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Volatility Modeling of Karachi Stock Exchange

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Abstract: Modeling volatility of emerging and developed capital markets of the world has always been an imperative area of research in the field of Statistics, Finance, and Economics with the detection of volatility clustering, time-varying volatility and asymmetric response of volatility to market activities and trends. The present study is attempted to investigate the most important features of Karachi stock exchange (KSE-100 Index) using the data of daily closing index from Jan 1, 1999 to Dec 31, 2008. For empirical investigation of data, ARMA, ARCH, GARCH and EGARCH models are fitted to model the conditional mean and conditional variance of KSE-100 Index. The results indicate that GARCH (1,1) model is the good fittedmodel but not able to capture the "leverage effect". So, EGARCH is applied to cope with this problem. EGARCH(1,1) model gives the best fit to data as compared to other applied techniques. Detailed analysis of data escort us to report that KSE-100 Index is highly volatile and positive returns are associated with higher volatility than negative returns of equal magnitude and it is clear that past residuals highly influence the current volatility.

Keywords: ARMA, ARCH, GARCH, EGARCH

1. INTRODUCTION

Financial markets play a decisive role in the development of any country's economy. Financial markets act as the channel for flow of long-term financial resources from the capital's savers. Hence in this era of growing globalization, proficient capital markets are of crucial need for economic prosperity and growth. Karachi Stock Exchange (KSE) is the most liquid and largest exchange in Pakistan. It trades the KSE-100 Index. In 2005, 88% of the KSE's total market capitalization was represented by the KSE-100 Index. Bloomberg declared the KSE as the best performing world stock market for the year 2002, and "Business Week" consistently ranked it for more than six years as best performing market of the World. It constantly revised the index as it is a good indicator of the overall Exchange performance over a period of time. On March, 2007, 754 companies were listed with the market capitalization of Rs. 3,200.182 billion (US $ 52.00) having listed capital of Rs. 495.968 billion (US $ 8.27 billion) and on December, 2009, 652 companies were listed with the market capitalization of Rs. 2.561 trillion (US$ 30.5 Billion) having listed capital of Rs. 717.3 billion (US$ 12 billion). As it is a well known fact that financial markets of a country reflect the country's economic health and progress. Changes in the financial zone of country also influence the other factors of economy such as economic growth and development, monetary policies, price indices, GDP, national income, foreign exchange rates, interest rates, etc. A variety of pragmatic studies have established the relationship among above mentioned variables. So, time series study is of great importance in this modern era of growing globalization, where proficient capital markets are a more integrated and essential factor of economy. Financial time series often reveal the fat taildness, skewness towards positive and negative values, sharp peakedness i.e. high kurtosis and leptokurtic trend in values. One more important characteristic of financial time series is the "Leverage effect". The term "Leverage Effect" refers to the fact that changes in stock prices have a propensity to be negatively correlated with changes in volatility. Practitioners and researchers have analyzed these properties of financial returns and successfully identify the way, that how this theoretical information can be utilized empirically for estimation and forecast of conditional mean and variances for practical implications. The most influential and powerful techniques developed for this purpose are the ARMA modeling to model the conditional mean and the family of generalized conditional autoregressive heteroskedastic (GARCH) models to model the conditional variances of financial time series. These volatility measures such as mean and variances are further applied in financial decisions such as risk

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analysis, trading volume and derivative pricing, and portfolio management etc. Pasha *et al.*, (2007) had compared the forecasting performance of heteroscedastic models i.e. GARCH, EGARCH, GJR and APARCH using the KSE-100 index data from January 1, 2002 to August 31, 2006. All the models were estimated using normal and fat-tailed distributions such as student-*t* and GED for the innovations of these models and the forecast performance had been compared on the basis of statistical loss functions. Results showed that asymmetric GARCH model proved better for the volatility forecast of the KSE-100 index return and the statistical loss functions supported the EGARCH and APARCH models than GJR model. Models with GED innovations showed better forecast than other distributions for innovations and GJR model with all the three distributions i.e. normal, students-t and GED provided superior forecasting. Finally, it was concluded that non-normal distributions provide better out-of-sample results than the normal distribution. Rashid & Ahmed (2008) had conducted a research study to examine the performance of linear vs. non-linear heteroscedastic models and forecast performance was also evaluated using the RMSE criterion. Data of KSE-100 Index was analyzed ranged from January 2001 to November 2007. Linear models included the Random walk model, AR model, MA and exponential smoothing models and in non-linear category GARCH, EGARCH, APARCH and GARCH-in-Mean models were used for comparison based on forecast abilities. The study concluded that the in case of linear model, exponential smoothing models proved to be best and in class of non-linear models, GARCH-in-Mean proved to be superior comparatively on the basis of RMSE criterion. Extensive work has been done on the modeling of financial time series, this study is also a preliminary effort to shed some light on the charisma of volatility pattern and leverage effect of Karachi Stock Exchange. The aim of present study is characterizing the fundamental and vital behavior of volatility and the documentation of stylized properties of KSE-100 Index data and modeling of conditional variances. This paper is organized as follows. Section II contains the data description. In Section III, methodology is discussed and in section IV empirical discussion has been done.

