Shortcommunication

Sindh Univ. Res. Jour. (Sci. Ser.) Vol.44 (2) 273-274 (2012)

SINDH UNIVERSITY RESEARCH JOURNAL(SCIENCE SERIES)

A Planar Hypotraceable Graph with 17 Vertices

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Received 17thDecember 2011 and Revised 03rdMarch 2012

Abstract: Purpose of this note is to show that $\overline{P_1^3} \leq 17$. Later an example with just 12 vertices.

Keywords: Traceable path, Hypo-Traceable graphs, longest path.

1. INTRODUCTION

Graph considered in this note is undirected; loopless and without multiple edges. For terminology and notation we will be following (Zamfirescu, 1972). A graph G is traceable if there exists a path which passes through every vertex of G. A path which contains all the vertices of G is called Hamiltonian path of the graph. A cycle which contains all the vertices of G is called Hamiltonian cycle of the graph and the graph is called the Hamiltonian graph. A graph which is not Hamiltonian is called Non-Hamiltonian and a graph which is not traceable is called non-traceable graph.

The hypo-property $\mathcal{P}$ of a graph G is a property that G does not have property $\mathcal{P}$ but $G - v$ has that property. A graph which is non-Hamiltonian but $G - v$ is Hamiltonian for all vertices v is called a hypo-Hamiltonian graph and similarly a graphwhich is non-traceable but $G - v$ is traceable for all vertices v is called a hypo-traceable graph. Petersen's graph is a well known example of a hypo-Hamiltonian graph.

2. MATERIAL AND METHOD

HYPO-TRACEABILITY

Motivation to study hypo-traceable and hypo-Hamiltonian graph comes from the question raised by TiborGallai in 1966 about the existence of a graph with the property that every vertex is missed by some longest path. The answers to the Gallai's question immediately started poured in as early as 1969 first by (Walther,1969) who found a (planar) graph on 25 vertices enjoying Gallai's requirement. (Zamfirescu, 1972).Proposed the generalisation of the question and asked more questions which reappeared.

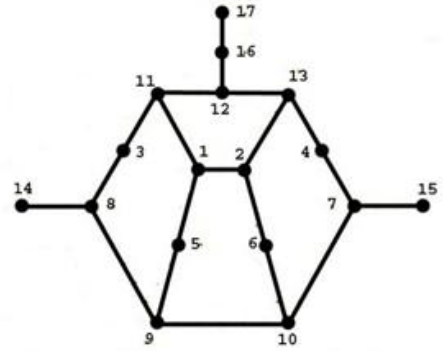

Fig.1-Hypo-Traceable Graph

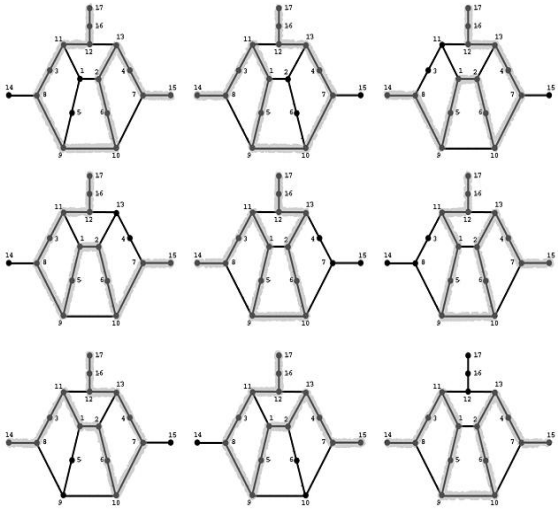

Fig.2- Longest Paths in Hypo-Traceable Graph

3. RESULTS AND DISCUSSION

Let $P_k^j = \infty$, $(\bar{P}_k^j = \infty)$ if there is no $k-$connected graph (planar graph) suchthat each set of j vertices are missed by some longest path. If $P_k^j \neq \infty$, $(\bar{P}_k^j \neq \infty)$, Let $P_k^j, (\bar{P}_k^j)$ denote the minimum number of vertices the $k-$connected graph mayhave, such that each set of $j$vertices are missed by some longest path.

Purpose of this note is to show that $\bar{P}_1^3 \leq 17$ .Later an example with just 12 vertices was found by (Walther1969), independently by (Zamfirescu1972). For planar graphs, the smallest example so far with 17 vertices was found by Schmitz in 1975. The first $2-$ connectedgraph constructed in 1972 by (Zamfirescu1972) had 82 vertices, while the smallest exampleknown today has 26 vertices.

If we impose further restrictions on the graphs under considerations, Gallai's question may get a negative answer. If we ask for example that the graphs be planar and $4-$connected, all vertices lie on every longest path, as these graphs are by a well-known result of Tutte. In this note we ask the graph tobe embeddable into the equilateral triangular lattice $\mathcal{T}$ of the plane. We find outthat, under this strong restriction too, there exist graphs with the property askedby Gallai.

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