Sindh Univ. Res. Jour. (Sci. Ser.) Vol.45 (4) 737-742 (2013)

SINDH UNIVERSITY RESEARCH JOURNAL (SCIENCE SERIES)

A Novel Mesh adaptation Technique for the computation of Stereo Depth Using Texture less Simple Stereo Pairs

K. B. AMUR , M. S. CHANDIO*, E. ALI

Quaid-e-Awam University of Engineering, Science and Technology, Nawabshah Pakistan

Abstract: A novel mesh adaptive strategy for a stereo vision depth for texture-less image pairs is proposed in this paper which is based on a posteriori error estimators. A Variational approach is proposed to solve one directional disparity map problem using TV regularization model. An adaptive P_1 finite Element method is used as a simulation device for the numerical solution of PDEs based model which is obtained from the minimization of the chosen energy model. This mesh adaptation procedure is performed by a posteriori error indicators in appropriate norm. Our locally adaptive methodology allows us the local and automatic choice of smoothing parameters.

Keywords: Mesh Adaption, A Posteriori estimates, Finite element Method, Stereo Depth.

1. INTRODUCTION

PDEs based techniques play a major role in the area of image processing and computer vision. Variety of approaches is available in the literature to process the image data using mathematical imaging techniques. A literature survey of relevant numerical approaches based on PDEs is given below.

TV regularization is one of the robust and discontinuity preserving approaches to compute the image motion in single direction (stereo Depth) and in two directions called ("Optic Flow"). Some of the research contributions on image motion problems are given in Amur(2011), Amur(3013), Belhachmi and F.Hecht (2011), Aubert et. el. (1995), Horn and Shunk (1981), Alvarez(1999), Issaque (1990) Weickert et.al.(2005) bruhn (2006). Generally it is observed that the quadratic regularizer does not perform very well in the neighborhood of moving discontinuities like edges in the image motion. Extra smoothness property of quadratic regularizer attracts the researchers in this area to work on a regularization approach. This approach is suitable to cope with discontinuities and to regularize the solution of ill-posed problems simultaneously. TV regularizer has such properties to solve these difficulties of extra smoothness along edges. We propose a novel smoothing strategy for one directional disparity map from a given stereo pair, this novel approach was appeared in Belhachmi and Hecht (2011), Amur(2011), Amur(2012), Amur(2013). Amur(2011) has discussed these mesh adaptive procedures in detail. Various applications in image processing and computer vision problems were associated to this mesh adaptive approach which is completely new idea of regularization from usual grid techniques available in literature specifically in image processing. Aubert *et.al.*(1999), Bruhn *et.al.* (2005), Weickert *et.al* (2005) used finite difference methods taking rectangular grids for image processing problems. In this work, an adaptive finite element method is considered as simulation device with triangular grid as computational domain.

2. PROBLEM DESCRIPTION

The variational method based on TV regularization seems to be more appropriate than quadratic regularization approach, since it does not enforce the disparities to be very smooth everywhere along the edges of moving objects and reconstruct the dense disparity map while preserving its corners. The energy model is considered to extract one directional disparity map "u" from the given stereo image pair as follow:

$$E(u) = \int_{\Omega} (\alpha(x)(\psi(|\nabla u|)) + D(I(x),u))dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The weight $\alpha(x)$, is strictly positive scalar valued continuous smoothing function on domain Ω, $x = (x_1, x_2)^T \in \Omega$, $\nabla = (\partial x_1, \partial x_2)^T$ denotes the gradient operator and $|\cdot|$ denotes the Euclidean norm. The structure of the given energy functional equation (1) is based on the following two parts, i.e. the regularization term $\alpha(x)\psi(|\nabla u|)$ which is generally known as regularizer or smoothness term which plays the role of smoothing the disparity u by its