DATA: For the analysis, the data used is the daily closing prices of KSE-100 index and it has been taken from the site of www.brecorder.com/index.php?stocks=index, covering the period from January 1, 1999 to December 31, 2008. The daily return series will be generated as follows:

$$R_t = 100 * \log(P_t/P_{t-1}).$$

Where P_t is the current index closing price at time stand P_{t-1} is showing the index closing price of previous day at time t-1.

Descriptive Statistics: Fig.1 is showing the prices of KSE-100 Index. The trend is non stationary and upward increasing from year 2002 to onward but started decreasing from 2008.



Fig 1.Daily Closing Prices of KSE-100 Index

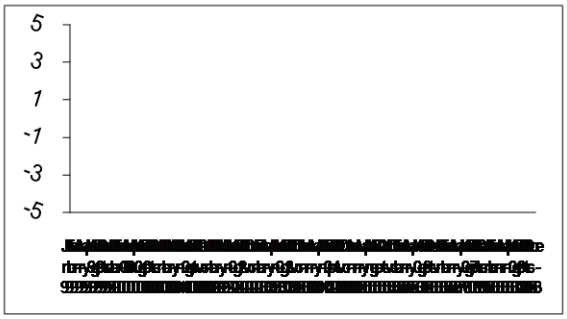


Fig 2: Returns Series of KSE-100 Index

Fig. 2 is showing the graph of returns series, which clearly exhibits volatility clustering, i.e. "Large changes tend to followed by the large changes of any sign and small changes tend to be followed by the small changes" and time varying characteristics of volatility.

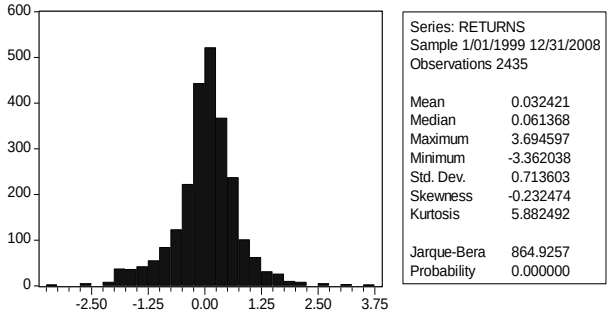


Fig 3: Descriptive Statistics of Returns of KSE-100 Index

Descriptive statistic is shown in Figure 3 along with table of statistics. The statistics exhibit that the returns are negatively skewed i.e. price series is highly leptokurtic and strongly skewed to the left, which is the indication that the KSE-100 index has non-symmetric returns and asymmetry towards negative values. kurtosis is greater then 3 signifying the heavier tail than standard normal distribution. The JB test has also rejected the hypothesis of normality.

2. MATERIAL AND METHOD

Methodology

Conditional mean of returns series is model by using ARMA model technique. The generalized form of ARMA (p, q) model is:

$$R_t=\varphi_0 + \acute{\varepsilon}_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \sum_{i=1}^{p} \varphi_i R_{t-i} .$$

ARMA (p, q) refers to the model with p autoregressive terms and q moving average terms.

