



On Twin Primes Dedicated to the Memory of Professor Dr K. D. Soomro

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Abstract: Two numbers p and $p + 2$ are twin primes if they both are primes. Using Wilson's number theoretic result., it is proved that the sequence is infinite.

Keywords: twin primes if they both are primes

1. INTRODUCTION

The problem of obtaining a complete list of primes less than some number n has been an area where many mathematicians have flexed their knowledge but the problem has proved to be of great labour and it never completed. Several, not complete but, reliable lists of prime numbers has been published so far. Probably the most famous one is of D. N. Lehmer. Even though the number of prime numbers is infinite, mathematicians discovered that the number of prime numbers, $\pi(n)$, upto the number n behaves like

$$\frac{n}{\log_e n} \text{ for very large values of } n. \text{ Legendre (1752-1833) and Gauss (1777-1855) independently conjectured the relationship between } \pi(n) \text{ and } \frac{n}{\log_e n} \text{ as}$$

$$\lim_{n \rightarrow \infty} \frac{\pi(n) \log_e n}{n} = 1$$

However the above relationship was proved later in 19th century by Hadamard (1865-1963) and de la Valle-Poussen (1866-1962) and this relation is known as the Prime Number Theorem.

An examination of the table of Sieve of Eratosthenes suggests that many consecutive primes differ by 2; that is if only one composite number exists between them. So, two consecutive odd integers, both of which are prime, are called twin Primes. (3, 5), (5, 7), (11, 13), (17, 19), ((29, 31), (41, 43) are some examples of the twin prim numbers. It is still open question whether or not there are infinitely many pairs of twin prime numbers.

In (1979), Robert Baillie computed all the prime numbers of the form $k2^n + 1$ for odd k where $0 \leq k < 150$, $0 \leq n \leq 1500$. In (1999), Karl-HeinzIndlekofer and AntalJarai found the numbers $242206083 * 2^{38880} \pm 1$ to betwin prime.

Purpose of this note is to prove the infiniteness of the twin primes and dedicate this result to the great memories of Professor Dr Khuda Dino Soomro.

2. MAIN RESULTS

Theorem. The sequence of twin primes is infinite.

Proof. Let p and $p + 2$ be twin primes, then by Wilson's theorem.

$$(p-1)! \equiv -1 \pmod{p} \dots (i)$$

then we have

$$(p-1)! = m_1 p - 1, m_1 \in \mathbb{Z}$$

$$(p+1)! \equiv -1 \pmod{p+2}$$

or

$$(p+1) (p-1)! \equiv -1 \pmod{p+2}$$

then by equation i) above, we have

$$(p+1) (m_1 p - 1) \equiv -1 \pmod{p+2}$$

or

$$(p+1) (m_1 p - 1) = m_2 (p+2) - 1 \dots (ii)$$

where $m_1, m_2 \in \mathbb{Z}$ and we have from equation (iii).

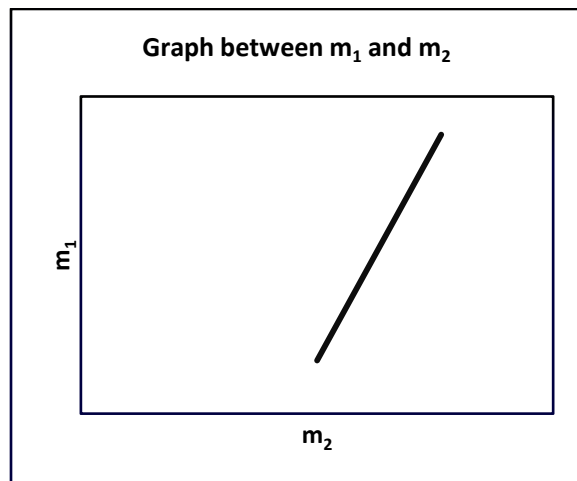
$$m_2 = \frac{(p+1)(m_1 p - 1) + 1}{p+2}$$

since $p \rightarrow \infty$ and also $m_1 \rightarrow \infty, m_2 \rightarrow \infty$

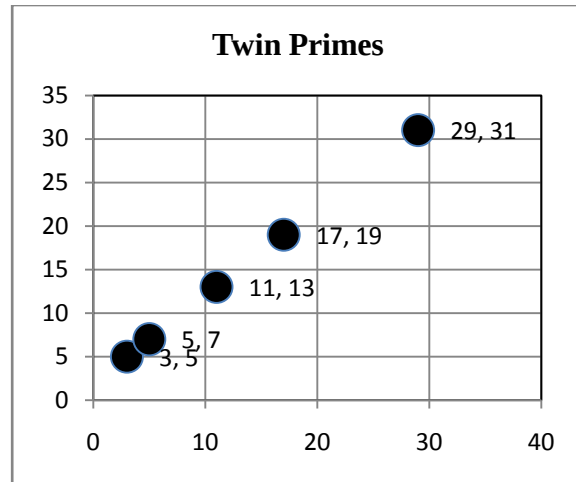
hence we have $p+2 \rightarrow \infty$ which proves the infiniteness of the twin primes **Table 1 and graph.**

Table 1. Proves the infiniteness

p	3	5	11	...	∞
$p+2$	5	7	13	...	∞
m_1	1	5	...	...	∞
m_2	5	10	...	...	∞



Graph 1



Graph 2

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