



A Probabilistic Tolerance Analysis for Mechanical Assembly of Rotating Machines

T. HUSSAIN, Z. ALI, J. DAUDPOTO, J.H.KHALIQDINA, I.A. MEMON*

Department of Mechanical, Mehran University of Engineering and Technology, Jamshoro, Sindh, Pakistan

Received 02th June 2012 and Revised 05th September 2012

Abstract: Due to inherent nature of manufacturing variability, the actual dimensions of manufactured components lie anywhere in the range of permissible variation randomly. The random manner by which the dimensional inaccuracies fall produce a pattern in which the variations are distributed within their tolerance zones. Mathematical modelling methods are presented and used to predict the statistical distributions of variations present in the complete assembly. This paper considers assemblies composed of axi-symmetric components and proposes a method to control assembly variation propagations using relative orientation technique. A relative orientation technique is a method in which individual components are re-orientated on a stage-by-stage basis to minimise the eccentricity from table-axis during the assembly of an engine rotor. Two straight-build assembly models are considered: i) Direct Build; and ii) Best Build for assemblies consisting of axi-symmetric components. Analytical expressions are determined for the probability density function (pdf) of the table-axis error for the complete assembly, and numerical comparisons are made with Monte Carlo simulation for the purposes of validation.

Keywords: Straight-build assembly; Tolerance analysis; Monte Carlo simulation; Variation propagation control; Probabilistic approach

1.

INTRODUCTION

Random variations from the nominal geometry always exist in manufactured components due to imperfections in manufacturing processes. As components are assembled, these variations propagate and accumulate, and can quickly drive assembly dimensions out of specification (Mantripragada and Whitney, 1999).

Traditional methods for studying assembly tolerance stack up are usually based on engineering experience, worst on worst tolerance analysis (WOW) method (Dantan and Qureshi, 2009, Spotts, 1971, Arron *et al*, 1975), or the root-sum-square tolerance analysis (RSS) method (Lin *et al*, 1997, He, 1991, Nigam and Turner, 1995). The WOW method often gives results that are overly pessimistic, while the RSS method only gives results for the mean square variation. These methods are not suitable for the analysis of geometric tolerances, and are very difficult to use in practice (Thimm *et al*, 2001, Wu *et al*, 2009).

The simplest and most popular method for statistical tolerance analysis is the Monte Carlo simulation (MCS) method. The main drawback of this method is that it is necessary to generate very large samples to achieve accurate predictions (Toft *et al*, 2007, Brooks and Gelman, 1998, Mustafa, 2007, Jungemann *et al*, 1997), which may require intensive computation.

This paper considers the special case of mechanical assemblies composed of axi-symmetric

components (Roy and Li, 1998, 1999, Cai *et al*, 2004, Whitney, 2004). These assemblies are used in aero-engines, and possess the special property that the assembly can be changed by rotating components relative to each other about the central axis of symmetry. Connective Assembly Models are used to describe the assembly process mathematically and a probabilistic approach is used to predict the probability that the eccentricity of the complete assembly is within required threshold values. The aim is to predict the probability density function (pdf) for the eccentricity of the complete assembly, avoiding the need to use Monte Carlo simulation.

Three different straight build strategies are considered: i) Direct Build; ii) Best Build. Direct Build Assembly (DBA) corresponds to the standard straight build process in which the axi-symmetric components are assembled without giving any consideration to controlling the eccentricity of the build. Best Build Assembly (BBA) takes advantage of the axi-symmetric property of the components and aims to minimise the eccentricity of the build. This is achieved by rotating the component being added at each stage about its nominal axis of symmetry.

2.

MATERIAL AND METHODS

Modelling straight-build for an axi-symmetric assembly

Straight-build assembly is the process of assembling axi-symmetric rigid components so as to achieve the assembly structure as straight as possible. In

++ Corresponding author, Z. ALI, email: engsam68@yahoo.com Ph: +92 (0) 22 277 1382

*Institute of Physics, University of Sindh, Jamshoro, Pakistan

this work, the eccentricity of the complete assembly is used as a measure of the quality of the build.

Connective model is presented and applied to nominally axi-symmetric components and assemblies. It is shown that connective model can be linearised for the case when the components are axi-symmetric and the dimensional variations are small.