++Corresponding Author: amurkb@gmail.com
*IMCS Sindh University Jamshoro

regularization effect, and the smoothing term is considered as $\psi(|\nabla u|) = |\nabla u|$. From the computational point of view, a widely used function is $\psi_\beta(|\nabla u|) = \sqrt{\beta^2 + |\nabla u|^2}$, where $(\beta > 0)$ is a small regularization parameter. The data term is modeled using brightness consistency assumption on grey values of pixels during the camera motion which yields

$$I(x+u, t+1) = I(x,t) \quad \ldots\ldots(2)$$

Using the Taylors expansion of first order, the data term is formed as

$$D(I(x),u) = (I_{x_1} + I_t)^2 \quad \ldots\ldots(3)$$

Let we set $f = -I_{x_1} I_t$ where I_{x_1} *and* I_t stand for derivatives with respect to x_1 *and* t respectively. Using the direct methods from calculus of variations the energy functional $E(u)$ satisfies the following Euler-Lagrange equation

$$0 = -\mathrm{div}\left(\frac{\alpha(x)\nabla u}{\sqrt{\beta^2 + |\nabla u|^2}}\right) + (I_{x_1}{}^2 u + I_{x_1} I_t)\ , in\ \Omega. \quad (4)$$

Where $\frac{\partial u}{\partial n} = 0,\ on\ \partial\Omega$, where we are looking for a steady state solution of a gradient system

$$\partial_\rho u = -\mathrm{div}\left(\frac{\alpha(x)\nabla u}{\sqrt{\beta^2 + |\nabla u|^2}}\right) + (I_{x_1}{}^2 u + I_{x_1} I_t) \quad (5)$$

Variational formula for (4) can be derived as

$$b(u,v) = l(f,v) \quad \ldots\ldots(6)$$

One is therefore interested to compute $u \in H^1(\Omega)$ where

$$\begin{cases} b(u,v) = \int_\Omega \left(\frac{\alpha(x)\nabla u \cdot \nabla v}{\sqrt{\beta^2 + |\nabla u|^2}}\right) dx + \int_\Omega I_{x_1}{}^2 u \cdot v\ dx \\ l(u,v) = \int_\Omega f \cdot v\ dx \ \ \forall\ v \in H^1 \subseteq X \end{cases} \quad ..(7)$$

3. NUMERICAL SCHEME

3.1 Discrete formulation:

The discrete formulation of the problem (6) is given as

$$b(u_h, v_h) = l(f, v_h)\ , \quad \forall\ v_h \in X_h \ldots\ldots\ldots\ldots (8)$$

Where

$$X_h := \left\{ v_h \in C^0(\overline{\Omega}) \,\middle|\, \forall K \in T_h,\ v_h|_K \in P_1(K) \right\}$$

Here $X_h \subset X$ is the discrete space with P_1 finite elements. The computational domain is considered as triangular grid T_h with maximum size of each element $h > 0$.

Where $P_1(K)$ denotes space of all polynomials functions having degree equal to one. We have

$$\begin{cases} b(u_h, v_h) = \int_\Omega \frac{\alpha(x)\nabla u_h \cdot \nabla v_h}{\sqrt{\beta^2 + |\nabla u_h|^2}} dx \\ + \int_\Omega I_{x_1}{}^2 u_h \cdot v_h\ dx \qquad \ldots\ldots\ldots(9) \\ l(u_h, v_h) = \int_\Omega f \cdot v_h\ dx \ \ \forall\ v_h \in X_h \end{cases}$$

Proposition: For any $\alpha_h > 0$ and $\lambda > 0$ there exist a unique solution of the discrete problem (9)

3.2 A Posteriori Error estimate

We apply a posteriori error estimate [Verfürth (1996), Amur(2011), Amur(2012), Amur(2013), Amur et.al (2013), Belhachmi and F.Hecht (2011)] which play an important role in our new mesh adaptive strategy. For each triangle (element) $K \in T_h$,

$$\eta_K = \alpha_K^{-\frac{1}{2}} h_K^2 \left\| I^2{}_{x_1} u_h - f_h \right\|_{L^2(K)} + \frac{1}{2} \sum_{e \in \varepsilon_h} \alpha_e^{-\frac{1}{2}} h_e^{-\frac{1}{2}} \left\| \left[\alpha_e \frac{\nabla u_h}{\sqrt{\beta^2 + |\nabla u_h|^2}} \cdot n_e \right]_e \right\|^2_{L^2(e)} \quad .(10)$$

Where $\alpha_e = \max(\alpha_{K_1}, \alpha_{K_2})$, is the maximum value on K_1 *and* K_2 are two triangles corresponding to

the edge e. $[\varphi]_e$ the jump of the φ across the edge e in the direction of normal vector n_e. h_K is denoted as the diameter of K and h_e is the length of $e \in \varepsilon_K$ (set of edges).

