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Domination Number of Prism over Cycle C_n

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Abstract : Prism over graph G is the Cartesian product of the graph $G$and the complete graph on two verticesK_2. The aim of this note is show the domination number of the Cartesian product of the cycle $C_n$andK_2, where $n \geq 3$ that is,
$\gamma\left(C_n \square K_2\right) = \frac{n}{2}$, if $n \equiv -4\ (mod\ 4)$ 2. $\gamma\left(C_n \square K_2\right) = \frac{n}{2} + 1$, if $n \equiv -2\ (mod\ 4)$ 3 `$\gamma\left(C_n \square K_2\right) = \frac{n+1}{2}$, if $n \equiv -1\ (mod\ 2)$,

1. INTRODUCTION

A graph $G$is triple consisting of a vertex set $V(G)$, an edge set $E(G)$and a relation that associates with each edge two vertices called endpoints. A complete graph on two vertices is denoted as K_2 having only two vertices and an edge.

A set $D$of vertices of a simple graph G, that is the graph without loops and multiple edges, is called dominating set if every vertex $u \in V(D) - D$is adjacent to some vertex $v \in D$. The **domination number** of a graph G, symbolically $\gamma(G)$is the order of a smallest dominating set of G. A dominating set $D$with $|D| = \gamma(G)$is called a minimum dominating set. The Cartesian product $G \times H$of graphs G and H is the graph with vertex set $V(G) \times V(H)$and $(a,x)(b,y) \in E(G) \times E(H)$ whenever $x = y$and $ab \in E(G)$, or $a = b$ and $xy \in E(H)$. For $x \in V(H)$ set $G_x = G \times \{x\}$ and for $a \in V(G)$set $H_a = \{a\} \times H$. $G_x$andH_a are called the layers of G or Hrespectively.

A prism over a cycle C_n, where $n \geq 3$is the Cartesian product $(C_n \square K_2)$ over n vertices. Prism over cycles is planar and polyhedral graph called the $n-$prism; the 3-prism, 4-prism, and 5-prism is commonly called the triangular prism, cube and the pentagonal prism. Earlier in year 2000 G. Chartrand and others shown results regarding claw domination for prisms and following results were obtained. We, in the following, state the theorem without proof.

Theorem 1. For $n \geq 3$, let G be the prism $C_n \square K_2$ then

1. $\gamma Y_1(G) = 2\left\lceil \frac{n}{4} \right\rceil$
2. $\gamma Y_2(G) = 3\left\lceil \frac{n}{2} \right\rceil$
3. $\gamma Y_3(G) = n$
4. $\gamma Y_4(G) = 2\left\lceil \frac{n}{5} \right\rceil, if\ n \equiv 0,3,4\ (mod\ 5), n \equiv 2,6\ (mod\ 10)\ or\ \gamma Y_4(G) = 2\left\lceil \frac{n}{5} \right\rceil - 1\ if\ n \equiv 17\ (mod\ 10)$
5. $\gamma Y_5(G) = 2\left\lceil \frac{n}{4} \right\rceil$
6. $\gamma Y_6(G) = 2,\ if\ n = 3\ or\ n \equiv 2,\ 6\ (mod\ 10)\ or\ \gamma Y_6(G) = 2\left\lceil \frac{n}{5} \right\rceil + i\ if\ n \geq 4\ and\ n \equiv i\ (mod\ 4)$
7. $\gamma Y_1(G) = 2\left\lceil n\frac{n}{4} \right\rceil$

Similarly in year 2005 M. A. Henning and others, determined the X-domination number of the prisms when X is the 2-stratified cycle. We, here too, in the following, state here the theorem without proof.

Theorem 2. For $n \geq 3$, let G be the prism $C_n \square K_2$ then

1. $\gamma X_1(G) = \left\lceil \frac{n}{2} \right\rceil + \left\lceil \frac{n}{4} \right\rceil - \left\lceil \frac{n}{4} \right\rceil$
2. $\gamma X_2(G) = 2n\ unless\ n =$ 4 $in\ which\ case\ \gamma X_2(G) = 2$
3. $\gamma X_3(G) = n$
4. $\gamma X_4(G) = 2\left\lceil \frac{n}{3} \right\rceil$
5. $\gamma X_5(G) = 2\left\lceil \frac{4n}{3} \right\rceil$

2. RESULTS

We describe here in this note that the prism is embedded in the usual Cartesian grid with origin at $(0,0)$.

Lemma 3. For $\boldsymbol{n \geq 3}$, for the Cartesian product $\boldsymbol{C_n \square K_2}$, the domination set $\boldsymbol{D = \{I \times 0\} \cup \{J \times 1\}}$ where $\boldsymbol{I = \{i \mid i \equiv -4\ (mod\ 4),\ 0 \leq i \leq n\}}$ and $\boldsymbol{J = \{j \mid j \equiv -2\ (mod\ 4),\ 0 \leq j \leq n\}}$.

