Sindh Univ. Res. Jour. (Sci. Ser.) Vol.44 (3) 427-432 (2012)

SINDH UNIVERSITY RESEARCH JOURNAL (SCIENCE SERIES)

A Study On Tolerance Representation, Variation Propagation Analysis And Control In Mechanical Assemblies

T. HUSSAIN, Z. ALI, J. LARIK

Department of Mechanical Engineering, Mehran University of Engineering and Technology, Jamshoro,

Received 21st May 2012 and Revised 16th July 2012

Abstract: Tolerance modelling, assembly variation modelling, statistical tolerance analysis and variation propagation control are the constituent part of the assembly design and these are very important for quality manufacture of mechanical products. Various methods have been developed for tolerance modelling, assembly variation modelling, statistical tolerance analysis and variation propagation control. This paper thus provides a review of the literature in the area of assembly modelling; mainly focusing on the literature related tolerance representation, modelling assembly variation propagation and assembly variation control.

Keywords: Tolerance modelling, Tolerance analysis, Variation propagation control in mechanical assemblies.

1. INTRODUCTION

Variations in parts and assemblies are unavoidable. These variations are the physical result of manufacturing processes. The parts and assemblies that are supposed to be identical, differ from each other in reality. In order to model components variations, one must have clear understanding of Geometric Dimensioning and Tolerancing (GD&T) and tolerance specifications. GD&T is a symbolic language for communicating engineering design specifications. It is used to specify the size, shape, form, orientation, and location of features on a part (Cogorno 2006). Variations in a manufactured component can be divided into two types: dimensional variations and geometric variations. Dimensional variations refer to variation in the size of a particular dimension (such as length, width, diameter etc), whereas, geometric variations refer to the variation in the shape or form of a component feature from its nominal form (Villeneuve *et al.* 2010).

Due to advancement of technology and increase of quality demands in the manufactured products, geometric variations require special attention in the design process of a mechanical assembly. These variations propagate and accumulate as parts are assembled together and the excessive amount of variations in an assembly may cause improper functionality of the product being assembled. Effectively controlling the propagation of variations in the assembly process is one way to help achieve the desired assembly quality (Thimm *et al.* 2001). In high-value low-volume complex assemblies, it is desired to perform assembly analysis to calculate assembly variation propagations before putting parts together in a real assembly. This will save assembly and dis-assembly time and cost, and will also enable the assembly engineer to take effective measures to control assembly variation propagations. Improving assembly quality and reducing the assembly time and cost are the main objectives of a successful manufacturing design process. Therefore, it is essential to have a quantitative design tool for specifying tolerances and estimation of tolerance stack-up in assembly before any control strategy is applied. This paper is aimed to give insight of up-to-date methods for tolerance modelling, tolerance analysis and variation propagation control in mechanical assemblies. This paper is thus divided into four main sections each describing: (1) tolerance modelling, (2) assembly variation propagation modelling, (3) tolerance analysis and (4) variation propagation control.

Initially, tolerance modelling and the current practices in modelling component variations are described in Section 2. Various approaches of assembly modelling are described in Section 3. Statistical methods for analysing assembly variation are referred in Section 4. A review of the current practices in minimising and controlling assembly variation is given in Section 5. Finally, a summary of the whole survey can be found in Section 6.

2. MATERIALS AND METHODS

2.1 Modelling Component Variations/Tolerance Modelling.

It is not practically possible that parts are produced to exact dimensions. This necessitates the designer to describe some allowable variation limits from the nominal conditions. These permissible but undesirable variations from basic dimensions are called tolerances.

Two types of tolerances are described in the dimensioning and tolerancing standards. Traditional plus/minus tolerances (also known as dimensional tolerances) that limit the variation in the size of a part,

++ Corresponding author, Z. ALI, email: engsam68@yahoo.com Ph: +92 (0) 22 277 1382

and geometric tolerances that control form, orientation, location and run-out of a feature (Juster 1992). In order to specify geometric variations in a component, the component is considered to be composed of features (geometric elements) such as planes, lines, etc. (Henzold 2006, Mäntylä *et al.* 1996).