2.1. Connective models for straight-build assembly

Connective assembly models (Whitney, 2004) are usually used to quantify the propagation of component variations through the assembly. In these models matrix transformations are used to relate the location and orientation of different features on one component to another component, and for this purpose coordinate frames are attached to each of the mating features on each component. The transformation matrix represents the operations of translation and rotation acting on a coordinate frame (Whitney, 2004):

$$\mathbf{T} = \begin{bmatrix} \mathbf{R} & \mathbf{p} \\ \mathbf{0}^T & 1 \end{bmatrix} \quad (1)$$

where $\mathbf{R}$ is a 3×3 rotational matrix indicating the orientation of one frame relative to other, $\mathbf{p}$ is a 3×1 displacement vector indicating the position of one frame relative to other, and superscript T indicates a vector or matrix transpose.

For a single component, $\mathbf{p}$ defines the coordinates of the origin of the mating feature on the upper surface of the component relative to the origin of the reference frame located at the mating feature on the lower surface of the component, while $\mathbf{R}$ represents the relative orientation of the upper surface of the component relative to the lower surface.

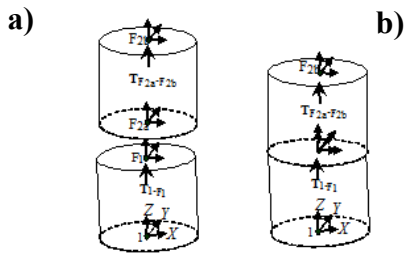


Fig. 1. An example two-component assembly: a) unassembled, b) assembled.

The assembly model assumes the components are assembled by joining mating features to each other (Whitney, 2004) and transformation matrices are used to relate the location of the mating features. The assembly can be described by a chain of transforms. For the two component axi-symmetric assembly shown in Figure 1b), the transform matrix $\mathbf{T}_{1-2}^N$ relating the location and orientation of the coordinate frame at the top of part 2 to

coordinate frame at the base of part 1 can be expressed as follows:

$$\mathbf{T}_{1-2}^N = \mathbf{T}_{1-F_1}^N \mathbf{T}_{F_1-F_{2a}}^N \mathbf{T}_{F_{2a}-F_{2b}}^N, \quad (2)$$

where $\mathbf{T}_{1-F_1}^N$, $\mathbf{T}_{F_1-F_{2a}}^N$ and $\mathbf{T}_{F_{2a}-F_{2b}}^N$ are transform matrices relating the location and orientation of the coordinate frame at mating feature F_1 to the coordinate frame at the base of Part 1, the coordinate frame at mating feature F_{2a} to the coordinate frame at mating feature F_1 , and from the coordinate frame at mating feature F_{2b} to the coordinate frame at mating feature F_{2a} on Part 2, respectively.

The superscript N indicates that the transform matrices relate to the nominal geometry of the component and there is perfect alignment between mating features. Using these definitions in the terms on the right hand side of equation (2), it can be shown that:

$$\mathbf{T}_{1-F_1}^N = \begin{bmatrix} \mathbf{I} & \mathbf{p}_1^N \\ \mathbf{0}^T & 1 \end{bmatrix}, \quad \mathbf{T}_{F_{2a}-F_{2b}}^N = \begin{bmatrix} \mathbf{I} & \mathbf{p}_2^N \\ \mathbf{0}^T & 1 \end{bmatrix},$$

$$\mathbf{T}_{F_1-F_{2a}}^N = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix}, \quad \mathbf{p}_i^N = \begin{bmatrix} 0 \\ 0 \\ Z_i \end{bmatrix}$$

where Z_i is the height of the i 'th component and $i=1,2$. Using these equations, in equation (2), transform matrix for the assembly of two components can be written as:

$$\mathbf{T}_{1-2}^N = \begin{bmatrix} \mathbf{I} & \mathbf{p}_1^N + \mathbf{p}_2^N \\ \mathbf{0}^T & 1 \end{bmatrix}. \quad (3)$$

In a similar way, the transform matrix for an assembly consisting of n axi-symmetric components can be expressed as:

$$\mathbf{T}_{1-n}^N = \begin{bmatrix} \mathbf{I} & \sum_{i=1}^n \mathbf{p}_i^N \\ \mathbf{0}^T & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \sum_{i=1}^n Z_i \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (4)$$

Where $\mathbf{T}_{1-n}^N$ is the transform matrix relating the location and orientation of the coordinate frame at the top of Part n to the coordinate frame at the base of Part 1.