To model the conditional variance, Autoregressive Conditional Heteroskedasticity (ARCH) models were introduced by Engle (1982) and generalized as GARCH (Generalized ARCH) by Bollerslev (1986). Since ARCH (q) model introduced by Engle (1982), the conditional variance is postulated to be a linear function of the past squared innovations,

$$\sigma_t^2 = Var_{t-1}(\varepsilon_t) = E_{t-i}(\varepsilon_t^2)^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2$$

All the parameters must satisfy $\alpha_i > 0$, $\omega > 0$ for all as procedure is to deal with variance The process is said to covariance stationary if and only if $\sum_{i=1}^{q} \alpha_i$ <1. In empirical applications, ARCH process often required large number of parameters to explain the dynamic structure of financial phenomenon. To evade these problems, Bollerslev (1986) proposed the generalized ARCH, or GARCH(p, q), model,

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma^2_{t-j} .$$

Same restriction of positivity of parameters is imposed on simple GARCH(l, 1) model. i.e. $\alpha_i > 0$, $\omega > 0$ and $\beta_j > 0$ for all i.The virtue of this approach is that GARCH model with a small number of terms appears to perform better than ARCH with many terms. For GARCH(1,1) sum of parameters should less than 1, as $\alpha_1 + \beta_1 < 1$. But practically, in many applications with high frequency financial data the estimates α_1 and β_1 turns out equal to unity, as $\alpha_1 + \beta_1 = 1$. This equality of parameters indicates that unconditional variance doesn't exist and the GARCH (p, q) process is integrated in variance i.e.

EGARCH (p, q). The integrated GARCH (p, q) model introduced by Engle and Bollerslev (1986). An IGARCH (1,1) model can be written as:

$$\sigma_t^2 = \omega + \beta_1 \sigma^2_{t-1} + (1-\beta_1)\varepsilon_{t-1}^2 \; as \; \alpha_1 = 1 - \beta_1,$$

where α_1 and β_1 are coefficients the of GARCH model.Since IGARCH model is infinite in unconditional variance.

The EGARCH model proposed by Nelson (1991) has no such constraint of parameters. The variance equation specified by Nelson (1991) has an advantage over the ARCH and GARCH that log of σ_t^2 is used and variance tends to be automatically positive without the imposition of restriction of non-negativity on parameters and it also captured the "Leverage Effect".

Table 2. Results of AR(1), MA(1) and ARMA(1,1) Model

Models	c	φ_1	θ_1	AIC	SBC
AR(1)	0.03256 (0.041)	0.0938** (0.000)		2.155701	2.160464
MA(1)	0.0324 (0.039)		0.0889 (0.000)	2.155930	2.160692
ARMA(1,1)	0.0325 (0.110)	0.8514** (0.000)	-0.7905** (0.000)	2.152030	2.159175

$$\log(\sigma_t^2) = \omega + \sum_{i=1}^{p} \alpha_i ((|Z_t| - E(|Z_t|) + \gamma Z_t) + \sum_{j=1}^{q} \beta_j ln(\sigma^2_{t-j})$$

Where $Z_t \sim N(0,1)$. Since α show the asymmetric function i.e. current volatility is influenced by the past standardized residuals. The term $(|Z_t| - E(|Z_t|)$ shows the effect of magnitude, and γZ_t is for the representation of sign effect. The volatility persistence is equal to $\sum_{j=1}^{q} \beta_j$ and for unconditional variance the sum of β should be less than 1 i.e.

$\sum_{j=1}^{q} \beta_j$<1.

EMPIRICAL RESULTS:

Stationary Tests: In table.1 the statistic of ADF and PP tests are reported for stationary test of returns series. Results tend to be significant leading towards the straight forward rejection the null hypothesis of unit root.

Table 1.Results of Unit Root Tests

ADF(Level)	-44.86156 (0.0001)	1% level	-3.432842*	Reject H_0 (Stationary)
		5% level	-2.862527*	
ADF(1st Differencing)	-20.53132 (0.0000)	1% level	-3.432861*	Reject H_0 (Stationary)
		5% level	-2.862535*	
PP(Level)	-45.84166 (0.0001)	1% level	-3.4323842	Reject H_0 (Stationary)
		5% level	-2.862527*	
PP(1st Differencing)	-564.3858 (0.0001)	1% level	-3.432843*	Reject H_0 (Stationary)
		5% level	-2.862527*	

Mean and Variance Model: In **(Table.2)** the estimated parameters of ARMA (1,1) are given along with other candidates model. Both AR and MA parameters of ARMA (1,1) model are turn to be statistically significant, AIC &SBC measures turned to be lowest as compare to other candidates models. So, the ARMA (1,1) seems best according to the goodness of fit criterions. Diagnostic tests of selected model has also verified ARMA (1,1) as appropriate model. Financial time series is stationary but variance in such cases is not constant over time. So, ARCH effects are suspected .The correlogram of squared such cases is not constant over time. So, ARCH effects are suspected .The correlogram of squared innovations of ARMA model is reported in table 3, which are showing the pattern of autocorrelation and indicating the volatility clustering. ARCH-LM test also supported the presence of ARCH effects.