Modern geometric tolerances provide the foundation for creating mathematical models that are essential for the development of computational tools. The technique of mathematically representing component variations is also known as tolerance modelling or modelling component variations (Hu *et al.* 2005, Koura *et al.* 2002). Work in the area of mathematically describing part variations was initially done by Sutherland (1963). Sutherland used constraints as a design aid in the creation of a part by defining constraints between the coordinates of a part to the error introduced into the system. He did not use constraints to define the part geometry. The work in the area of describing part variations mathematically can be classified into the following categories: (1) offset method; (2) parametric space method; (3) algebraic methods; (4) vectorial tolerancing method; and, (5) homogeneous transformation matrix method.

2.1 Offset Method

The offset model proposed by Requicha (1980) is one of the earliest methods. Requicha introduced a mathematical technique for representation of part variations based on the tolerance zone approach. An example of tolerance zone for geometric tolerance specification is illustrated in (Pasupathy *et al.* 2003). **Fig. 1 (a)** illustrates a part with geometric tolerance specification. The shaded area in **Fig. 1 (b)** indicates the corresponding tolerance zone, as per specifications, within which the manufactured part should lie.

Etesami (1988) by using a very similar approach to that of Requicha based on the concept of offset solids, proposed another theory for modelling geometric tolerances. Etesami suggested in his theory that the manufactured part model is composed of a boundary model for part surfaces, a model for axes and curves, and a datum model.

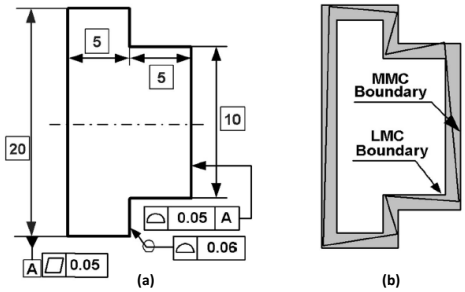


Fig. 1 (a) A part with geometric tolerance callouts, (b) An instance of the actual part that is acceptable as per specification provided in (a) (Pasupathy, Morse and Wilhelm 2003).

Srinivasan and Jayaraman (1989) state that Requicha theory is inadequate for describing tolerance zone limits. They refined Requicha theory and proposed a tolerancing framework based on the virtual boundary requirement (VBR). The virtual boundary serves as the maximum material envelope for two surfaces. One of its surface lie completely inside the boundary and the other surface lie completely outside the virtual boundary In virtual boundary, the individual tolerances of each component are derived from these virtual boundary requirements. Due to this reason the virtual boundary requirement is not an adequate representation of the part functional requirement, which require separate and individual tolerance specifications.

2.2 Parametric Space Model

The parameter space is one of the first tolerance modelling concepts to evaluate tolerances on geometric models. (Hillyard and Braid 1978) proposed a general concept that uses geometric constraints between part coordinates and the variational surface as specified by the tolerances. (Lin *et al*. 1981) extended and refined the above constraint-based variational-geometry method to improve the computational efficiency.

An example taken from (Pasupathy, Morse and Wilhelm 2003) is presented in Fig. 2 to describe the concept of parametric approach of tolerance modelling. **Fig. 2 (a)** shows a 2D part with variance feature that is free to orient within the tolerance zone, and **Fig. 2 (b)** shows the extent of feature variation.

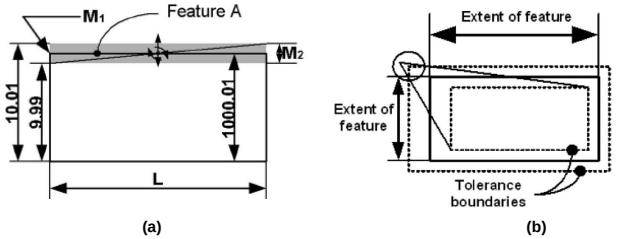


Fig. 2: (a) Variability of feature A, (b) Errors in worst cases in parametric space models(Pasupathy, Morse and Wilhelm 2003).