If a feature on a part is not placed at its nominal design position, the relative rotation and location errors need to be included in the definition of the relevant matrix transforms. This is achieved by introducing a matrix transform $\mathbf{D}$ having the same form as equation (1) for each matrix transform between two mating features on the same part, that takes account of the error only. Applying this procedure to an assembly

consisting of n nominally axis-symmetric components, the transform matrix T_{1-n} relating the location and

orientation of the coordinate frame at the top of Part n to the coordinate frame at the base of Part 1 gives:

$$T_{1-n} = T_{1-F_1} D_1 T_{F_1'-F_{2a}} T_{F_{2a}-F_{2b}} D_2 T_{F_{2b}'-F_{3a}} T_{F_{3a}-F_{3b}} D_3 \cdots T_{F_{(n-1)b}'-F_{na}} T_{F_{na}-F_{nb}} D_n \quad (5)$$

where

$$T_{F_{ia}-F_{ib}} = \begin{bmatrix} \mathbf{I} & \mathbf{p}_i^N \\ \mathbf{0}^T & 1 \end{bmatrix}, \quad D_i = \begin{bmatrix} d\mathbf{R}_i & d\mathbf{p}_i \\ \mathbf{0}^T & 1 \end{bmatrix}, \quad d\mathbf{p}_i = \begin{bmatrix} dX_i \\ dY_i \\ dZ_i \end{bmatrix}, \quad T_{F_1'-F_{2a}} = \begin{bmatrix} \mathbf{R}_2 & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix}, \quad T_{F_{(i-1)b}'-F_{ia}} = \begin{bmatrix} \mathbf{R}_i & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix},$$

$$\mathbf{R}_i = \begin{bmatrix} \cos(\theta_{Zi}) & -\sin(\theta_{Zi}) & 0 \\ \sin(\theta_{Zi}) & \cos(\theta_{Zi}) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Here $\mathbf{R}_i$ is a rotation matrix.

$$T_{1-n} = \begin{bmatrix} \prod_{i=1}^n (\mathbf{R}_i d\mathbf{R}_i) & \sum_{i=1}^n \left(\prod_{j=2}^i \mathbf{R}_{j-1} d\mathbf{R}_{j-1} \right) \mathbf{R}_i (\mathbf{p}_i^N + d\mathbf{p}_i) \\ \mathbf{0}^T & 1 \end{bmatrix} \quad (6)$$

Comparing Eqs. (2) and (6), the variation propagation in the x-, y-, and z-directions ($dpx(n)$, $dpy(n)$, $dpz(n)$) can be calculated as follows.

$$\begin{bmatrix} dpx(n) \\ dpy(n) \\ dpz(n) \end{bmatrix} = \sum_{i=1}^n \left(\prod_{j=2}^i \mathbf{R}_{j-1} d\mathbf{R}_{j-1} \right) \mathbf{R}_i (\mathbf{p}_i^N + d\mathbf{p}_i) - \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^n Z_i \end{bmatrix} \quad (7)$$

where the table-axis error is:

$$w(n) = \sqrt{dpx(n)^2 + dpy(n)^2} \quad (8)$$

2.2. Linearized straight-build assembly models

In practical assembly, the orientation imperfections are small (so that $\cos(d\theta_{Xi}) \cong 1$, $\cos(d\theta_{Yi}) \cong 1$, $\cos(d\theta_{Zi}) \cong 1$, $\sin(d\theta_{Xi}) \cong d\theta_{Xi}$, $\sin(d\theta_{Yi}) \cong d\theta_{Yi}$, $\sin(d\theta_{Zi}) \cong d\theta_{Zi}$). The models can be given as follows for the different assembly procedures.

2.2.1. Direct Build Assembly

For the Direct Build Assembly, the components are not rotated; it means that the mating feature frames between two components have the same position and orientation, i.e. $\theta_z(i) = 0$. The variation matrix can be rewritten as:

$$D_i \cong \begin{bmatrix} \mathbf{I} + \delta\mathbf{R}_i & d\mathbf{p}_i \\ \mathbf{0}^T & 1 \end{bmatrix} \quad (9)$$

Where

$$\delta\mathbf{R}_i = \begin{bmatrix} 0 & -d\theta_{Zi} & d\theta_{Yi} \\ d\theta_{Zi} & 0 & -d\theta_{Xi} \\ -d\theta_{Yi} & d\theta_{Xi} & 0 \end{bmatrix}$$