To model the variance for further analysis, estimated parameters of ARCH(5) are shown in table 4 All parameters are positive and tends to be significant. ARCH-LM test is also supporting the appropriateness of this model. But for the problem of parsimony, it is essential to estimate GARCH(1,1) mode with less no. of parameters as an better alternative of ARCH model. So in table.5, GARCH (1,1) model is estimated and results reveals that estimated parameters of GARCH(1,1) are highly significant. The parameter α_1 is positive and significant and showing the measure of risk which is measured by own conditional variance and indicating that the positive and significant autocorrelation in returns. The parameter β_1 is also significant. Furthermore, the condition $\alpha_1+ \beta_1$ is measure of volatility persistence and is also an estimation of the decay rate of response function on daily basis. This condition reflects that how the historical conditional volatility affected the current conditional variance of stock returns. Sum of coefficients should be less then one i.e. $\alpha_1+ \beta_1 <1$ but in this case the values of α_1 and β_1 sum up to unity i.e. $\alpha_1+\beta_1 = 1$. It indicates that estimated conditional variance is an integrated GARCH (IGARCH) process and is non-stationary. In addition, the persistence of volatility will remain forever and a shock has indefinite influence on volatility level. This result of integrated conditional variance process is further attributed to structural change in KSE-100 Index performance and its high growth with sharp pace as volatility persistence decreased. It implies that new shocks will have influence on returns for the longer periods and further it is also an indication of non existence of unconditional variance. AIC and SBC turned to be lowest in case of GARCH as compared to ARCH. For the judgment of "Leverage Effect" normality test of standardized residuals has been done and JB-Statistic has rejected the hypothesis of normality which is due to the existence if "Leverage Effect". So, EGARCH (1,1) model is fitted and reported in table.5.The coefficient α_1=0.445092 corresponds to the asymmetric function which is statistically significant and indicating that current volatility is highly influenced by the past standardized residuals. The leverage effect term γ is negative and significant, demonstrating the presence of negative leverage effect. Thus it implies that, negative returns are associated with higher volatility than positive returns of equal magnitude. The coefficient $\beta_1$is also significant, and implies higher degree of persistence. So, estimated EGARCH model successfully captures the asymmetry effect and the news impact in KSE-100 index returns.

Table 3.Correlogram of the Squared Innovations of ARMA(1,1) Model

Autocorrelation	Partial Correlation	Lags	ACF	PACF	Q-Stat	Prob
|** |	|** |	1	0.301	0.301	221.00	
|** |	|* |	2	0.247	0.172	369.71	
|** |	|* |	3	0.203	0.102	470.57	0.000
|* |	| |	4	0.148	0.039	523.91	0.000
|* |	| |	5	0.115	0.021	556.46	0.000
|* |	| |	6	0.100	0.023	580.70	0.000
|* |	| |	7	0.097	0.032	603.51	0.000
|* |	| |	8	0.116	0.058	636.47	0.000
|* |	|* |	9	0.137	0.070	682.10	0.000
|* |	| |	10	0.120	0.034	717.44	0.000

Table 4. Estimation of ARMA(1,1)-ARCH(5) model

Mean Equation						
Parameters	**c**		**φ_1**		**θ_1**	
Estimates	0.1299** (0.000)		0.9889** (0.000)		-0.9727** (0.000)	
Variance Equation						
Parameters	**ω**	**α_1**	**α_2**	**α_3**	**α_4**	**α_5**
Estimates	0.10331** (0.000)	0.30780** (0.000)	0.24031** (0.000)	0.08562** (0.019)	0.14917** (0.000)	0.18082** (0.000)
AIC	1.792921	**ARCH-LM TEST**				
SBC	1.816739	**F-Statistic**		1.384224 (0.226999)	**Obs* R^2**	6.918495 (0.226773)