In Fig. 2 (a), model variables are introduced to parameterise the possible variations of the vertices of the edges in terms of position or orientation. In the example, model variables and represent size and orientation deviations, respectively. The model variations are defined as:

$$-0.1 \le M_1 \le 0.1\,, \qquad (1)$$

$$\text{and, } -0.2 \le M_2 \le 0.2\,, \text{ respectively.} \qquad (2)$$

2.3 Algebraic Methods

In order to overcome the limitations of parametric models, Inui *et al.* (1991) proposed a method of interpreting tolerances algebraically. Here the variational model for tolerances is developed using algebraic constraints. Mullins and Anderson (1995) later

proposed a method that interprets the geometric tolerances into a form of algebraic relations. Another development in algebraic method was the research by Roy and Li (1999, 1998). They proposed a similar approach and considered the resultant zones from multiple controls, such as size and form tolerances, on a single feature. Their work realises the construction of resultant zones as proposed by Fleming (1988).

To demonstrate the concept of tolerance representation based on algebraic method, an example of 2D part (taken from (Pasupathy, Morse and Wilhelm 2003)) is shown in Fig. 3. The 2D part shown in Fig. 3 is similar to one given in Fig. 3. The feature of the 2D part in **Fig. 3 (a)** has a simultaneous size and form tolerance specification. The tolerance zone is obtained by offsetting the nominal feature corresponding to specification as shown in Fig. 3 (b). An extreme instance of the resultant variation due to simultaneous effect of size and form tolerance is shown in **Fig. 3 (b).**

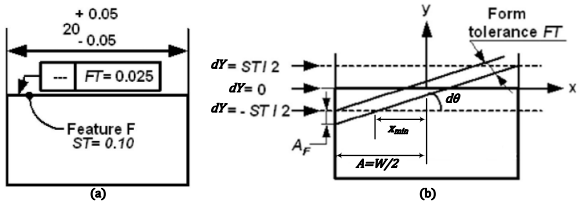


Fig. 3: (a) Feature with simultaneous tolerances, (b) An extreme instance of the resultant zones specified for feature F in (a) (Pasupathy, Morse and Wilhelm 2003).

Here, the variational model for tolerances is developed using algebraic constraints. The algebraic constraints for variables and are defined by the inequalities:

$$-d\theta_{\max} \le d\theta \le d\theta_{\max}, \quad (3)$$

$$-(ST/2) - A_F \le dY \le ST/2, \quad (4)$$

The calculated extreme variations about the extent of a feature are neither accurate nor conservative, because the intersections extend beyond the boundary specified by the design intent.

A form tolerance zone boundary and the extent of feature variation defined as above, for the studies conducted in the domain of parametric and algebraic models, contradict with the definition given in current design standards (ASME 1994, BSI 2005). According to these design standards, the extent of feature variation always lies within the tolerance zone limits and the boundary of form tolerance zone is obtained by offsetting the nominal feature.

2.4 Vectorial Tolerancing Method

Vectorial tolerancing is an alternative way for representing component tolerances. Vectorial tolerancing represents a chain of dimensions and tolerances as a link of vectors. Fortini (1967) initially introduced the vector tolerancing concept. A comprehensive study of vector tolerancing applied to 2D assemblies was further extended by (Bjorke 1978). The major development in vectorial method was done by (Wirtz 1993). An overall perspective of the application of vectorial tolerancing is described in (Wirtz 1993), deviations between the nominal part and the actual part are represented by using the vectors characterising their relative orientation and location. Thus, the deviation vector between these vectors represents the tolerance.

Vector models have been utilised extensively for both 2D and 3D assemblies by a group of researchers in Brigham Young University Gao *et al.* 1998). Their work showed that vector tolerancing methods can be applied to define individual parts composed of different surfaces. Based on vectorial representation of part variations, they developed a vector loop method to calculate assembly variation propagations.