Eq. (6) can be approximately rewritten as:

$$T_{1-n}^{approx} = \begin{bmatrix} \prod_{i=1}^n (\mathbf{I} + \delta\mathbf{R}_i) & \sum_{i=1}^n \prod_{j=2}^i (\mathbf{I} + \delta\mathbf{R}_{j-1}) (\mathbf{p}_i^N + d\mathbf{p}_i) \\ \mathbf{0}^T & 1 \end{bmatrix} \quad (10)$$

where all second and higher order products of small quantities $\delta\mathbf{R}_k$ ($k=1, \dots, n$) and $d\mathbf{p}_l$ ($l=1, \dots, n$) have been neglected.

$$T_{1-n}^{approx} = \begin{bmatrix} \mathbf{I} + \sum_{i=1}^n \delta\mathbf{R}_i & \sum_{i=1}^n (\mathbf{p}_i^N + d\mathbf{p}_i) + \sum_{i=2}^n \left(\sum_{j=2}^i \delta\mathbf{R}_{j-1} \right) \mathbf{p}_i^N \\ \mathbf{0}^T & 1 \end{bmatrix} \quad (11)$$

Then

$$T_{1-n}^{approx} = \begin{bmatrix} 1 & -\sum_{i=1}^n d\theta_{Zi} & \sum_{i=1}^n d\theta_{Yi} & \sum_{i=1}^n dX_i + \sum_{i=1}^{n-1} d\theta_{Yi} \sum_{j=i+1}^n Z_j \\ \sum_{i=1}^n d\theta_{Zi} & 1 & -\sum_{i=1}^n d\theta_{Xi} & \sum_{i=1}^n dY_i - \sum_{i=1}^{n-1} d\theta_{Xi} \sum_{j=i+1}^n Z_j \\ -\sum_{i=1}^n d\theta_{Yi} & \sum_{i=1}^n d\theta_{Xi} & 1 & \sum_{i=1}^n (dZ_i + Z_i) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (12)$$

Comparing Eqs.(2) and (12), the variation propagation in x-, y- and z-direction is

$$dpx(n) = \sum_{i=1}^n dX_i + \sum_{i=1}^{n-1} d\theta_{Yi} \sum_{j=i+1}^n Z_j \quad (13)$$

$$dpy(n) = \sum_{i=1}^n dY_i - \sum_{i=1}^{n-1} d\theta_{Xi} \sum_{j=i+1}^n Z_j \quad (14)$$

$$dpz(n) = \sum_{i=1}^n Z_i \quad (15)$$

The table-axis error is:

$$e_n^{Direct} = \sqrt{\left(\sum_{i=1}^n dX_i + \sum_{i=1}^{n-1} d\theta_{Yi} \sum_{j=i+1}^n Z_j \right)^2 + \left(\sum_{i=1}^n dY_i - \sum_{i=1}^{n-1} d\theta_{Xi} \sum_{j=i+1}^n Z_j \right)^2} \quad (16)$$

2.2.3. Best Build Assembly

In the best build assembly, the build eccentricity is minimised by rotating each component

about its axis of symmetry and selecting the assembly configuration that provides the minimum assembly eccentricity. The minimum eccentricity for a one-component assembly is given by:

$$\varepsilon_1^{\min} = \sqrt{dX_1^2 + dY_1^2}. \quad (17)$$

For two components assembly in the best build, the second component must be rotated until its centricity is in the opposite direction to the equivalent centricity of the first component at the top of the second, as shown in (Fig. 2).

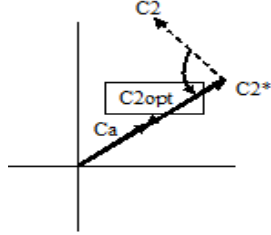


Fig. 2. Best Build Assembly with two components

In Fig. 2, C2* is centricity at rear plane of assembly considering the first component; C2 is original centricity of the second component; C2opt is the result of C2 rotated to the opposite direction of C2*; and Ca is the centricity of assembly.