3.3 Computational Algorithm and Mesh Adaption

The mesh adaptive strategy given in this paper is based on the idea of both refinement and coarsening the grid. The a posteriori errors estimate η_K is the metric for the given mesh building. We use isotropic mesh and unstructured grid which allow the local adaptive choice of smoothness. The mesh adaptive process is automatic which shows the dramatic regularization effects on the disparity mesh, consequently a dense disparity mesh with reduction of error density at each step of adaptation is observed. The adaptive algorithm [(Amur2011, Amur (2012), Amur (2013), Amur et.al.(2013), Belhachmi and F. Hecht (2011)] is based on the following steps:

(i) Associate the initial grid T_h^0 the given image pair.

(ii) Do :

- Compute the stereo depth on T_h^0 grid to the noisy image with $\alpha = \alpha_0$ a large constant.
- Build T_h^1 as the isotropic triangular mesh using the metric as a posteriori error estimate given in equation (10).
- Choose locally the optimal value for α_1 on T_h^1.

(iii) GO to step (ii) and compute u_h on T_h^1 for α_1

For more details the reader is referred to our previous work [(Amur2011, Amur (2012), Amur (2013), Amur *et.al* .(2013), Belhachmi and F.Hecht (2011)].

For the automatic adaptive choice of optimal regularization parameter α, during adaptation process for each triangle K, following formula is proposed

$$\alpha_K^{k+1} = \max\left(\frac{\alpha_K^k}{1+\beta*\left(\left(\frac{\eta_K}{\|\eta\|_\infty}\right)-0.1\right)^+}, \gamma_\alpha \right) \quad \ldots(11)$$

Where γ_α is a threshold and the β is a used for controlling the regularization by α in the non smooth regions of the disparity map. The mesh adaptive control automatically identifies the damaged regions where the a posteriori error density η_K is large. The values of α are decreased by the given algorithm to refine the grid. The smoothing process decreases the error density η_K from each approximation at every adaptive iteration consequently the improvement in the quality of disparity map and corresponding meshes is observed as shown in image results Figure (1). Initially the value of α is selected as large number. The results of convergence for three mesh adaption iterations are shown in the given **Table 1, and Figure 1.**

4. RESULTS AND DISCUSSION

Here, we have considered a simple texture less stereo pair named as house taken from the Vision and Autonomous Systems Center's Image Database http://www.ius.cs.cmu.edu/idb/. We have performed four mesh adaptive iterations of our adaptive algorithm which is designed using a posteriori error estimate. Process converges very fast on third iteration. Adaptive procedure is very clear from the results as meshes at various adaptive iterations, disparity map at first and third iterations, the error density (a posteriori estimate) at first three iterations and the effects of the regularization by the weight function $\alpha(x)$. One can observe from this novel computational strategy for computer vision problems that solution starts from very worst visual quality to a very good numerical approximation. The disparity mesh shown after first adaptive iteration looks dense everywhere, whereas on the converged solution (third iteration) it is refined only on the computed disparity map (control of refinement) and observed as coarsen in the regions away from the converged solution which is our goal. We are interested to move the mesh points towards the regions where it is required to regularize; consequently very fine and high quality mesh associated to the disparity map is obtained. This all magic is performed by a posteriori error indicator and the regularization weight $\alpha(x)$. One can compare the disparity map results after first and third iteration that how this regularization strategy improves the visual quality of the disparity image dramatically. The third type of the results is based on the images for the computed error density η_K, the maximum norm of η_K showing the fast convergence process is given in table1 and Figure 1. The error density image after first iteration is very large and decreases on second and third iterations which in result give rise to the better image for the approximate disparity map. Last three images show the effects of regularization by $\alpha(x)$. The color

images are given for the error indicator η_K, scales are given in dark colors which show the greater values and lighter colors show the smaller values for η_K. Results are shown in **Figure 2.**

Table 1. Results A Posteriori error Estimator η_K at various adaptive iterations in ∞ norm