2.5 Homogenous Transformation Matrix Method

The homogeneous transforms representation of part variations is a combined concept of the parameter space and the offset solid modelling approach (Roy *et al..*, 1991, Whitney *et al.* 1994). The work of representing part tolerances using homogeneous transforms was initially seen in the thesis of Bernstein (1989). The inspiration of representing part variations using transform matrices came from the field of robot manipulators (Jastrzebski 1991) and Whitney *et al.* 1994) represented part tolerances using matrix transforms to model part variations in a form compatible with assembly tolerance analysis model. In (Whitney, Gilbert and Jastrzebski 1994), the variations with respect to nominal dimensions are expressed by a transformation matrix obtained by the first order perturbation analysis of rotation and translation matrices.

In (Desrochers and Rivière 1997) elementary surfaces are introduced and are called topologically and technologically related surfaces (TTRS). A homogeneous matrix transform represents six degrees of freedom (three translational and three rotational degrees of freedom of error) of variation of a toleranced feature as:

$$\mathbf{DT} = \begin{bmatrix} 1 & -d\theta_z & d\theta_y & dX \\ d\theta_z & 1 & -d\theta_x & dY \\ -d\theta_y & d\theta_x & 1 & dZ \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (5)$$

The method of representing part variation using homogeneous transform is also called as degrees of freedom (DOF) representation method. In this

method, a small variation torsor is defined as a 6D vector which contains three translational and three rotational values as under:

$$\tau = \left[dX, dY, dZ, d\theta_X, d\theta_Y, d\theta_Z \right]^T . \quad (6)$$

In all above cited literature, very little attention is given towards modelling variation (such as run-out variations) for axi-symmetric components, and no work has been found that describes a variational model for axi-symmetric components capable of expressing component variation along degrees of freedom of component variation.

3. Assembly Variation Propagation Modelling

A variation propagation model is an analytical model for calculating the accumulation of tolerances in a mechanical assembly during individual assembly stages and is the basic tool for statistical tolerance analysis. Several models have been proposed to represent the variation propagation in assembly processes. The models developed in broad spectrum can be grouped into two categories, depending on whether the model considers the assembly of rigid parts or flexible/compliant parts. Compliant refers to non-rigid parts like sheet metals used in the assembly of automotive body parts.

In the early stage of variation analysis research Veitschegger and Wu (1986) performed calculations for the prediction of robot uncertainty. In their work they used the common method of matrix transform (Paul 1981) for the representation of relative locations of robot joints.

In later research, (Jastrzebski 1991) generalised the derivations of Veitschegger and Wu and applied those calculations to the case of assembly modelling. In the work (Veitschegger and Wu 1986), position and orientation are described by five degrees of freedom, whereas, Jastrzebski considered six degrees of freedom to fully define the transformation in space. Jastrzebski provided the basic calculation for variation propagation in assemblies without looking into any type of tolerance. (Gilbert 1992 and Whitney *et al.* 1994) extended the work of (Jastrzebski 1991) and included geometric feature variation using matrix transform for assemblies compatible with tolerance analysis based on a closed form algorithm.

Chase *et al.* (1995, 1998) developed a vector loop-based assembly model for the analysis of tolerance stack-up in an assembly. They used vector assembly models based on 2D and 3D vector loops which represented the dimensional chains. This method allowed an assembly to be described in terms of vector chains added tip-to-tail, with kinematic joints inserted to represent the degrees of freedom between mating parts. Small dimensional variations were applied to the vectors, to calculate error accumulation.