The minimum eccentricity is given by:

$$\varepsilon_2^{\min} = \sqrt{(dX_1 + d\theta_{Y1}Z_2)^2 + (dY_1 - d\theta_{X1}Z_2)^2} - \sqrt{dX_2^2 + dY_2^2} \quad (18)$$

For three components assembly, to minimize the eccentricity, in theoretically, the three components are considered simultaneously until the resultant centricity of the three components achieves the minimum value. Thus, the modelling for the three components assembly in the paper follows these two steps: i) minimizing the resultant centricity of the first component and the second component at the top of the third component; and ii) minimizing the resultant eccentricity of the third component and the minimized resultant centricity from the first two components. Therefore, in step i), the second component must be rotated until the centricity of the second at the top of the third component is in the opposite direction to the equivalent centricity of the first component at the top of the third, and in step ii), the third component is rotated until its eccentricity is opposite direction to the resultant eccentricity of the first and the second components. Therefore, the minimum eccentricity is given by:

$$\varepsilon_3^{\min} = \sqrt{\frac{[dX_1 + d\theta_{Y1}(Z_2 + Z_3)]^2 + [dY_1 - d\theta_{X1}(Z_2 + Z_3)]^2}{[dX_2 + d\theta_{Y2}Z_3]^2 + [dY_2 - d\theta_{X2}Z_3]^2}} - \sqrt{dX_3^2 + dY_3^2} \quad (19)$$

In a similar way, the table-axis error at Stage n can be described as:

$$\varepsilon_n^{\min} = \sqrt{\frac{[\sqrt{dX_1^2 + dY_1^2} + d\theta_{Y1}Z_1]^2 + [\sqrt{dX_2^2 + dY_2^2} + d\theta_{Y2}Z_2]^2 + \dots + [\sqrt{dX_{n-1}^2 + dY_{n-1}^2} + d\theta_{Y_{n-1}}Z_{n-1}]^2}{[\sqrt{dX_n^2 + dY_n^2} + d\theta_{Yn}Z_n]^2}} \quad (20)$$

3. Probabilistic approach for straight-build assembly

In this section the dimensional variations dX_i , dY_i , $d\theta_{Xi}$, and $d\theta_{Yi}$ for each component are modelled as being random variables and pdfs are developed for the build eccentricities arising from Direct Build, and Best Build.

3.1. Direct Build Assembly

Since the component variations are Gaussian distributed (3D translation and rotation), From Eqs. (10) and (11), the variation propagation in x-, and y-direction is Gaussian distributed and can be described as follows.

$$dp_x(n) \sim N(0, \sum_{i=1}^n \sigma_{Xi}^2 + \sum_{i=1}^{n-1} \sigma_{\theta Yi}^2 (\sum_{j=i+1}^n Z_j)^2) = N(0, \sigma_n^2) \quad (21)$$

$$dp_y(n) \sim N(0, \sum_{i=1}^n \sigma_{Yi}^2 + \sum_{i=1}^{n-1} \sigma_{\theta Xi}^2 (\sum_{j=i+1}^n Z_j)^2) = N(0, \sigma_n^2) \quad (22)$$

where assuming $\sigma_{Xi} = \sigma_{Yi}$, $\sigma_{\theta Xi} = \sigma_{\theta Yi} = \sigma_{\theta}$, and

$$\sigma_n^2 = \sum_{i=1}^n \sigma_{Xi}^2 + \sum_{i=1}^{n-1} \sigma_{\theta}^2 (\sum_{j=i+1}^n Z_j)^2 \quad (23)$$

Therefore the table-axis error is Rayleigh distributed. Using Eq. (16) and a change of variables it can be shown easily that the pdf for the build eccentricity $\varepsilon_n^{\text{Direct}}$ for an assembly consisting of n axis-symmetric components using Direct Build is given by:

$$p(\varepsilon_n^{\text{Direct}}) = \begin{cases} \frac{\varepsilon_n^{\text{Direct}}}{\sigma_n^2} \exp(-\frac{(\varepsilon_n^{\text{Direct}})^2}{2\sigma_n^2}), & \varepsilon_n^{\text{Direct}} \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (24)$$

3.2. Best Build Assembly

For a one component assembly, the minimum eccentricity $\varepsilon_1^{\min}$ is given by Eq. (17). Recalling that dX_1 and dY_1 are zero mean Gaussian with standard deviation σ_{X1} or σ_{Y1} , it can be shown that using a change of variable the pdf for $\varepsilon_1^{\min}$ is given by:

$$p(\varepsilon_1^{\min}) = \begin{cases} \frac{\varepsilon_1^{\min}}{\sigma_{X1}^2} \exp(-\frac{(\varepsilon_1^{\min})^2}{2\sigma_{X1}^2}), & \varepsilon_1^{\min} \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (25)$$