No. of Iteration	Error Estimator η_K
1	0.0516
2	0.03136
3	0.00340

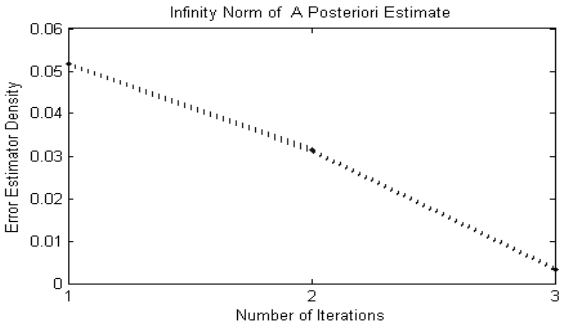
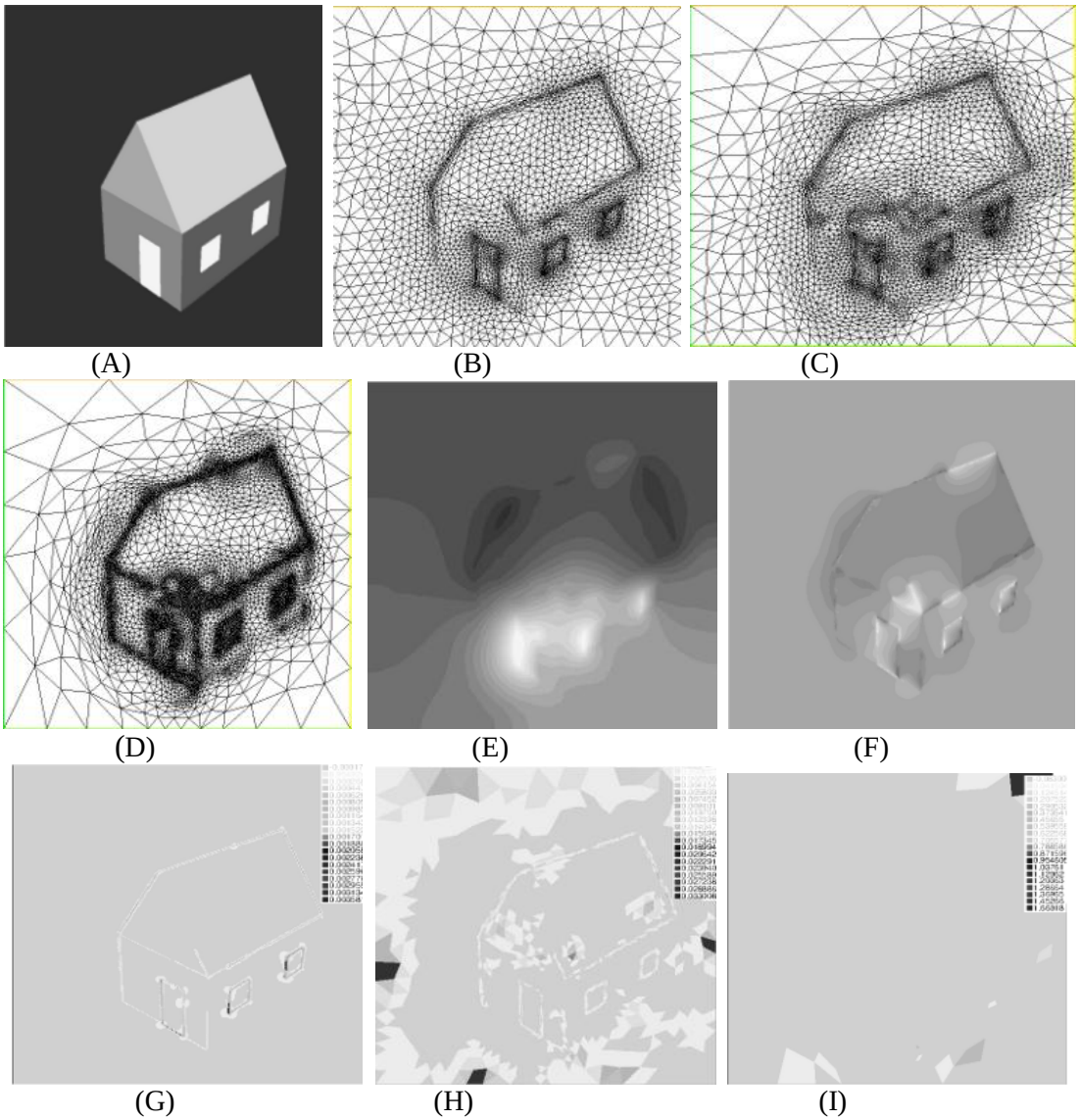
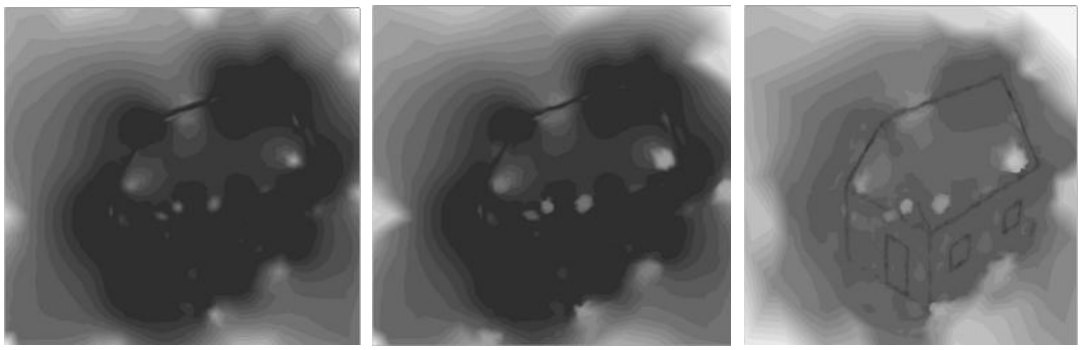