Mantripragada and Whitney (1999) used state transition models to predict variation propagations in mechanical assemblies. In state transition model, the propagation of variations is treated as the propagation of errors during transformations of Cartesian frames attached to mating features on every part. Mathematically, they described the state of an assembly at any assembly station by six degrees of freedom vector as follows:

$$\tilde{\mathbf{X}}(k) = \begin{bmatrix} \mathbf{dp}_k \\ \mathbf{dR}_k \end{bmatrix} = \begin{bmatrix} dp_k^X \\ dp_k^Y \\ dp_k^Z \\ d\theta_k^X \\ d\theta_k^Y \\ d\theta_k^Z \end{bmatrix}. \quad (7)$$

The state transition form given by (Mantripragada and Whitney 1999) for rigid assemblies is as follows:

$$\tilde{\mathbf{X}}(k+1) = \mathbf{A}(k)\tilde{\mathbf{X}}(k) + \mathbf{F}(k)\tilde{\mathbf{w}}(k), \quad (8)$$

Mantripragada and Whitney (1999) also employed a state transition model to describe the propagation of variation in assembly of flexible parts and they refer to the flexible assemblies as Type-2 assemblies. In their work, the fixture is assumed to be perfect, implying they actually modelled the variation accumulation in assembly caused by part-fabrication imperfection. The work of Mantripragada and Whitney determines the in-process adjustments in Type-2 assemblies by the type of interface features between parts being assembled which are modelled as control inputs to the dynamic system.

Zha *et al.* (2001) proposed a knowledge-based expert system to support top-down design for assembled products. Their research involves the integration of artificial intelligence with the designer intelligence with the aim to increase productivity with improved quality. However, artificial intelligence in the manufacturing process industries requires advances equipment for the assembly process which may result in considerably increased capital cost of the industry.

Zhou *et al.* (2003) refined the state transition model equation given by (Mantripragada and Whitney 1999) and introduced a new term of un-modelled system noise

The state-transition model equation given by Zhou *et al*. is given as:

$$\tilde{\mathbf{X}}(k+1) = \mathbf{A}(k)\tilde{\mathbf{X}}(k) + \mathbf{F}(k)\tilde{\mathbf{w}}(k) + \mathbf{U}(k). \quad (9)$$

Most of above mentioned literature use only linear model to calculate assembly variation propagation out of which the recent linear approach the exact solutions is simple in calculation compared to all others.

4. RESULTS AND DISCUSSION

Tolerance Analysis

Tolerance analysis is a method of predicting and analysing assembly variation due to tolerance of individual components and assembly operations. Tolerance analysis is carried out when the tolerances of individual parts are known and the designer intends to find out or allocate the dimensions for assembly. This involves: first, gathering data on the individual component variations; second, creating an assembly model to identify which dimensions contribute to the final assembly dimensions; third, applying the manufactured component variations to the model to predict the variations in assembly dimension (Chase *et al.* 1995). Tolerance analysis verifies the proper functionality of a design while considering variations associated with the individual parts in an assembly process. Tolerance analysis can be based on either deterministic method or statistical method for the analysis of one-, two-, or three-dimensional design models (Hong *et al.* 2002). Deterministic tolerance analysis can also be termed as variation propagation modelling, which is carried out for existing assemblies. However, statistical tolerance analysis uses suitable statistical methods for predicting assembly variability or tolerance.

6. Summary and Perceived Knowledge Gaps

In this paper, the literature has been reviewed in the area of modelling component variation, modelling assembly variation propagation, and statistical tolerance analysis. In last five years a handsome amount of research has carried out in the field of tolerance modelling and tolerance analysis. During this time most of the issues needing improvement earlier have been discussed and new modelling strategies have been discussed, especially for modelling geometric variations in components For modelling component variation, homogeneous transform matrix (HTM) method is a robust method and is capable to capture the component variations along the three degrees of translation errors and the three degrees of rotation errors. HTM method is also compatible with most of the assembly variation propagation modelling approaches reviewed in the literature.

Tolerance analysis is performed in order to predict the effects of manufacturing variations present in each assembly component on final assembly variation. Various tolerance analysis methods have been reviewed in this paper, and in the current year, a robust tolerance analysis model have been produced that calculate exact solutions by (Hussain 2012) and is capable to incorporate for all sources of assembly variation propagation.