Table 5. Results of ARMA (1,1)-GARCH (1,1) Model and ARMA (1,1)-EGARCH (1,1)

	Mean Equation			Variance Equation					
Parameters	**c**	**φ_1**	**θ_1**	**ω**	**α_1**	**β_1**	**γ**	**AIC**	**SBC**
GARCH (1,1)	0.1330* (0.000)	0.9905* (0.000)	-0.9773** (0.000)	0.0179* (0.000)	0.2524* (0.000)	0.7491* (0.000)		1.783	1.799
EGARCH (1,1)	0.1226 (0.0000)	0.9870 (0.000)	-0.9722 (0.000)	-0.3944 (0.000)	0.4451 (0.000)	0.9286 (0.032)	-0.0510 (0.032)	1.760	1.780
ARCH-LM Test of GARCH(1,1)			**F-Statistic**		0.5921(0.706)	**Obs*R^2**		2.9643 (0.705)	

3. CONCLUSIONS

This study is attempted to investigate the imperative and essential features of KSE-100 index using the data of return series from January 1, 1999 to December 31, 2008. For the modeling of volatility of KSE-100 Index returns, ARMA, ARCH, GARCH and EGARCH models are fitted. Returns series exhibits stylized characteristics such as volatility clustering, time-varying conditional heteroskedasticity, excess kurtosis, and fat-tiredness, and leverage effect, which are the common characteristics of financial returns. The results indicate that GARCH (1,1) model is good to fit the returns data but as the parameters estimates of GARCH(1,1) sum up to unity, this further indicates that the GARCH is an Integrated GARCH process in variance. Conditional variance is non-stationary, and this result of integrated conditional variance process is further attributed to structural change in KSE-100 Index performance and its high growth with sharp pace as volatility persistence decreased. EGARCH (1, 1) is also fitted for leverage effect in returns series and it gives the best fit to data as compared to other applied techniques.

REFERENCES:

Aggarwal, R., C. Inclan*and* and R.. Leal, (1999) Volatility in emerging stock market.Journal of Financial and Quantitative Analysis, 34 **(1)**: 33-55.

Ahmed, S. and O. Farooq, (2008) The effect of 9/11 on the stock market volatility dynamics: Empirical Evidence from a front line stat. International Research Journal of Finance and Economic. ISSN 1450-2887 Issue 16 (2008) Euro Journals publishing, Inc.2008.

Brockwell, P. J. and R. A. Davis, (2002) Introduction to Time Series and Forecasting, 2nd. ed., Springer-Verlag.

Bollerslev, T., (1986) Generalized Autoregressive Conditional Heteroscedasticity GARCH. Journal of Econometrics, 31 **(3)**: 307-327.

Butt, B.Z., K.U. Rehman, M.A. Khan, and N. Safwan, (2010) Do economic factors influence stock returns A firm and industry level analysis. African Journal of Business Management, 4 **(5)**: 583-593.

Farid, A., and J. Ashraf, (1995) Volatility at Karachi stock-Exchange. The Pakistan Development Review 34 **(4)**: 651-657.

Pasha, R. G., T. Qasim and M. Aslam, (2007) Estimating and Forecasting Volatility of Financial Time Series in Pakistan with GARCH-type Models. The Lahore Journal of Economics, 115-149.

Peter, H and H. Björn (2005) Forecasting Variance Using Stochastic Volatility and GARCH.The European Journal of Finance, 11 **(1)**: 33–57.

Rashid, A. and S. Ahmad, (2008) Predicting stock Returns Volatility: An Evaluation of Linear vs. Nonlinear Method, International Research Journal of Finance and Economics ISSN 1450-2887 Issue 20.

Saleem, K., (2007) Modeling Time varying Volatility and Asymmetry of Karachi Stock Exchang (KSE), International J. Economic Perspective, **(1)**: 1-9.

Tsay S. and N. Ruey, (2005) *Analysis of Financial Time series*. 2nd ed., A John Wiley and Sons, Inc., Publications.

William H. and A. Greene, (1993) *The Econometrics Analysis*.2nd ed., Macmillan Publishing company. USA.