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LP Model for Priority based Water Allocation

A.D. JUMANI, N. KHAN*

Department of Mathematics Shah Abdul Latif University Khairpur 66020, Pakistan

Corresponding author, Ali Dino Jumani, email: alidino.jumani@salu.edu.pk Cell No. 00923138101439

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Abstract: Reservoir storage is necessary to use the highly variable water resources of a river basin for beneficial purposes such as city water supply, industrial water supply, irrigation, and hydroelectric power generation. Dams and other appropriate structures also regulate rivers to reduce damages and wastage of water caused by floods, drains to sea and by inadequate allocation. Water quality, environmental resources, protection and enhancement of fish and wildlife, and other environmental resources are important considerations in managing reservoir (river) system. We present here the linear programming model for three reservoirs for the allocation of scarce water keeping in view the priority factor.

Keywords: Firm yield, Linear programming, Mass balance, Ripple diagram, Simulation,

1. INTRODUCTION

Population and economic growth in various regions of the nation are accompanied by increased needs for water supply, energy and other services. Depleting groundwater reserves are resulting in an increase reliance on surface water in many areas. Concerns have grown in recent years regarding maintenance of river flows and each occurrence of major flood or drought in a region motivates reevaluation of water management practices and each time, like chalk and blackboard, we delete some of the old practices and add some new ones.

With an aging number of dams and reservoirs being operated in an environment of change and constant demands with restricted resources, an operational improvement is imminent and inevitable. Traditionally, Water allocation is regarded as a sub-problem of reservoir system analysis and has been solved through simulation of the reservoir system. Simulation, alone, does not guarantee the optimum solution and consequently linear programming approach is the tool to be used for optimum allocation.

2. MATERIAL AND METHODS

Reservoir Operations

An operating plan (release policy) is a set of rules for determining the quantities of water to be stored and to be released from a reservoir under various conditions. Typically, a regulation plan includes a set of quantity criteria within which significant flexibility exists for operator judgment. The operating rules provide guidance to the water managers who make the actual release decisions. In modeling, the reservoir model contains some mechanism for making release decision within the framework of operating rules. Reservoir operating rules and the operating decisions made within the framework of these rules involve firstly allocating storage capacity between multiple users and type of use secondly minimizing the risks and consequences of water shortages and flooding and lastly optimizing the beneficial use of water, energy and land resources.

Firm Yield analysis

Firm yield is commonly used measure of water supply and hydroelectric power generation capabilities. Firm yield is the estimated maximum release or diversion rate, or hydroelectric energy production rate, which can be estimated continuously during a hypothetical period of hydrology. Firm yields are commonly determined repeatedly executing a simulation model and the Ripple diagram is also an alternative approach to the determination of the firm yield. Linear programming is one of the several alternative approaches for performing the firm yield analysis; and we will be emphasizing the LP approach for the determination of firm yield for the reason that simulation and Ripple diagram does not guarantee the optimum solution in the priority scenario.

*Department of Mathematics University of Karachi, Karachi

LP Model

Optimization and Mathematical Programming are used synonymously to refer to a modeling approach in which a formal algorithm computes values for a set of decision variables that minimizes or maximizes an objective function subject to constraints.

In Optimizing models, the objective function and constraints are represented by mathematical expressions as a function of the decision variables, which typically includes release and storage volumes.

Linear programming consists of finding values for a set of *n* decision variables that maximize or minimize an objective function $x_1, x_2, ..., x_n$ Z of the form:

subject to a set of m $Z = c_1 x_1 + c_2 x_2 + ... + c_n x_n$

constraints of the form:

$$a_{11}x_{11} + a_{12}x_{12} + ... + a_{1n}x_{1n} \le b_1$$
$$a_{21}x_{21} + a_{22}x_{22} + ... + a_{2n}x_{2n} \le b_2$$
$$. \qquad .$$
$$. \qquad .$$
$$. \qquad .$$
$$a_{m1}x_{m1} + a_{m2}x_{m2} + ... + a_{mn}x_{mn} \le b_m$$

$x_j \ge 0 \quad for \quad j = 1,2,...,n$ and a_{ij}, b_i, c_j are constants.

