work of Bharadvaj *et al*. (1982), Perktold *et al*. (1991) and Baaijens *et al*. (1993). The schematic diagram of the problem geometry is shown in figure 1. Here, the walls of the carotid artery bifurcation are taken as rigid-walls. Fully developed flow is considered at the inlet. No slip boundary conditions are imposed on the walls. Pressure at the outlet is set to zero.

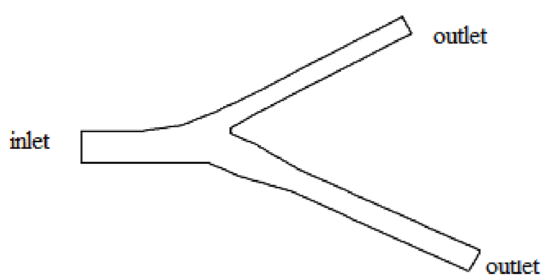

Figure 1. Schematic geometry of carotid artery bifurcation

4. RESULTS DISCUSSIONS

In this article, the physical nature of human blood described, particularly velocity and shear stresses for both Newtonian and generalized Newtonian fluid. For both cases the flow considered, is steady, laminar, isothermal and incompressible. Viscosity is taken as 0.0035 Pa and density is 1050 kg/m$^{3.}$ The results are obtained for Reynolds number 10 (*Re* =10). The characteristic dimension is determined in mm. For generalized Newtonian fluid simulation a normal carotid artery is considered. Results are compared with Newtonian Fluid. Thus flow becomes Newtonian at high shear rates; this is the case in our study, where high shear rates are observed. Researchers have used different viscosity models for Newtonian, generalized Newtonian and non-Newtonian fluid. We have used FEM due to its advantage over the other methods such as FDM and FVM in dealing with irregular shaped boundaries. Blood flow through large arteries is laminar but chaotic or turbulent some times in the complex flow paths as well as in irregular sizes of arteries. The turbulent behavior of blood creates many complicated factors, such as 2D or 3D geometries and non-Newtonian viscoelasticity.

In figure 2, line plots of wall shear stress are shown at ECA and ICA. It is observed that the wall shear-stresses for Newtonian fluid are slightly higher than that of Power-Law. This depicts that the Newtonian fluid may produce high stresses in that region. Similar sort of profile is seen for ICA.

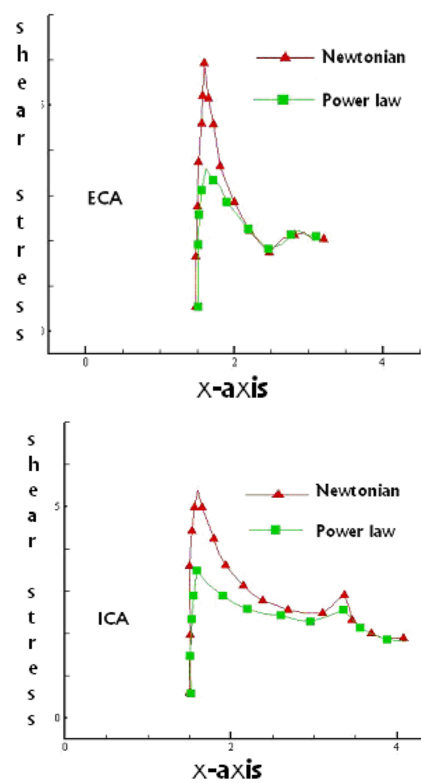

Figure 2: Shear stress on the ECA and ICA.