Figure 1. Line Plot Of Error Estimator In Maximum Norm





(J) (K) (L)

Figure 2. (A) The Left image of stereo Pair named as House, (B) Mesh adaption after first regularization iteration, (C) Mesh adaption after second adaptive iteration, (D) Mesh adaption after third adaptive iteration, (E) stereo Depth after first adaptive iteration, (F) stereo depth after third adaptive iteration, (G) Error estimator density η_K after first adaptive iteration, (H) Error estimator density η_K after second adaptive iteration, (I) Error estimator density η_K after third adaptive iteration, (J) Effects of regularization α after first iteration, (K) Effects of regularization α after second iteration and (L) Effects of regularization α after third iteration.

5. CONCLUSION

In this paper we have applied a novel mesh adaption method for the computation of stereo depth from texture less stereo pairs. It is observed that the computational strategy developed in this paper performs efficiently for the given simple stereo pair. The local adaptive strategy by the regularization function $\alpha(x)$ shows significant regularization effects on the disparity map domain (Figure 2. J-L) and the adaptive process yields the sharp edges. Though the idea of the TV regularization is very old in the image processing and computer vision research which is famous for Hypersurfaces recovery but the computation of grey value at all locations of moving image is still a challenge for image practitioners, because the variational techniques have this drawback of grey value recovery at all locations.

The novel idea of triangular grids and the adaptive finite element method with a posteriori control of adaptation provides efficient approximations for disparity map computations as observed from disparity image meshes (Figure 2). The method discussed in this paper performs well in the case of the simple stereo pairs. Future work will be devoted to brightness changes and textured patterns in the textured image pairs using mesh adaption technique which is proposed in this paper.

REFERENCES

Amur K.B (2011). Contrôle adaptatif, techniques de régularisation et applications en analyse d'images. P.hD thesis LMAM University of Metz France.

Amur K.B (2012). Some regularization Strategies for an Ill-Posed Denoising Problem. International Journal of Tomography and Statistics, 19(1), 46-59.

Amur K. B (2013). A Posteriori Control of Regularization for Complementary Image motion Problem. SURJ (Sci. Ser.), 45(3): 546-552.

Amur K. B, S. F. Shah, A.A. Sheikh (2013). An Adaptive control for Tikhonov Regularization on Unstructured Grid for A Variational Denoising Problem. SURJ (Sci. Ser.), 45(3):553-558.