This pdf is a Rayleigh distribution

4. RESULTS AND DISCUSSION

Numerical examples

The performance of the proposed method is determined by its accuracy, consistency and efficiency. To investigate the accuracy, consistency and efficiency of the proposed methods, a three-component assembly considering the components to be nominally identical is analysed. Each component is axi-symmetric (cylindrical) with nominal height $H = 70\text{mm}$ and nominal diameter $W = 100\text{mm}$ - so the nominal mating feature on the upper surface has coordinates $[0, 0, 70]$ relative to the mating feature on the lower surface, i.e. $X_i = 0\text{mm}$, $Y_i = 0\text{mm}$, $Z_i = 70\text{mm}$. Dimensional tolerances for each component are specified for the location (dX_i , dY_i and dZ_i) and orientation ($d\theta_{Xi}$, $d\theta_{Yi}$ and $d\theta_{Zi}$) of the mating feature on its upper surface relative to its lower surface, and the tolerances are assumed to have a zero-mean Gaussian distribution, where the standard deviation (σ) is taken to be one third of the specified tolerance. The influence of different component tolerances on the eccentricity of the build is assessed and the tolerance values chosen for the component heights h_i ($= dX_i = dY_i = dZ_i$) are: 0.1mm, 0.5mm and 2mm.

Numerical results obtained using the proposed probabilistic methods are compared with the standard Monte Carlo simulation method, which is based on using the exact transformation matrices described in Eq. (7). Convergence studies have been performed to determine the number of simulations and orientations required by Monte Carlo simulation to obtain accurate results. For the example considered, it was found that 50000 simulations and 16 orientations were required to obtain predictions for the mean, standard deviation, skewness and kurtosis that were accurate to within 2%.

To confirm the accuracy of the proposed approach, the pdfs have been plotted on both linear and log axes in (Figs. 3-5). Probability density functions (pdf's) for build eccentricity arising for three component assemblies with different tolerances are shown in (Figs. 3, 4 and 5).

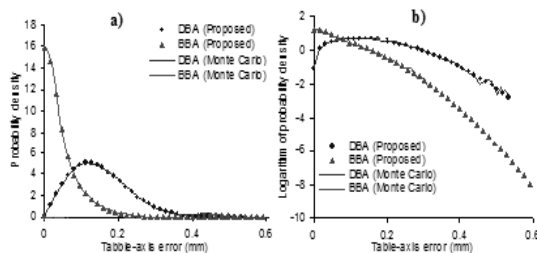


Fig. 3. Probability density function for three-component-build eccentricity with tolerance 0.1mm: a) linear scale, b) semi-log scale.

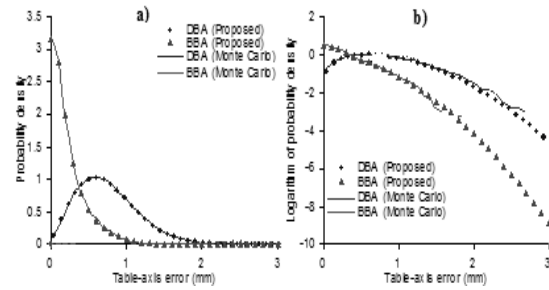


Fig. 4. Probability density function for three-component-build eccentricity with tolerance 0.5mm: a) linear scale, b) semi-log scale.

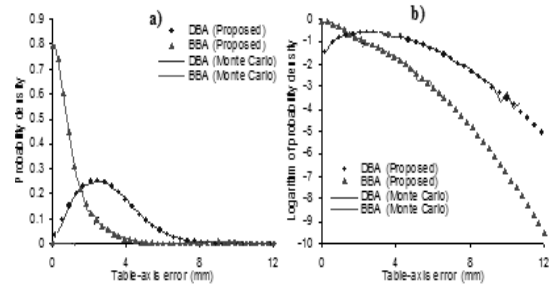


Fig. 5. Probability density function for three-component-build eccentricity with tolerance 2.0mm: a) linear scale, b) semi-log scale.

From these figures it is clear that all statistical distributions are highly non-Gaussian, and the results obtained using the proposed probabilistic approach are in excellent agreement with Monte Carlo simulation. It should be noted that the agreement at the tails of the log-pdf is less accurate because Monte Carlo simulation becomes less accurate at high eccentricities. This deficiency of the Monte Carlo method serves to highlight the advantages offered by the derived pdf expressions, which are valid for all eccentricity values.