In the field of variation propagation control in mechanical assemblies, relatively less work have been found. In the current year, only the research group in the University of Nottingham UK have carried a good deal of research in controlling assembly variation propagations in the assembly of rotating machines. They proposed a relative orientation method of minimising variation propagation in rotor assembly containing axi-symmetric components. One of the member of the research group in the University of Nottingham have proposed another variation propagation control method for non-rotating structure (casing) of high speed rotating machines. This method used optimum mating sequence using combinatorial approach.

REFERENCES:

Bernstein, N. X. (1989) Representation and Analysis of Dimension and Tolerance Information in Solid Modeling. MSc Thesis, University of the Negev, Beer Shava, Israel.

Bjorke, O., (1978) Computer-Aided Tolerancing. Tapir Publishers, Norway.

BSI, (2005) European Standard BS EN ISO 1101:2005, Geometric Product Specifications (GPS)-Geometrical Tolerance: Tolerances of Form, Orientation, Location and Run-out, London, UK.

Chase, K. W., J. Gao, and S. P. Magleby (1995) General 2-D Tolerance Analysis of Mechanical Assemblies with Small Kinematic Adjustments. Journal of Design and Manufacture, (**5**): 263-274.

Chase, K. W., J. Gao, S. P. Magleby, and C. D. Sorensen (1996) Including Geometric Feature Variations in Tolerance Analysis of Mechanical Assemblies. IIE Transactions, (**28**): 795-807.

Cogorno, G., (2006) Geometric Dimensioning and Tolerancing for Mechanical Design. McGraw-Hill Professional Publishing, Blacklick, OH, USA.

Denavit, J., and R. S. Hartenberg (1955) A Kinematic Notation for Lower Pair Mechanisms Based on Matrices. Journal of Applied Mechanics, (**22**): 215-221.

Desrochers, A., and A. Rivière (1997) A Matrix Approach to the Representation of Tolerance Zones and Clearances. The International Journal of Advanced Manufacturing Technology, (**13**): 630-636.

Etesami, F. (1988) Tolerance Verification through Manufactured Part Modeling Journal of Manufacturing Systems, (**7**): 223-232.

Fleming, A. (1988) Geometric Relationships Between Toleranced Features. Artificial Intelligence, (**37**): 403-412.

Fortini, E., (1967) Dimensioning for Interchangeable Manufacture. Industrial Press Inc., New York.

Gao, J., K. W. Chase, and S. P. Magleby (1998) Generalized 3D Tolerance Analysis of Mechanical Assemblies with Small Kinematic Adjustments. IIE Transactions, (**30**): 367-377.

Gilbert, O. L. (1992) Representation of Geometric Variations using Matrix Transforms for Statistical Tolerance Analysis in Assemblies. MSc Thesis, Massachusetts Institute of Technology, Cambridge, Massachusetts, USA.

Henzold, G., (2006) Geometrical Dimensioning and Tolerancing for Design, Manufacturing and Inspection. Elsevier, Oxford, UK.

Hong, Y. S., and T. C. Chang (2002) A Comprehensive Review of Tolerancing Research. International Journal of Production Research, (**40**): 2425-2459.

Hu, J., and G. Xiong (2005) Dimensional and Geometric Tolerance Design Based on Constraints. International Journal of Advanced Manufacturing Technology, (**26**): 1099-1108.

Hussain, T., Z. Yang, A. A. Popov, and S. McWilliam (2011) Straight-Build Assembly Optimization: A Method to Minimize Stage-by-Stage Eccentricity Error in the Assembly of Axisymmetric Rigid Components (Two-Dimensional Case Study). ASME Journal of Manufacturing Science and Engineering, (**133**): 031014.

Inui, M., and F. Kimura, (1991) Algebraic Reasoning of Position Uncertainties of Parts in an Assembly. ACM, Proceedings of the first ACM symposium on Solid modeling foundations and CAD/CAM applications, Austin, Texas, United States: 419-428.