Proof. Let $(i,0) \in D$then the vertices$(i,1), (i+1,0)$ and $(i+n-1,0)$are adjacent to the vertex $(i,0)$and does not belong to the domination set D. The next contendervertex belonging to the domination set D on layer $C_n \square \{0\}$ would be $(i+4,0)$and vertices adjacentto this vertex would be $(i+3,0), (i+5,0)$ and $(i+4,1)$and are not the memberof D. As i runs through 0to n; i.e., $0 \leq i \leq n$, we have $I = \{i \mid i \equiv -4\ (mod\ 4), 0 \leq i \leq n\}$ whenever $(0,0) \in D$.

The corresponding vertices of the form $(j,1) \in D$from the second layer $C_n \square \{1\}$would be $(j+2,1),(j+6,1)$ we$J = \{j \mid j \equiv -2 \pmod 4, 0 \leq j \leq n\}$. Hence we have $(i,0)$ and $(j,1)$ each belonging to the domination set D.

Theorem 4. For $n \geq 3$, if $n \equiv -4 \pmod 4$then the domination number$\gamma\left(C_n \square K_2\right) = \frac{n}{2}$.

Proof. Let the domination set D of the Cartesian product $C_n \square K_2$contain the vertices of the form $(i,0)$and $(j,1)$; where $n \equiv -4 \pmod 4$ and $j \equiv -2 \pmod 4$respectively. It is an obvious fact to confirm that D contains the dominating vertices of the Cartesian product $C_n \square K_2$and $|D| = \frac{n}{2}$.

Now we will show that $C_n \square K_2 \geq \frac{n}{2}$. Let $n$be of the form$4k+t$, that is, $n = 4k+t$,where $k > 0$ and $t \geq 2$. For the minimal dominating set D from lemma above, wehave only two C_n −layers which are not empty and are adjacent. Since no two adjacent layers are empty and hence for empty $s$layers we have

$s \leq 2\left\lfloor \frac{n}{2} \right\rfloor = 4k + \left\lfloor \frac{t}{2} \right\rfloor$

As dominating vertices are contained in each layer so

$|D| = \frac{s + (4k + \left\lfloor \frac{t}{2} \right\rfloor)}{2}$

that is

$|D| = \frac{4k + \left\lfloor \frac{t}{2} \right\rfloor}{2}$

$|D| = \frac{n}{2}$

Whenever $n \equiv -4 \pmod 4$ for $n \geq 3$.

Theorem 5. For$n \geq 3$, if $n \equiv -2 \pmod 4$then the domination number$\gamma\left(C_n \square K_2\right) = \frac{n}{2} + 1$.

Proof. Let the domination set D of the Cartesian product $C_n \square K_2$contain the vertices of the form $(i,0)$and $(j,1)$where $i \equiv -4 \pmod 4$and$j \equiv -2 \pmod 4$respectively. It is an obvious fact to confirm that Dcontains the dominating vertices of the Cartesian product $C_n \square K_2$and $|D| = \frac{n}{2} + 1$.

Now we will show that $C_n \square K_2 \geq \frac{n}{2} + 1$. Let n be of the form$4k+t$, that is, $n = 4k+t$,where $k > 0$ and $t \geq 2$. For the minimal dominating set D from lemmaabove, we have only two C_n −layers which are not empty and are adjacent. Since no. two adjacent layers are empty and hence for empty $s$layers

$s \leq 2\left\lfloor \frac{n}{2} \right\rfloor = 4k + \left\lfloor \frac{t}{2} \right\rfloor + 1$

As dominating vertices are contained in each layer so

$|D| = \frac{s + (4k + \left\lfloor \frac{t}{2} \right\rfloor)}{2} + 1$

that is

$|D| = \frac{4k + \left\lfloor \frac{t}{2} \right\rfloor}{2} + 1$

$|D| = \frac{n}{2} + 1$

Whenever $n \equiv -2 \pmod 4$ for $n \geq 3$.

Theorem 6. For$n \geq 3$, if $n \equiv -1 \pmod 2$ then the domination number $\gamma\left(C_n \square K_2\right) = \frac{n+1}{2}$.

Proof. Let the domination set $D$of the Cartesian product $C_n \square K_2$contain the vertices of the form $(i,0)$and $(j,1)$where$i \equiv -4 \pmod 4$and$j \equiv -2 \pmod 4$respectively. Again; It is an obvious fact to confirm that Dcontains the dominating vertices of the Cartesian product $C_n \square K_2$and $|D| = \frac{n+1}{2}$.

Now we will show that $C_n \square K_2 \geq \frac{n+1}{2}$. Let $n$be of the form $4k+t$, that is, $n = 4k+t$,where $k > 0$ and $t \geq 2$. For the minimal dominating set $D$from lemmaabove, we have only two C_n −layers which are not empty and are adjacent. Since no two adjacent layers are empty and hence for empty $s$layers

$s \leq 2\left\lfloor \frac{n}{2} \right\rfloor = 2k + \left\lfloor \frac{t}{2} \right\rfloor + 1$

As dominating vertices are contained in each layer so

$|D| = \frac{s + (2k + \left\lfloor \frac{t}{2} \right\rfloor + 1)}{2}$

that is

$|D| = \frac{2k + \left\lfloor \frac{t}{2} \right\rfloor + 1}{2}$

$|D| = \frac{n+1}{2}$

Whenever $n \equiv -1 \pmod 2$ for $n \geq 3$.

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