6.

CONCLUSIONS

Models describing the straight-build assembly of rigid axi-symmetric components have been presented. These models are based on Connectivity Models and use a linearization procedure to develop analytical expressions for the eccentricity of the assembly. The developed expressions were used to analyse two different build scenarios: i) Direct Build and ii) Best Build, for the assembly of three axi-symmetric components. Best Build Assembly is based on rotating each component and selecting the orientations that minimise the eccentricity of the final assembly. In contrast, Direct Build does not attempt to optimise the assembly. By re-ordering the terms appearing in the derived linearised expressions for eccentricity, simplified expressions for the build eccentricity arising from both scenarios were developed. These expressions

were used in conjunction with the assumption that the dimensions of each component are independent Gaussian random variables and were used to develop analytical expressions for the probability density functions for the build eccentricity arising from the two different build scenarios. Numerical examples were used to validate the accuracy of the probabilistic approach in comparison to Monte Carlo simulation and assess its efficiency. Numerical results were obtained for a three-component assembly.

REFERENCES:

- Arron, D. D., and E. W. Charles, (1975) "Machine design theory and practices", MacMillan, New York.
- Brooks, S.P. and A. Gelman, (1998) "Alternative methods for monitoring convergence of iterative simulations" *Journal of Computational and Graphical Statistics*, Vol. (7): 434-455.
- Cai, M., J., and Z. T. Wu, (2004) "Mathematical model of cylindrical form tolerance", *Journal of Zhejiang University Science*, Vol. (5): 890-895.
- Dantan, J.Y. and A.J. Qureshi, (2009) "Worst-case and statistical tolerance analysis based on quantified constraint satisfaction problems and Monte Carlo simulation", *Computer-Aided Design*, Vol. (41): 1-12.
- He, J. R., (1991) "Estimating the distributions of manufactured dimensions with the beta probability density function", *International Journal of Machine Tools and Manufacture*, Vol. (31): 383-396.
- Jungemann, C., S. Yamaguchi, and H. Goto, (1997) "Convergence estimation for stationary ensemble Monte Carlo simulations", *International Conference on Simulation of Semiconductor Processes and Devices*, Cambridge, MA, USA.
- Lin, C.Y., W. H. Huang, M. C. Jeng, and J. L. Doong, (1997) "Study of an assembly tolerance allocation model based on Monte Carlo simulation" *Journal of Materials Processing Technology*, Vol. (70): 9-16.
- Mantripragada, R. (1999) "Modelling and controlling variation propagation in mechanical assemblies using state transition models", *IEEE Transactions on Robotics and Automation*, Vol. (15): 124-140.
- Mustafa, Y.A., (2007) "A convergence criterion for the Monte Carlo estimates", *Simulation Modelling Practice and Theory*, Vol. (15): 237-246.
- Nigam, S. D. and J. U. Turner, (1995) "Review of statistical approaches to tolerance analysis", *Computer-Aided Design*, Vol. (27): 6-15.
- Roy, U. and B. Li, (1998) "Representation and interpretation of geometric tolerances for polyhedral objects-I. Form tolerances", *Computer-Aided Design*, Vol. (30): 151-161.
- Roy U. and B. Li, (1999) "Representation and interpretation of geometric tolerances for polyhedral objects-II. Size, orientation and position tolerances", *Computer-Aided Design*, Vol. (31): 273-285.
- Spotts, M. F. (1971) "Design of machine elements", Prentice-Hall, Englewood Cliffs, New Jersey.
- Thimm, G., G. Britton, and F. S. Cheong, (2001) "Controlling tolerance stacks for efficient manufacturing", *The International Journal of Advanced Manufacturing Technology*, Vol. (18): 44-48.
- Toft, N., G. T. Innocent, G. Gettinby, and S.W. Reid, (2007) "Assessing the convergence of Markov Chain Monte Carlo methods: An example from evaluation of diagnostic tests in absence of a gold standard", *Preventive Veterinary Medicine*, Vol. (79): 244-256.
- Wu, F., J. Y. Dantan, and P. Martin, (2009) "Improved algorithm for tolerance allocation based on Monte Carlo simulation and discrete optimization", *Computers and Industrial Engineering*, Vol. (56): 1402-1413.