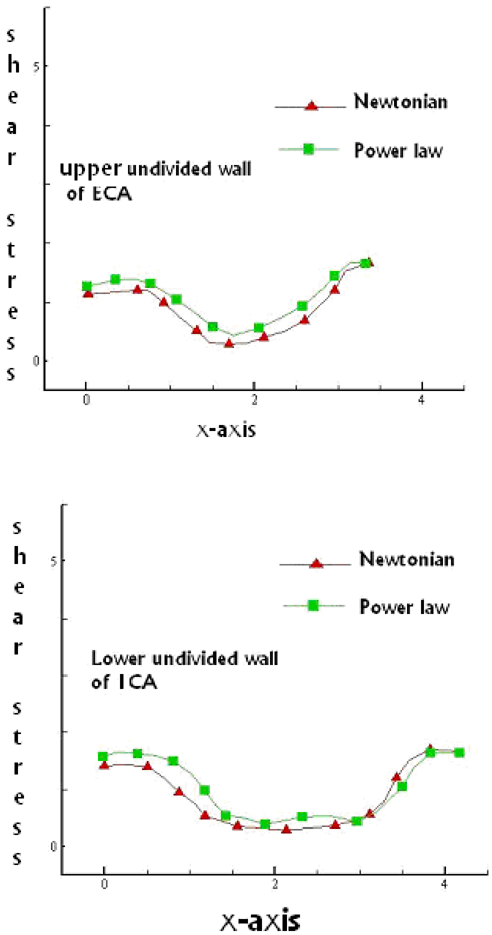

Figure 3: Shear stress on the upper undivided Wall of ECA and lower undivided wall of ICA

In figure 3, line plots of wall shear stresses are shown at undivided upper wall of ECA and lower wall of ICA. It is observed that the wall shear-stresses for Newtonian and generalized Newtonian fluids are same and are lower in value than that of divided walls. Thus it is concluded that wall shear stresses for both Newtonian and generalized Newtonian will behave same in the entire region of the CA.

In the following figures, secondary velocity profiles at various regions of CA are depicted. Here, we can see the level of U_{sec} at ICA wall for Power-Law is lower than that of Newtonian.

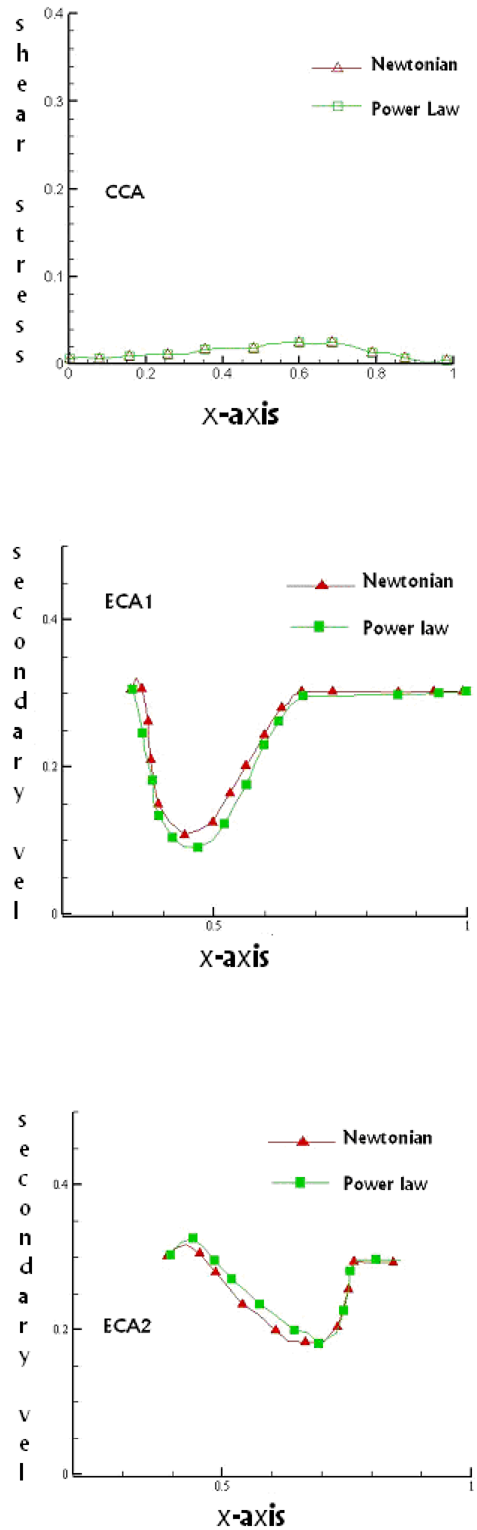

Figure 4: Secondary velocity profiles at various regions of ECA and CCA.