The linear programming model is expressed in more concise form as:

minimize (or maximize)

$$Z = \sum_{j=1}^{n} c_j x_j$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i \qquad for\, i = 1,2,\ldots,m$$

and

$$x_j \ge 0 \quad for \quad j = 1,2,\ldots,n$$

As we brought up earlier that firm yield is the maximum demand that can be met continuously a sequence of reservoir inflows. Now, here in the model, we say that the storage capacity *Z* is required to meet a specified set of water demands x_i representing a firm yield and can be computed with the LP formulation:

Maximize

$$Z$$

Subject to

$$y_t = y_{t-1} + i_t - x_t - r_i \qquad for\, t = 1,2,\ldots,T$$
$$y_t \le Z \qquad for\, t = 1,2,\ldots,T$$
$$y_t, x_t, r_t \ge 0 \qquad for\, t = 1,2,\ldots,T$$

where

Z = reservoir storage capacity

y_t = storage content at the end of period t

i_t = inflow to the reservoir during period t

x_t = demand during period t representing firm yield

r_t = all releases other than x_t during period t

T = number of time periods in the analysis

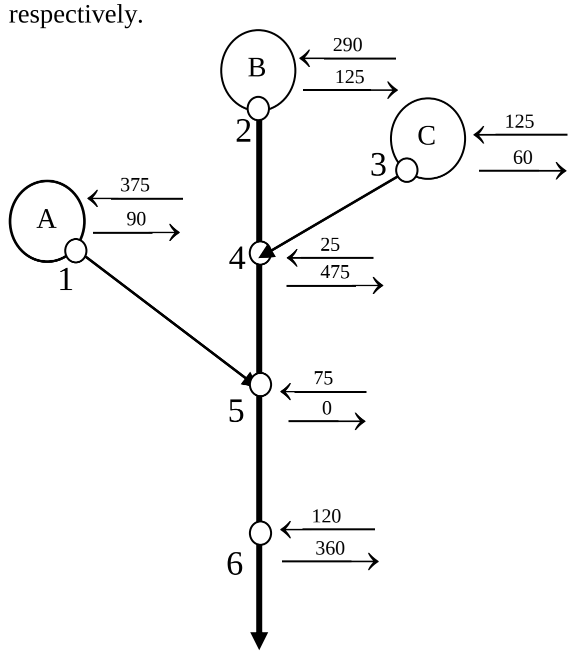
3. RESULTS AND DISCUSSION

Thus the reservoir storage capacity Z is minimized (maximized) subject to constraints, which include the reservoir mass balance and not allowing the storage content to exceed storage capacity, and not allowing the variables to have negative values.

Arrangement of three reservoirs A, B and C in the **(Fig.1)** below located at nodes 1, 2 and 3 have storage capacities of

$$750 \times 10^6 m^3, 900 \times 10^6 m^3 \; and \; 470 \times 10^6 m^3$$

respectively.



The initial storage in reservoirs A, B and C at the beginning of the time interval is $460\times10^{6}m^{3}$, $215\times10^{6}m^{3}$ *and* $230\times10^{10}m^{3}$; demand and supply at each node is indicated with in-arrow and out-arrow labeled with quantity respectively. Certain releases are necessary to maintain and meet the water supply requirement targets and the minimum required flow between the six nodes is:

Nodes	1-5	2-4	3-4	4-5	5-6	Below 6
Flow(10^6m^3)	0	5	10	10	20	30

Literal of Figure-1 is tabulated in **(Table-1)** in which the supply and demand for each node is shown; the total supply is presented in column four and the demand is presented in column five; the allocations are based on the relative priorities tabulated in the last column such as a flow with relative priority 5 at node five has the highest priority; lower priorities are met only to the extent that higher-priority supply are not affected.