Jastrzebski, M. J. (1991) Software for Analysis of 3-D Statistical Tolerance Propagation in Assemblies using Closed-Form Matrix Transform. MSc Thesis, Massachusetts Institute of Technology, Cambridge, Massachusetts, USA.

Juster, N. P. (1992) Modelling and Representation of Dimensions and Tolerances: A Survey. Computer-Aided Design, (**24**): 3-17.

Li, B., and U. Roy (2001) Relative Positioning of Toleranced Polyhedral Parts in an Assembly. IIE Transactions, (**33**): 323-336.

Lin, V. C., D. C. Gossard, and R. A. Light (1981) Variational Geometry in Computer-Aided Design. Computer Graphics, (**15**): 171-177.

Mantripragada, R., T.W.Cunningham,and D.E. Whitney, (1997) Assembly Oriented Design: A New Approach to Designing Assemblies, Proceedings of the Fifth IFIP TC5/WG5.2 International Workshop on Geometric Modeling in Computer-Aided Design, USA: 308-324.

Mantripragada, R and D. E. Whitney (1999) Modeling and Controlling Variation Propagation in Mechanical Assemblies using State Transition Model. IEEE Transactions on Robotics Automation, (**15**): 124-140.

Mäntylä, M., D. Nau, and J. Shah (1996) Challenges in Feature-Based Manufacturing Research. Communications of the ACM, (**39**): 77-85.

Mullins, S. H., and D. C. Anderson, (1995) Algebraic Representation of Geometric and Size Tolerances. 21st ASME Design Automation Conference, (**82**): 309-316.

Pasupathy, T. M. K., E. P. Morse, and R. G. Wilhelm (2003) A Survey of Mathematical Methods for the Construction of Geometric Tolerance Zones. Journal of Computing and Information Sci .in Eng. (**3**): 64-75.

Requicha, A. G. (1980) Representations for Rigid Solids: Theory, Methods, and Systems. ACM Comput. Surv., (**12**): 437-464.

Roy, U., C. R. Liu, and T. C. Woo (1991) Review of dimensioning and tolerancing: representation and processing. Computer-Aided Design, (**23**): 466-483.

Srinivasan, V., and R. Jayaraman (1989) Geometric Tolerancing: 2. Conditional Tolerances. IBM Journal of Research and Development, (**33**): 105-124.

Sutherland, I. E., (1963) Sketchpad: A Man-Machine Graphical Communication System. Technical Report No. MIT Lincoln Laboratory Technical Report No. 296, Lexington.

Thimm, G., G. A. Britton, and F. S. Cheong (2001) Controlling Tolerence Stacks for Efficient Manufacturing. The International Journal of Advanced Manufacturing Technology, (**18**): 44-48.

Veitschegger, W. K., and C. H. Wu (1986) Robot Accuracy Analysis Based on Kinematics. IEEE Journal of Robotics and Automation, (**2**): 171-179.

Villeneuve, F., and L. Mathieu, (2010) Geometric Tolerancing of Products. John Wiley & Sons Inc,

Whitney, D. E., O. L. Gilbert, and M. Jastrzebski (1994) Representation of Geometric Variations using Matrix Transforms for Statistical Tolerance Analysis in Assemblies. Research in Engineering Design, (**6**): 191-210.

Wirtz, A., (1993) Vectorial Tolerancing A Basic Element for Quality Control, Proceedings of 3rd CIRP Seminars on "Computer Aided Tolerancing", Ecole Normale Superieure of Cachan, Cedex, France: 115-128.

Zha, X. F., H. J. Du, and J. H. Qiu (2001) Knowledge-Based Approach and System for Assembly-Oriented Design, Part II: the System Implementation. Engineering Applications of Artificial Intelligence, (**14**): 239-254.

Zhou, S., Q. Huang, and J. Shi (2003) State Space Modeling of Dimensional Variation Propagation in Multistage Machining Process using Differential Motion Vector. IEEE Transactions on Robotics and Automation, (**19**): 296-309.