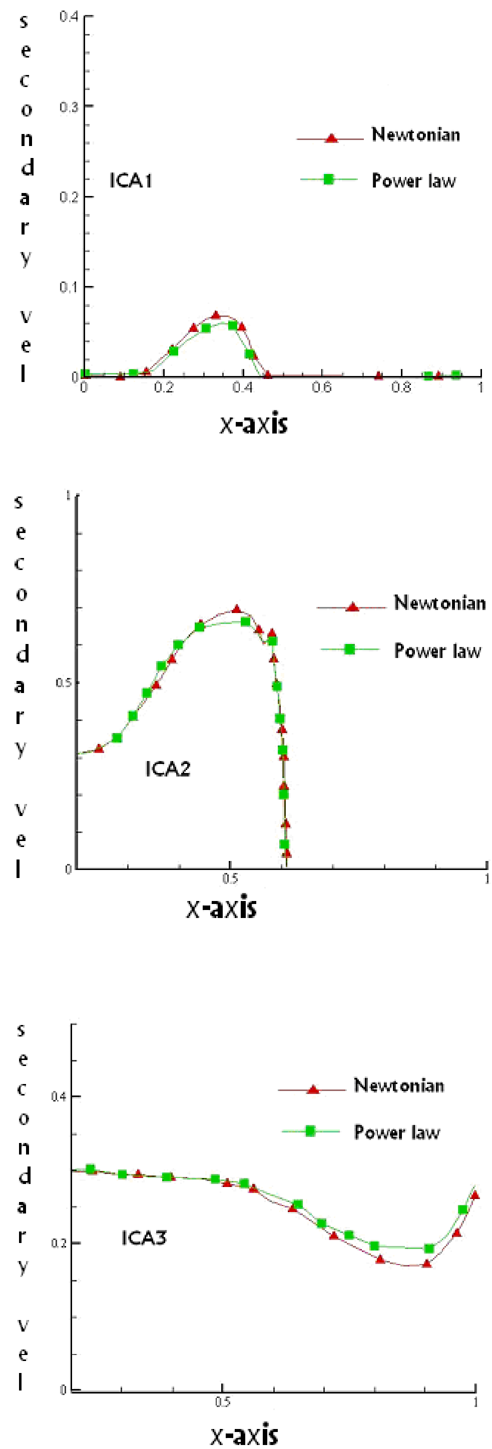

Figure 5: Secondary velocity profiles at various regions of ICA.

Line plots shown in figure 4 and 5, depicts that both Newtonian and Power-law fluids have same distribution trend and will result in the same shape of the stenosis in Carotid artery. In the CCA the velocity profiles are parabolic while at the divider walls the velocity profiles are changed due to increased shear stresses at that location.

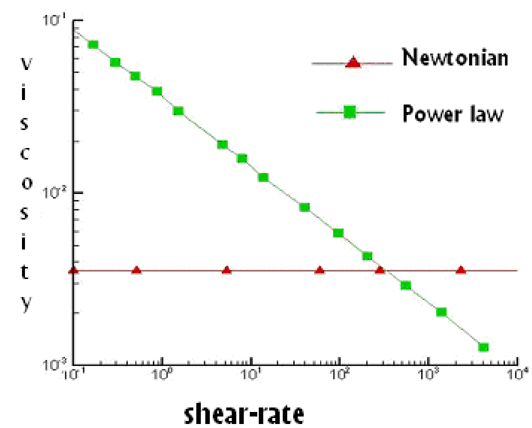

Figure 6: Comparison between Newtonian and Power law for viscosity and shear-rate

In figure 6, the line-plots show that for power-law model, the viscosity decreases with increasing shear-rates. Thus flow becomes Newtonian at high shear rates; this is the case in our study, where high shear rates are observed.

6. CONCLUSIONS

Finite Element Simulation of blood flow is performed considering both Newtonian and Generalized Newtonian fluid. Computations are taken at average velocity of the blood. Line plots shown in figures 2 and 3 depict that the shear stresses of Newtonian fluid flow for both the ECA and ICA are slightly higher as compared to the Power Law Model. The upper wall shear stress and the lower wall shear stress obtained for Power-Law model are almost same as to the Newtonian fluids, both coincides after a certain distance (figure 4 and figure 5 and 6). The effects of wall shear stress, shear rate and viscosity for various Reynolds numbers in the ICA are discussed. The results show that as shear rate increases viscosity decreases. From the secondary velocity profiles we observed that the high velocity gradient may cause the high shear rates in the narrowed region of the stenotic carotid artery that subsequently will increase wall shear stress at the location of occlusion. It is proved that the semi-implicit pressure correction algorithm is consistent and work well with blood flow simulation also. The obtained results are in agreement with the results available in literature.

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