Table-1 The total supply is presented in column

Node	Initial Storage (10^6m^3)	Local Inflow (10^6m^3)	Total Supply (10^6m^3)	Demand (10^6m^3)	Relative Priority
1	460	375	835	90	4
2	215	290	505	125	3
3	105	125	230	60	1.5
4	-	25	25	475	2
5	-	75	75	-	-
6	-	120	120	360	5

The decision variables and solution is presented in **(Table-2).** The decision variables include in-stream flows in each of five-river node $x_{1,}$ $x_{2,}$ $x_{3,}$ $x_{4,}$ x_5 and out-stream water supply at four nodes $x_{6,}$ $x_{7,}$ $x_{8,}$ $x_{9;}$ and the ending storage in the three reservoirs $x_{10,}$ $x_{11.}$

Table-2 The decision variables

Decision Variables	Purpose of Decision Variable	Solution Values
x_1	Flow from 1 to 5	330
x_2	Flow from 2 to 4	110
x_3	Flow from 3 to 4	10
x_4	Flow from 4 to 5	415
x_5	Flow from 5 to 6	475
x_6	Flow below node 6	30
x_7	Supply at node 1	90
x_8	Supply at node 2	125
x_9	Supply at node 3	60
x_{10}	Supply at node 4	475
x_{11}	Supply at node 6	305
x_{12}	Reservoir A ending storage	750
x_{13}	Reservoir B ending storage	900
x_{14}	Reservoir C ending storage	0

The total supply at each node consists of reservoir storage at the beginning of the time interval, and the local flow entering the river between the nodes. The model incorporates 14 decision variables defined in Table-2 include the input supply and output at the nodes. The objective function is formulated to reflect relative priorities between water users as:

maximize $Z = 4x_7 + 3x_8 + 1.5x_9 + 2x_{10} + 5x_{11} + x_{12} + x_{13} + x_{14}$

The objective function coefficients are used simply to assign relative priorities to guide the allocation of the limited water resources to the competing users.

Subject to:

$$
\begin{aligned}
& x_1 \geq 0,\ \ x_2 \geq 5, \\
& x_3 \geq 10,\ \ x_4 \geq 10, \\
& x_5 \geq 20,\ \ x_6 \leq 30, \\
& x_7 \leq 90,\ \ x_8 \leq 125 \\
& x_9 \leq 60,\ \ x_{10} \leq 475, \\
& x_{11} \leq 360,\ \ x_{12} \leq 750 \\
& x_{13} \leq 900,\ \ \ x_{14} \leq 470 \\
& x_1 + x_6 + x_{10} = 835 \\
& x_2 + x_7 + x_{11} = 505 \\
& -x_2 + x_3 + x_8 = 25 \\
& -x_1 - x_3 + x_4 = 75 \\
& -x_4 + x_5 + x_9 = 120
\end{aligned}
$$

4. CONCLUSION

A problem can either be solved by dynamic programming, DP, formulation or by LP. Linear programming has the advantage over DP of being more precisely defined and easier to understand. Another reason is the availability of generalized codes is much more limited for DP than for LP.

REFERENCE:

Klemes, V., (1982) *The Essence of Mathematical Model of Reservoir Storage*, Canadian Journal of Civil Engineering, Vol. **(9):** No.4. 32-34.

Liu, C. S., A. C. 1973) Tedrow, *Multi-lake River System Operating Rules*, Journal of the Hydraulics Division, ASCE, Vol. **(94):** No.HY9.

Wagner, H. M. (1975) *Principles of Operations Research*, Englewood Cliffs, NJ Prentice-Hall.

Becu, N., P. Perez, A. Walker, O. Barreteau, C. Le (2003) Page, Agent based simulation of a small catchment water management in northern Thailand: Description of the CATCHSCAPE model, Ecological Modeling, Vol. **(170):** Issues 2–3, 15 December 2003, 319-331.

Letcher, R. A., B. F. W. Croke, A. J. Jakeman (2007) Integrated assessment modeling for water resource allocation and management: A generalized conceptual framework, Environmental Modeling & Software, Vol. **(22):** Issue 5, May 2007, 733-742.

Li, Y.P., G.H. Huang, Y.F. Huang, H.D. Zhou, (2009) A multistage fuzzy-stochastic programming model for supporting sustainable water-resources allocation and management, Environmental Modeling & Software, Vol. **(24):** Issue 7, July 2009, 786-797.

Shangguan, Z., M. Shao, R. Horton, T. Lei, L. Qin, J. Ma, (2002) A model for regional optimal allocation of irrigation water resources under deficit irrigation and its applications, Agricultural Water Management, Vol. **(52):** Issue 2, 3 January 2002, 139-154.