Aubert, G., R. Derriche, P. Kornprobst (1995). Optic flow estimation while preserving its discontinuities a variational approach. In Proc. Second Asian Conference on Computer Vision, vol 2, singapore 290-295 and Springer 1996 Lecture Notes in Computer Science ISBN 3-540-60793-5.

Aubert G., R. Deriche, P. Kornprobst (1999). Computing optical flow via variational techniques. SIAM Journal on Applied Math., 60 (1), 156-182.

Aubert G., and Kornprobst (2006). Mathematical Problems in Image Processing: Partial Differential Equations and the Calculus of Variations. Springer-Verlag, second edition vol.147. Printed in USA.

Alvarez L., J. Esclarin, M. Lefebure, and J. Sanchez (1999). A PDE model for computing the optic flow. In Proc. XVI Congreso de Ecuaciones Diferenciales y aplicationes, Las Palmas de Gran Canaria, Spain, 1349-46.

Anandan P. (2000). A computational framework and an algorithm for the measurement of visual motion. International Journal of Computer Vision, vol. 39(1), 41-56.

Robert A. Adams and John J. F. Fourier (2003). Sobolev spaces. Academic press, Elsevier, second edition, Printed in Netherlands.

Brenner S. C and L. R. Scott (1994). The mathematical theory of finite element methods. Third Edition, Springer Science, Business Media, LLC, 233 Spring Street, New York, NY 10013, USA.

Belhachmi Z. and F.Hecht (2011). Control of Effects of the regularization on Variational optic flow computations. J. Mathematical imaging and vision, 40 (1):1-19.

Borzi A., K. Ito, K. Kunisch (2002). Optimal control formulation for determining the optical flow, SIAM J. Sci. Computing, 24 (3), 818-847.

Bruhn A. (2006). Variational Optic Flow Computation, accurate modeling and efficient numerics. PhD thesis in computer Science Saarbrcken, Germany. Saarland University.

Bruhn A., J. Weickert (2006). A confidence measure for variational optic flow methods. In R. Klette, R. Kozera, L. Noakes, J. Weickert (Eds.): Geometric Properties from Incomplete Data. Pages 283-297, Springer, Dordrecht, 2006.

Bruhn, A., J. Weickert, Christophschnorr (2005). Lucas/Kanade meets Horn/Schunck: Combining Local and Global Optic flow methods. International Journal of computer vision 61(3), 211-231.

Hecht F., Olivier Pironneau, Jacques Morice (2013). FreeFem++ .http://www.freefem.org/. September 05 2013.

Horn, B., B. Shunk (1981). Determining Optical flow. Artificial Intelligence, Vol.17, 185-203.

Issaque C. (1993). Nonlinear Variational Method for Optical Flow Computation. 8th SCIA, 523-530.

Nagel H.H (1990). Extending the oriented smoothness constraint into the temporal domain and the estimation of the derivatives of optical flow. Lectures notes in computer sciences, Springer Berlin, 139-148.

Schnörr C. (1994). Unique reconstruction of piecewise smooth images by minimizing strictly convex non-quadratic functional. Journal of Mathematical Imaging and Vision, 4, 189.

Slesareva N., A. Bruhn and J. Weickert (2005). Optic flow goes stereo; A variational method for estimating discontinuity-preserving dense disparity maps. In W. Kropatsch, R. Sablatnig and A. Hanbury, eds. Pattern Recognition. Volume 3663 of Lecture Notes in Computer Science, Springer, Berlin, 3340.

Verfurth R. (1996). A Review of a posteriori error Estimation and Adaptive mesh refinement Techniques. Advances in Numerical Mathematics, Wiely and Teubner (1996).

Weickert, Andres Bruhn, Thomas Brox and Nils Papenberg (2005). A Survey on Variational Optic Flow Methods for Small Displacements. FR 6.1-Mathematics University of Sarlandes, Postfach 15 11 50, 6641 Saarbr¨ucken Germany.

Zimmer, H. Andres Bruhn, Jaochim Weickert, Levi Valgaerts, Agustin Salgado, Bodo Rosenhahn, and Hans-Peter Seidel (2009). Complementary optic flow. 7th International Conference, EMMCVPR, Bonn, Germany, 24-27 and Springer, Lecture Notes in Computer Science Volume 5681, 2009, 207-220.